Activity Based Learning in Statistics. Why shouldn't statistics be fun?

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Outline

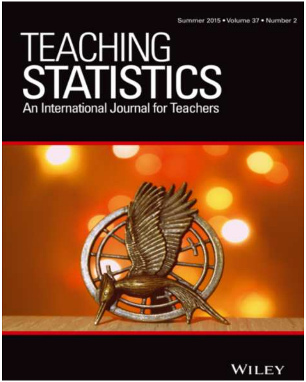
Introduction

- Activity Based Statistics - An intent to facilitate statistical understanding and intuition by using hands-on activities in the classroom.
- These activities generate interest, intuition and understanding of complex statistical concepts.
- Students like them!

Tintle, Nathan, et al. Introduction to Statistical Investigations: High School Binding. John Wiley, 2015.

Gnanadesikan, Mrudulla, et al. "An activity-based statistics course." Journal of Statistics Education 5.2 (1997).

Hunger Games



- Student senior research project (Erica Daniels)
- Analyzed the lottery in the Hunger Games
- Teaching Statistics article (March 2015)
- Won the 2015 Peter Holmes Prize
- Cause Webinar, June 2016
- Stochastik in der Schule (2016)

https://www.mcs.sdsmt.edu/kcaudle/downloads/HGames_1.0.tar.gz

Hunger Games

The Hunger Games is an annual event in the fictional country of Panem. Each year, 24 children (tributes) are chosen by lottery from 12 districts to fight to the death in the arena for the entertainment of the Capitol citizens.

Classroom exercise uses fictitious data from the Hunger Games lottery as an example of how to teach students the concept of a permutation goodness of fit test.

Book Details

- 2 children are chosen from each of the 12 districts.
- 7 (of the 24) children chosen in the lottery were careers (i.e. they were not selected by lottery).
- Katniss volunteered for her 12 year old sister Prim. Therefore, we use Prim's age and not Katniss'.
- A child's name is entered into the lottery once at 12 years of age. If they are not chosen, the next year they receive an additional entry for a total of two. This process continues until the age of 18, resulting in a total of 7 entries.
- A child's chance of selection in the lottery is based on two factors, their age and the number of tesserae they have claimed.

Lottery Proportions

Research Question: Based on these proportions, do we believe the "Game Makers" are following the lottery rules?

Table 1: Lottery Entries by Age

Age	Entries	Proportion by Age
12	1	1/28
13	2	2/28
14	3	3/28
15	4	4/28
16	5	5/28
17	6	6/28
18	7	7/28

The Tesserae

A child may receive an extra ration of food for his or her family. In doing so however, they receive one more entry in the lottery. We make the modest assumption that only the older children (Ages 16-18) take an extra ration. One for age 16, another at age 17, etc. Using this modest assumption we arrive at the following proportions.

Table 2: Lottery Entries by Age (with Tesserae)

Age	Entries	Proportion by Age
12	1	1/34
13	2	2/34
14	3	3/34
15	4	4/34
16	5	6/34
17	6	8/34
18	7	10/34

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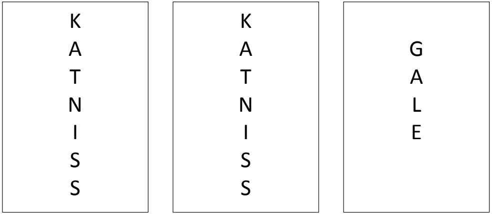
Randomization Tests

- We introduce a simple randomization test before returning to the more complicated lottery situation.
- Katniss, the heroine in the story, goes squirrel hunting with her friend Gale.
- Katniss kills four squirrels and Gale kills none.
- We would like to test the theory that Katniss is twice as likely to kill a squirrel as Gale.

Research Question: Is Katniss twice as likely to kill a squirrel as Gale?

Randomization Tests

To test the theory, take 3 index cards and write Katniss on two of them and Gale on the third card.

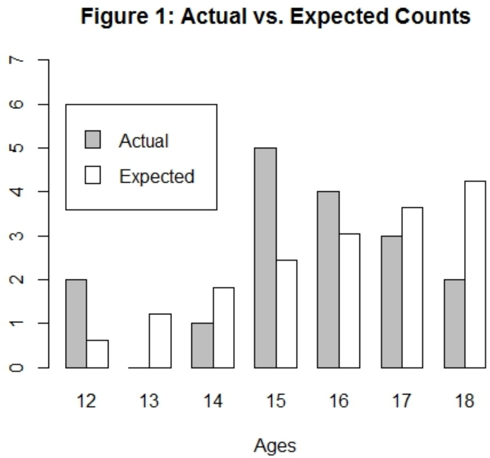


Randomization Tests

- You can perform this simulation by having a student pick a card to simulate the killing of a squirrel.
- To kill a second squirrel, have the student return the card to the stack so that it is in its original configuration and then redraw.
- There are $3^4 = 81$ possible outcomes.
- Of the 81 outcomes, 16 of those (2^4) represent a scenario where Katniss kills all 4 squirrels.
- Pr(4 kills by Katniss and none by Gale) equals 16/81 $\approx$ 20%
- Since this probability is fairly high, we cannot reject the assumption that Katniss kills twice as many squirrels as Gale.

Permutation Goodness of Fit Test

From the book, we get the actual ages of the 17 children who were selected by lottery. If we multiply the proportions from table 1 by 17 we get the expected counts.



Permutation Goodness of Fit Test

- Since it is not possible to look at all possible outcomes like it was with the squirrel hunting, we perform a randomization test by sampling from the expected distribution (previous slide).
- To simulate this, take 28 index cards and write the number 12 on one of the cards. Write the number 13 on two of two of them, etc. Repeat this until the numbers on the cards match the numbers in table 1.
- Next, have students draw a random sample of size 17 (with replacement) from this deck.
- Facilitate a discussion amongst the students regarding how many of each age they should have in the sample of size 17.

Permutation Goodness of Fit Test

- A standard technique for determining closeness is the chi-square goodness-of-fit test.
- A chi-squared goodness-of-fit test is not permissible here because all but one of the cell counts is less than five.
- We can still use the chi-squared test statistic as a measure of closeness.

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

O_i is the observed count in categories $i = 1, 2, .., k$

$E_i = np_i$ is the expected number in each category

Permutation Goodness of Fit Test

$$TS = \frac{(2-0.61)^2}{0.61} + \frac{(0-1.21)^2}{1.21} + \ldots + \frac{(2-4.25)^2}{4.25} = 9.11$$

Table 3: Actual vs. Expected Counts

Age	Actual	Expected
12	2	$17 \times 1/28 \approx 0.61$
13	0	$17 \times 2/28 \approx 1.21$
14	1	$17 \times 3/28 \approx 1.82$
15	5	$17 \times 4/28 \approx 2.43$
16	4	$17 \times 5/28 \approx 3.04$
17	3	$17 \times 6/28 \approx 3.64$
18	2	$17 \times 7/28 \approx 4.25$

Permutation Goodness of Fit Test

- Have several students perform the simulation with the index cards and ask them how their test statistic compares to the test statistic that was calculated using the book data (i.e. 9.11).
- Using an R package that I developed we can run a simulation. Package: Link Manual: Link
- Once installed, the function: rtesthg(m,d) will run a permutation goodness of fit test.
- The function performs a random sample of size 17 from the hunger games distribution and calculates the test statistic. It then repeats the process m times and determines the proportion of times the random sample test statistic exceeds 9.11.

d=1 $\Rightarrow$ no tesserae
d=2 $\Rightarrow$ use tessarae

Permutation Goodness of Fit Test

Some examples (No tessare):

rtesthg(10000,1)
0.1589

rtesthg(10000,1)
0.1537

rtesthg(10000,1)
0.156

We see that roughly 15% of the test statistics exceed 9.11.

Permutation Goodness of Fit Test

We recalculate the test statistic using expected values using the tessare distribution (table 2). Some examples (with tessare):

rtesthg(10000,2)
0.0528

rtesthg(10000,2)
0.0483

rtesthg(10000,2)
0.0543

We see that roughly 5% of the test statistics exceed 12.55.

Shuffling

- How many times should you riffle shuffle a deck of cards?
- Bayer and Diaconis (1992) show that an upper bound on the number of shuffles needed is 8.55.
- Useful cutoff is 7 shuffles

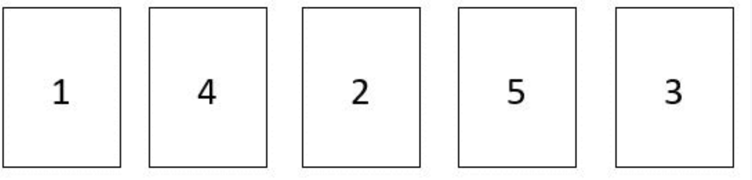
Caudle, K. A. (2018). You betcha it's random: riffle shuffling in cards gameswhen is enough, enough?. Teaching Statistics. (Thank you Robert Morse)

Riffle Shuffling

- A human (typically) doesn't typically perform a perfect riffle shuffle.
- Gilbert-Shannon-Reeds (GSR) model simulates a random riffle shuffle. (Gilbert/Shannon 1955, Reeds 1981)
- Simulate the cutpoint (K) with a binomial distribution: rbinom(1,n,0.5)
- Riffle shuffling preserves ordering.
- The GSR model assumes all arrangements are equally likely so the individual position of a card follows a discrete uniform probability distribution.
- Shuffling code can be found in the "shuffleCI" package posted on CRAN.

Determining Randomness of Deck

- Kendall's Tau distance $(\kappa) = \frac{\text{\# Dissimilar Pairs}}{\binom{n}{2}}$
- Rising sequences
- Example:

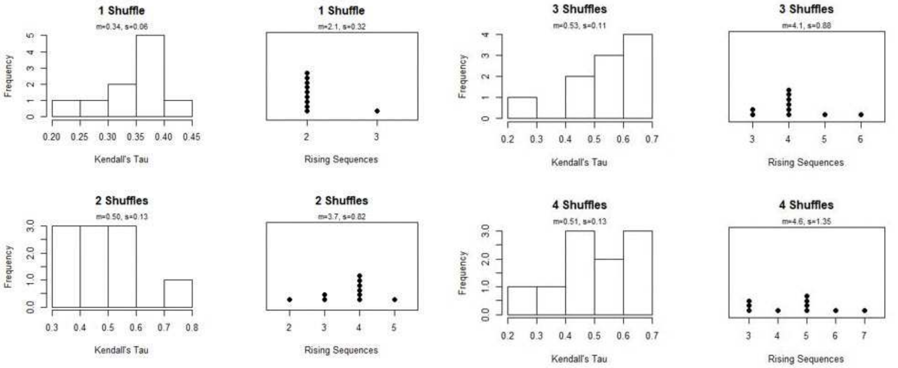


- Compare (1,4,2,5,3) to (1,2,3,4,5), $\kappa = \frac{3}{10}$
- 2 Rising sequences

Collecting the Data

- Start with 10 index cards numbered 1-10
- Have students place cards in numerical order
- Shuffle cards 1 time
- Determine κ and number of rising sequences
- Repeat with 2,3 and 4 shuffles
- Make class-wide frequency histograms and dot plots.

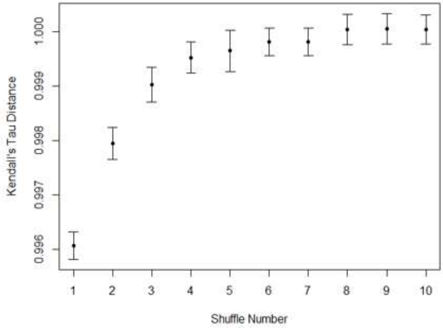
Example Board Arrangement

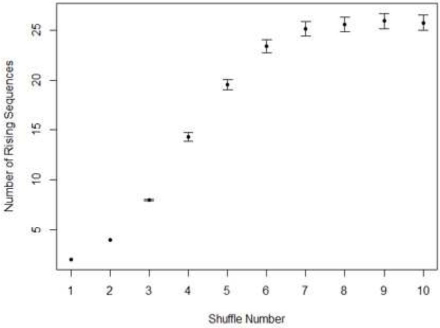


Normal Deck (52 cards)

- Using the shuffleCI package on CRAN you can perform the same simulation exercise with a standard deck of 52 playing cards.
- The sigtest() function in the shuffleCI package performs a comparison between Kendall's Tau and rising sequences with regards to answering the question how many times should one riffle shuffle a deck of cards.
- The output is 2 plots. The confidence intervals for each method and the p-values for 2 sample t-tests.

Confidence Interval Comparison





2-sample t-test

```
> sigtest(30)
     Intervals KTtest   RStest
[1,] "3-4"     "0.0208" "0"
[2,] "4-5"     "0.5907" "0"
[3,] "5-6"     "0.4629" "0"
[4,] "6-7"     "0.9928" "6e-04"
[5,] "7-8"     "0.229"  "0.3933"
[6,] "8-9"     "0.9386" "0.5299"
[7,] "9-10"    "0.9439" "0.7572"
```

Conclusions

- Use pop culture or relevant activities in class.
- Engage students by involving them in exercises.
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Caudle, Kyle, and Erica Daniels. "Did the Gamemakers Fix the Lottery in the Hunger Games?." Teaching Statistics 37.2 (2015): 37-40.

Caudle, Kyle A. "You betcha it's random: riffle shuffling in cards gameswhen is enough, enough?." Teaching Statistics (2018).