Introductory Statistics – Day 10

Hypothesis Testing and Errors

Activity 1: Put the following steps in order:

A Write the alternative hypothesis

B Create the sampling distribution (using simulations for now)

C Gather data

D Either reject or fail to reject the null hypothesis

E Write the null hypothesis

F Decide if your data is extreme

Activity 2: The average age of first marriage in the U.S. is 28. Does completing college affect this average age? You gather data from 100 randomly selected college graduates who are or have been married and you find the average age of first marriage to be 27.6 years. Which of these is a correct way to state the null hypothesis? (select all that are correct)

A $H_0 = 28$

B $H_0 : \mu = 27.6$

C $H_0 : \bar{x} = 27.6$

D $H_0 : \mu = 28$

E $H_0 : \mu < 28$

F H_0 : College graduates will have the same average first marriage age as non-college graduates.

Activity 2, continued:
The average age of first marriage in the U.S. is 28. Does completing college affect this average age? You gather data from 100 randomly selected college graduates who are or have been married and you find the average age of first marriage to be 27.6 years. Which of these is a correct way to state the alternative hypothesis? (select all that are correct)

A $H_A > 28$

B H_A : College graduates have a higher average first marriage age than non-graduates.

C H_A : College graduates have a different average first marriage age than non-graduates.

D $H_A : \bar{x} \neq 27.6$

E $H_A : \mu \neq 28$

F $H_A : \mu < 28$

Activity 3:
According to the National Center for Education Statistics, about 80% of students in the US change their major at least once. Do freshmen entering college with a health science major have a lower chance of changing their major? A random sample of 200 college students who had started as a health science major found that 70% changed their major at least once. Which of these is a correct way to state the null hypothesis? (select all that are correct)

A $H_0 = 80\%$

B $H_0 : \mu = 160$

C $H_0 : p = 0.8$

D $H_0 : p = 0.7$

E $H_0 : p = 140$

Activity 3, continued:
Which of these is a correct way to state the alternate hypothesis? (select all that are correct)

A $H_A : p \neq 140$

B $H_A : p \neq 0.70$

C $H_A : p < 0.80$

D $H_A : p \neq 0.80$

E H_A : Health science majors have a lower chance of changing their majors than students with other majors.

What if we are wrong?

There are two different ways to be wrong when we conclude a hypothesis test.

- First, we could reject the null hypothesis when the null hypothesis is actually true. This is called a Type I error, and is sometimes denoted α (alpha).

  When setting up a hypothesis test, the researcher generally chooses the α level when they decide how strong the evidence needs to be. This α is also called the significance level. While $\alpha = 0.05$ is common, α can be set at 0.01 or 0.10 or whatever value is acceptable in the specific field in which you are investigating.

- Second, we could fail to reject the null hypothesis, when the null hypothesis is actually not true. This is called a Type II error, and is sometimes denoted β (beta).

Test conclusion

	You have not rejected H_0	You have rejected H_0
Actually H_0 is true	good	Type I Error
Actually H_A is true	Type II Error	good

Example. A US court considers two possible claims about a defendant: she is either innocent or guilty. If we set these claims up in a hypothesis framework, which would be the null hypothesis and which the alternative?

- The null hypothesis is that the person is innocent. Only if we have sufficient evidence should she be declared guilty. If we have some evidence, but not enough (beyond a shadow of doubt), we might not think she's really innocent, but we do not declare her guilty. If the evidence isn't strong enough against her, she is considered "not guilty".
- Statisticians describe this situation with a double negative. We say "Don't reject the null hypothesis" (not guilty), but we do not say "Accept the null hypothesis" (innocent).

Activity 4. Questions about Errors

1. In the criminal justice system, what is a Type 1 error and what is a Type 2 error?

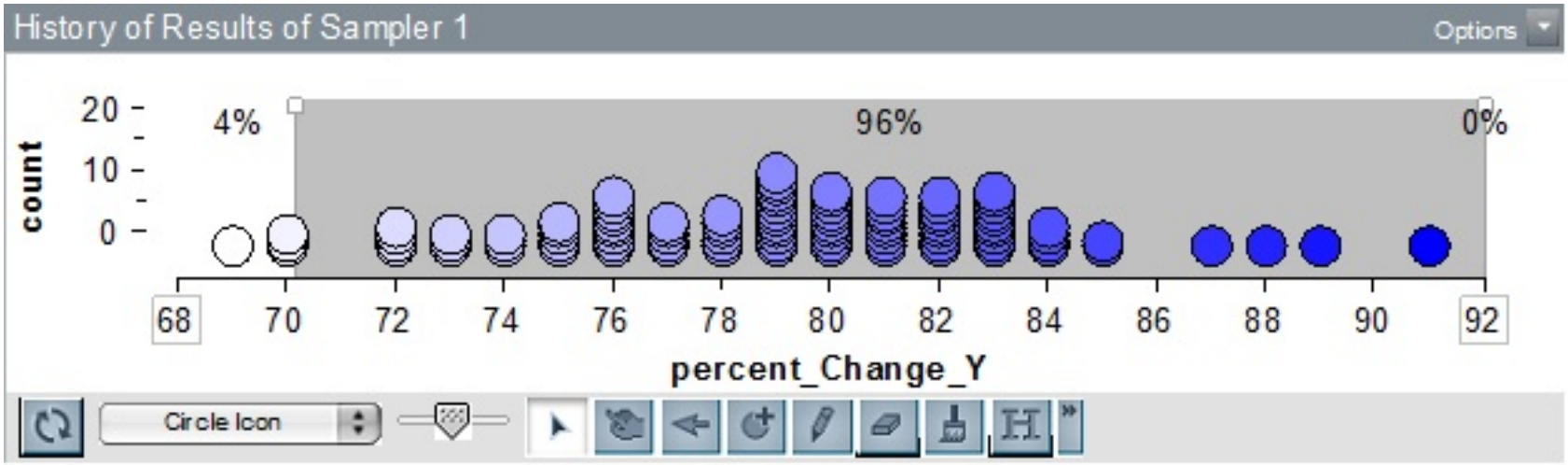
2. True or False: It is easier to reject the null hypothesis if the researcher uses a smaller alpha level. Defend your answer.

3. True or False: You are more likely to make a Type I error when using a small sample than when using a large sample. Defend your answer.

4. A gambler is trying to determine if a coin is weighted, but unknown to him the coin is actually fair. He flips the coin 100 times, find the percent heads, and determines that the probability of finding that many heads on a fair coin is 0.04. Using an alpha level of 0.05 does the gambler make an error? If so, what type?

If we reject the null hypothesis, what do we do next?

Example. According to the National Center for Education Statistics, about 80% of students in the US change their major at least once. Do freshmen entering college with a health science major have a lower chance of changing their major? A random sample of 100 college students who had started as a health science major found that 70% changed their major at least once.



The picture above provides the sampling distribution relevant for this hypothesis test (simulated in TinkerPlots). What should our conclusion be?

Example, continued. What should we do next?
If we reject someone else's claim, we should follow-up with a better claim of our own.
Our claim: a *confidence interval*

- indicates where we think the population parameter is.
  In this case, we would make a claim about the proportion of freshmen health science majors who will change their majors.

Our best guess is 70%, because that is what our sample data showed us. But it would be better to provide a range of values.

- Point estimate: 70% of health science majors will change majors.
- Interval estimate: Between _____ and _____ of health science majors will change majors.

  In order to fill in the blanks for the interval, we can return to our simulator and reset the spinner to 70%. After collecting many samples, we can find the interval that contains 95% of our data. This will approximate our 95% confidence interval for the true proportion of freshmen health science majors who will change their majors. This concept is the focus of the next section.