# Introductory Statistics - Day 20 

Comparing Two Proportions

Open the NCBabySmoke data set from Moodle (from OpenIntro).

- Previously, we've worked with single proportions.
- Today, we want to compare proportions from two subgroups of a population and determine if there is a statistically significant difference between those proportions.

Is a difference in smoking rates among mature and younger moms.

■ What should our null and alternative hypotheses be?

$$
\begin{gathered}
H_{0}: p_{\text {mature }}=p_{\text {younger }} \\
\text { or } \\
H_{0}: p_{\text {mature }}-p_{\text {younger }}=0 \\
H_{A}: p_{\text {mature }}-p_{\text {younger }} \neq 0
\end{gathered}
$$

Use a pivot table in Excel to organize your data.

- Find the proportion of mature moms and younger moms who smoke, as well as the point estimate (the difference between them).

$$
\hat{p}_{\text {mature }}=\frac{11}{133}=0.0827 \text { and } \hat{p}_{\text {younger }}=\frac{115}{867}=0.1326
$$

The difference in rates of smoking based on age is

$$
\hat{p}_{\text {mature }}-\hat{p}_{\text {younger }}=-0.0499
$$

- What do we still need in order to answer our research question?

$$
S E_{\text {pooled }}=\sqrt{p_{\text {pooled }}\left(1-p_{\text {pooled }}\right)\left(\frac{1}{n_{1}}+\frac{1}{n_{2}}\right)}
$$

where the pooled proportion is $p_{\text {pooled }}=\frac{x_{1}+x_{2}}{n_{1}+n_{2}}=\frac{p_{1} n_{1}+p_{2} n_{2}}{n_{1}+n_{2}}$
For this problem, $p_{\text {pooled }}=\frac{11+115}{133+867}=\frac{126}{999}=0.126$.
Therefore $S E_{\text {pooled }}=\sqrt{0.126(1-0.126)\left(\frac{1}{133}+\frac{1}{867}\right)}=0.0309$

- Let's find the p-value.
- Let's find the p-value.

$$
2 \times \operatorname{norm} \cdot \operatorname{dist}(-0.0499,0,0.0309,1)=2 \times 0.05317=0.1063
$$

- Conclusion?

With a p-value of 0.1063 , we do not have sufficient evidence to say the difference we found was significant. We cannot claim that there is a significant difference in the rates of smoking between younger and more mature new moms.

## Confidence Intervals with 2 proportions:

In order to follow-up with a confidence interval, we can use the same confidence interval strategy as before, but we will need a different formula for the standard error SE.
Confidence Interval:

$$
\begin{aligned}
& \text { point estimate } \pm z_{\text {critical }} \times S E \\
& \left(\hat{p_{1}}-\hat{p_{2}}\right) \pm z_{\text {critical }} \times S E
\end{aligned}
$$

Why is the standard error that we used in our hypothesis test not appropriate for a confidence interval?

$$
S E_{\text {unpooled }}=\sqrt{\frac{p_{1}\left(1-p_{1}\right)}{n_{1}}+\frac{p_{2}\left(1-p_{2}\right)}{n_{2}}}
$$

## Comparing proportions

Use the NCBabySmoke data set to answer each of the questions on the slides for today.

| column name | description and units |
| :--- | :--- |
| fage | father's age |
| mage | mother's age |
| mature | under 35 vs. 35 or older |
| weeks | length of pregnancy |
| premie | premie or full term |
| visits | number of doctor visits |
| marital | married or not married |
| gained | weight gained by mom (lbs) |
| weight | weight of baby (lbs) |
| lowbirthweight | low is $\leq 5.5$ lbs |
| gender | baby's gender |
| habit | smoking habit of mom |
| whitemom | white or not white |

Activity A. Is there a difference in smoking prevalence between new moms who are married and not married?

1 State the null and alternative hypotheses
2 Find and label the proportions ( $p_{\text {married }}$ and $p_{\text {notmarried }}$ ) and the sample size $n_{\text {married }}$ and $n_{\text {notmarried }}$ using a pivot table in Excel.

3 Find the difference in proportions, the standard error, and the p-value. Label each in your Excel sheet. Note: You will need to use the pooled proportion in your calculations for SE because your null hypothesis is that there is no difference between the two proportions.

4 State your conclusions in a complete sentence related to the context of the problem.

5 Are you surprised by the conclusions? Did you expect something different?

6 If you rejected the null hypothesis, what should you do to follow-up?

Activity B. Is there a difference in rates of low weight babies between smoking moms and non-smoking moms?

1 State the null and alternative hypotheses
2 Find and label the proportions ( $p_{\text {smoking }}$ and $p_{\text {nonsmoking }}$ ) and the sample size $n_{\text {smoking }}$ and $n_{\text {nonsmoking }}$.
3 Find the difference in proportions, the standard error, and the p-value. Label each in your Excel sheet.
4 State your conclusions in a complete sentence related to the context of the problem.
5 Are you surprised by the conclusions? Did you expect something different?

6 If you rejected the null hypothesis, what should you do to follow-up?

Activity C. Generate two more research questions that you could ask of this data. Choose one question which could be answered with a one proportion hypothesis test and choose a second question that requires a two proportion hypothesis test.

