

# MathQuest: Difference Equations

## Differences and Derivatives

1. The first difference,  $f(n + 1) - f(n)$ ,
  - (a) is exactly the same as the first derivative.
  - (b) is a discrete approximation of the first derivative.
  - (c) is not very helpful for learning about functions.
  - (d) None of the above
  
2. The first difference can
  - (a) pinpoint exactly where a function has a critical point.
  - (b) pinpoint exactly where a function changes concavity.
  - (c) approximate the location of a function's critical points.
  - (d) approximate the location of a function's inflection point.
  - (e) None of the above
  
3. The second difference can
  - (a) pinpoint exactly where a function has a critical point.
  - (b) pinpoint exactly where a function changes concavity.
  - (c) approximate the location of a function's critical points.
  - (d) approximate the location of a function's inflection point.
  - (e) None of the above
  
4. If we sample  $f(x) = 3x + b$  using step sizes of 1, what are the first differences?
  - (a) 0
  - (b) 1
  - (c) 1.5
  - (d) 3
  - (e) Cannot determine without knowing  $b$ .
  
5. If we sample  $f(x) = 3x + b$  using step sizes of 0.5, what are the first differences?

- (a) 0
- (b) 1
- (c) 1.5
- (d) 3
- (e) Cannot determine without knowing  $b$ .

6. If we sample  $f(x) = 3x + b$  using step sizes of 0.5, what are the second differences?

- (a) 0
- (b) 0.5
- (c) 1
- (d) 3
- (e) Cannot determine without knowing  $b$ .

7. If we sampled a second degree polynomial in step sizes of 1, what would the second differences be?

- (a) 0
- (b) 0.5
- (c) 1
- (d) 2
- (e) Cannot determine without knowing the polynomial.

8. If we sampled the function  $f(x) = x^2 + bx + c$  in step sizes of 0.5, what would the second differences be?

- (a) 0
- (b) 0.5
- (c) 1
- (d) 2
- (e) 4
- (f) Cannot determine without knowing the polynomial.

9. The phrase “ $y$  is proportional to  $x$ ” means

- (a)  $y = kx$
- (b) if  $x$  doubles, then  $y$  doubles.

- (c) a graph of  $y$  versus  $x$  would always go through the origin.
- (d) All of the above
- (e) None of the above
10. If the change in population is proportional to the population size, with proportionality constant  $k$ , we can say
- (a)  $a_{n+1} = ka_n$
- (b)  $\Delta a_n = ka_n$
- (c)  $a_n = k$
- (d) All of the above
- (e) None of the above
11. In the year 2000 the population of the US was 281 million, and our population grows by about ten percent every decade. Formulate a difference equation to model the population of the US.
- (a)  $\Delta a_n = 0.1a_n$
- (b)  $a_{n+1} = 1.1a_n$
- (c)  $\frac{\Delta a_n}{a_n} = 0.1$
- (d) All of the above
- (e) None of the above
12. A difference equation to model the population of frogs is  $\Delta f_n = 0.2f_n$ , where  $n$  is in years. What is a verbal description of this scenario?
- (a) The frog population is increasing at a rate of 20 percent per year.
- (b) The size of the frog population next year will be 120% of this year's population size.
- (c) The change in the frog population is proportional to the current population, with constant of proportionality equal to 0.2.
- (d) All of the above
- (e) None of the above