MathQuest: Difference Equations

Differences and Derivatives

1. The first difference, \( f(n + 1) - f(n) \),
   
   (a) is exactly the same as the first derivative.
   (b) is a discrete approximation of the first derivative.
   (c) is not very helpful for learning about functions.
   (d) None of the above

2. The first difference can
   
   (a) pinpoint exactly where a function has a critical point.
   (b) pinpoint exactly where a function changes concavity.
   (c) approximate the location of a function’s critical points.
   (d) approximate the location of a function’s inflection point.
   (e) None of the above

3. The second difference can
   
   (a) pinpoint exactly where a function has a critical point.
   (b) pinpoint exactly where a function changes concavity.
   (c) approximate the location of a function’s critical points.
   (d) approximate the location of a function’s inflection point.
   (e) None of the above

4. If we sample \( f(x) = 3x + b \) using step sizes of 1, what are the first differences?
   
   (a) 0
   (b) 1
   (c) 1.5
   (d) 3
   (e) Cannot determine without knowing \( b \).

5. If we sample \( f(x) = 3x + b \) using step sizes of 0.5, what are the first differences?
6. If we sample \( f(x) = 3x + b \) using step sizes of 0.5, what are the second differences?

(a) 0  
(b) 0.5  
(c) 1  
(d) 3  
(e) Cannot determine without knowing \( b \).

7. If we sampled a second degree polynomial in step sizes of 1, what would the second differences be?

(a) 0  
(b) 0.5  
(c) 1  
(d) 2  
(e) Cannot determine without knowing the polynomial.

8. If we sampled the function \( f(x) = x^2 + bx + c \) in step sizes of 0.5, what would the second differences be?

(a) 0  
(b) 0.5  
(c) 1  
(d) 2  
(e) 4  
(f) Cannot determine without knowing the polynomial.

9. The phrase “\( y \) is proportional to \( x \)” means

(a) \( y = kx \)  
(b) if \( x \) doubles, then \( y \) doubles.
(c) a graph of y versus x would always go through the origin.
(d) All of the above
(e) None of the above

10. If the change in population is proportional to the population size, with proportionality constant $k$, we can say
   
   (a) $a_{n+1} = ka_n$
   (b) $\Delta a_n = ka_n$
   (c) $a_n = k$
   (d) All of the above
   (e) None of the above

11. In the year 2000 the population of the US was 281 million, and our population grows by about ten percent every decade. Formulate a difference equation to model the population of the US.
   
   (a) $\Delta a_n = 0.1a_n$
   (b) $a_{n+1} = 1.1a_n$
   (c) $\frac{\Delta a_n}{a_n} = 0.1$
   (d) All of the above
   (e) None of the above

12. A difference equation to model the population of frogs is $\Delta f_n = 0.2f_n$, where $n$ is in years. What is a verbal description of this scenario?
   
   (a) The frog population is increasing at a rate of 20 percent per year.
   (b) The size of the frog population next year will be 120% of this year’s population size.
   (c) The change in the frog population is proportional to the current population, with constant of proportionality equal to 0.2.
   (d) All of the above
   (e) None of the above