MathQuest: Difference Equations

Discrete Dynamical System Models

1. You open a bank account with \$500. You add \$25 to your account each month, and the bank pays you 0.2% interest each month. Which discrete dynamical system describes your account balances?

   (a) $a_{n+1} = 0.2a_n + 25 + 500$ with $a_0 = 0$

   (b) $a_{n+1} = 0.002a_n + 25$ with $a_0 = 500$

   (c) $a_{n+1} = 1.002a_n + 500$ with $a_0 = 25$

   (d) $a_{n+1} = 1.002a_n + 25$ with $a_0 = 500$

   (e) None of the above

2. You've been spending too much money lately, and your credit card balance has risen to \$1200. You tear up the card and start paying \$40 per month to try to pay off the balance. Meanwhile, your credit card company charges you 1.5% interest on the balance each month. Which discrete dynamical system describes your account balances?

   (a) $a_{n+1} = 1.015a_n - 40$ with $a_0 = 1200$

   (b) $a_{n+1} = 1.015a_n - 40n + 1200$ with $a_0 = 0$

   (c) $a_{n+1} = 1.5a_n + 40$ with $a_0 = 1200$

   (d) $a_{n+1} = 0.015a_n - 40$ with $a_0 = 1200$

   (e) None of the above

3. You open a bank account that pays 3% annual interest, compounded monthly. Which difference equation describes your account balances, assuming that you do not make any further deposits?

   (a) $b_{n+1} = 3b_n$

   (b) $b_{n+1} = 1.3b_n$

   (c) $b_{n+1} = 1.03b_n$

   (d) $b_{n+1} = b_n + 0.03$

   (e) None of the above

4. The following difference equation describes the value of a car, where n is in years.

$$V_{n+1} = 0.86V_n$$

Which of the following is a true statement?

(a) The value of the car increases 86% each year.
(b) The value of the car decreases 86% each year.
(c) The value of the car decreases by 14% each year.
(d) The value of the car will eventually level out at \$8600.
(e) None of the above

5. The following difference equation describes the value of a car, where n is in years.

$$V_{n+1} = 0.86V_n$$

If $V_0 = \$20,000$, what is V_1?

(a) \$17,200
(b) \$19,914
(c) \$20,086
(d) Need more information.

6. The following difference equation describes the value of a car, where n is in years.

$$V_{n+1} = 0.86V_n$$

Which of the following is the correct interpretation of V_1?

(a) V_1 is the change in the value of the car after one year.
(b) V_1 is the value of the car after one year.
(c) $V_0 - V_1$ gives the value of the car after one year.
(d) None of the above

7. Which scenario below could be modeled with this difference equation?

$$a_{n+1} = 1.01a_n + 180$$

(a) A bank account earns 1% annual interest and receives deposits of \$180 per month.
(b) A population is increasing at a rate of 101% per year plus a yearly increase of 180.

(c) There are initially 180 cows on a ranch, and each year the population increases by 1%.

(d) The number of deer at a wildlife sanctuary naturally increases by 1% per year. In addition, 180 new deer are brought to the sanctuary each year.

(e) All of the above

(f) None of the above

8. If $a_n = 1.01a_n + 180$ with $a_3 = 100$, what is a_4?

(a) 101

(b) 281

(c) 282.8

(d) You can't figure out a_4 without knowing a_0.

(e) None of the above

9. A growing bookstore's inventory changes according to a regular pattern. Each week the bookstore sells 250 books, and each week it receives 265 new books. In addition, the price of books goes up about 2% each year. Customers prefer comic books to serious literature, buying 3 times as many comic books as classics. Which of the following difference equations describes how the bookstore's inventory changes from week to week?

(a) $a_{n+1} = a_n - 250 + 265$

(b) $a_{n+1} = 1.02a_n - 250 + 265 - 3a_n$

(c) $a_{n+1} = a_n + (0.02/52)a_n - 250 + 265$

(d) $a_{n+1} = 3a_n + 15 + 1.02$

(e) None of the above

10.

$$E_{n+1} = E_n + 0.3E_n\left(1 - \frac{E_n}{1000}\right)$$

This difference equation allows us to predict the growth of an elephant population in an African preserve, where n is in years.

Which of the following would be a true statement?

(a) The elephant population will grow at a rate of 30%.

(b) The preserve can support 1000 elephants.

(c) The elephant population will grow at a rate of 0.3%.

(d) The maximum number of elephants the preserve can support is 0.3 $E(n)$.

11.

$$E_{n+1} = E_n + 0.3E_n\left(1 - \frac{E_n}{1000}\right)$$

This difference equation allows us to predict the growth of an elephant population in an African preserve, where n is in years.

Which of the following would be a true statement?

(a) If $E_0 < 1000$ then the elephant population will shrink.

(b) If $E_0 > 1000$ then the elephant population will grow.

(c) If $E_0 = 1000$ then the elephant population will hold steady.

(d) None of the above

12.

$$S_{n+1} = S_n + k \times S_n\left(1 - \frac{S_n}{P}\right)$$

This difference equation allows us to predict how a disease will spread through a town of population P, where S_n is the number of people who are sick after n weeks.

A larger value of k means that...

(a) the town has a larger population.

(b) more people start out being sick.

(c) the disease will spread more quickly.

(d) the disease is more deadly.

13.

$$S_{n+1} = S_n + k \times S_n\left(1 - \frac{S_n}{P}\right)$$

This difference equation allows us to predict how a disease will spread through a town of population P, where S_n is the number of people who are sick after n weeks.

Suppose we calculate that $S_{10} > P$. This means that...

(a) people are getting sick more quickly.

(b) the town population has grown.

(c) no more people will get sick.

(d) our model has failed.

14.

$$B_{n+1} = B_n + 0.3B_n\left(1 - \frac{B_n}{100}\right)$$

This equation allows us to predict the number of Carroll faculty who have seen the movie "Harry Potter."

Suppose that $B_{11} = B_{10}$. This means that...

(a) $B_{10} = 0.3$

(b) $B_{10} < 0$

(c) $B_{10} > 100$

(d) $B_{10} = 100$

15.

$$B_{n+1} = B_n + 0.3B_n\left(1 - \frac{B_n}{100}\right)$$

This equation allows us to predict the number of Carroll faculty who have seen the movie "Harry Potter."

Suppose that $B_6 = 0$. This means that...

(a) $B_7 = 100$

(b) $B_7 = 0.3$

(c) $B_7 = 0$

(d) This is not possible.

16.

$$T_{n+1} = T_n + 0.005T_n\left(15,000 - T_n\right)$$

Suppose this equation allows us to predict the spread of a new computer technology, where T is the number of companies that have this technology during year n.

Suppose that $T_{10} = 5,000$. How many more companies will get the technology during the next year?

(a) $0.005 \times 5,000$

(b) $0.005 \times 10,000$

(c) $0.005 \times 5000 \times 10,000$

(d) $0.005 \times 5000 \times 15,000$

17.

$$T_{n+1} \quad = \quad T_n + 0.005T_n\,(15,000 - T_n)$$

Suppose this equation allows us to predict the spread of a new computer technology, where T is the number of companies that have this technology during year n.

Suppose that $T_5 = 10,000$. If the total number of companies who will eventually get the technology was increased from 15,000 to 20,000 this would...

(a) speed up the spread of the technology.

(b) slow down the spread of the technology.

(c) not change how the technology spreads.

(d) stop the spread of the technology.