

MathQuest: Difference Equations

Equilibrium and Long-Term Behavior

1. Suppose we have $a_{n+1} = 3a_n$ with $a_0 = 0$. What is a_{400} ?
 - (a) 0
 - (b) 3^{400}
 - (c) 1200
 - (d) It would take way too long to figure this out.

2. Suppose we have $a_{n+1} = 3a_n$ with $a_0 = 1$. What is a_3 ?
 - (a) 0
 - (b) 1
 - (c) 9
 - (d) 27
 - (e) None of the above

3. Suppose we have $a_{n+1} = 0.9a_n$. What is the equilibrium value?
 - (a) 0
 - (b) 0.9
 - (c) There is no equilibrium value.
 - (d) The equilibrium value cannot be determined without knowing the initial value.
 - (e) None of the above

4. Suppose we have $a_{n+1} = 0.9a_n$ with $a_0 = 1$. What is a_3 ?
 - (a) 0
 - (b) 1
 - (c) 0.9
 - (d) 0.729
 - (e) None of the above

5. Suppose we have $f_{n+1} = 3f_n - 10$ with $f_0 = 5$. What is f_3 ?
- (a) 0
 - (b) 5
 - (c) 15
 - (d) 125
6. Suppose we have $f_{n+1} = 3f_n - 10$ with $f_0 = 4$. What is f_3 ?
- (a) 0
 - (b) 2
 - (c) 4
 - (d) 12
 - (e) None of the above
7. Suppose $d_{n+1} = 0.9d_n + 2$. What is the equilibrium value?
- (a) 9
 - (b) 2
 - (c) 20
 - (d) 20/9
 - (e) None of the above
8. Suppose $g_{n+1} = -2g_n + 3$. Which statement describes the long-term behavior of the solution with $g_0 = 0$.
- (a) The solution stays at 0.
 - (b) The solution grows without bound.
 - (c) The solution grows and approaches the equilibrium value.
 - (d) The solution oscillates farther and farther from the equilibrium value.
 - (e) None of the above
9. **True or False** An equilibrium value can never be negative.

10. The following difference equation describes the population of a small town, where n is in years.

$$p_{n+1} = 0.9p_n$$

Which of the following is a true statement?

- (a) Eventually this town will die out.
 - (b) Eventually this town will have 1,000,000 people.
 - (c) This town's population will fluctuate, but the town will never grow substantially or die out.
 - (d) A big city modeled with this difference equation would have more people in the long run than this town will have.
11. The following difference equation describes the population of a small town, where n is in years.

$$p_{n+1} = 1.17p_n$$

Which of the following is a true statement?

- (a) Eventually this town will die out.
 - (b) Eventually this town will have 1,000,000 people.
 - (c) This town's population will fluctuate, but the town will never grow substantially or die out.
 - (d) This difference equation does not have an equilibrium value.
12. The difference equation $a_{n+1} = 1.08a_n$ might model the population of some species. If $a_0 = 5000$, which of the following statements is true?
- (a) $a_{10} > 5000$
 - (b) $a_{40} < 5000$
 - (c) It is possible that $a_{30} = 5000$
 - (d) More than one of these statements could be true.
 - (e) None of these statements has to be true.

13. The difference equation $a_{n+1} = 0.93a_n$ might model the population of some species. If $a_0 = 5000$, which of the following statements is true?
- (a) $a_{10} > 5000$
 - (b) $a_{40} < 5000$
 - (c) It is possible that $a_{30} = 5000$

- (d) More than one of these statements could be true.
 - (e) None of these statements has to be true.
14. When we are looking for an equilibrium value, why can we change both a_n and a_{n+1} to E and then solve the resulting equation?
- (a) a_n and a_{n+1} both represent amounts, so they're the same thing anyway.
 - (b) a_n and a_{n+1} are just symbols, so we can use a different symbol to represent them.
 - (c) At equilibrium, each term is the same as the one before.
 - (d) None of the above
15. A polluted lake has a percentage of its contaminants washed away each year, while factories on the lake dump in a constant amount of pollutants each year. The equilibrium value for this lake is 500 pounds. This means that
- (a) if there are initially 500 pounds of contaminants, there will always be 500 pounds of contaminants (assuming the conditions remain the same).
 - (b) when there are 500 pounds of contaminants in the lake, the amount of pollutants being washed out each year is exactly equal to the amount being dumped in.
 - (c) if there are currently 750 pounds of pollutants in the lake, next year there will be fewer pollutants in the lake.
 - (d) All of the above are correct.
 - (e) Some, but not all, of the above are correct.
16. Suppose the only equilibrium value of a difference equation is 10, and this is a stable equilibrium. **True or False** If $a_5 = 7$, then $7 < a_8 < 10$.
17. **True or False** A difference equation can only have one equilibrium value.
18. Consider the difference equation $a_{n+1} = ra_n$. If r is increased, what is the effect on the equilibrium value?
- (a) The equilibrium value increases.
 - (b) The equilibrium value decreases.
 - (c) The equilibrium value doesn't change.
 - (d) The effect depends on how much r is increased by and what it started at.

19. Consider the difference equation $a_{n+1} = ra_n + b$ where $r = 3$ and $b = -2$. If r is increased, what is the effect on the equilibrium value?
- (a) The equilibrium value increases.
 - (b) The equilibrium value decreases.
 - (c) The equilibrium value doesn't change.
 - (d) The effect depends on how much r is increased by.