MathQuest: Difference Equations

Solutions to Nonhomogeneous DEs with a Constant Term

1. **True or False** The function $a_n = r^n C$ solves the difference equation $a_{n+1} = ra_n + b$.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

2. The solution to $a_{n+1} = 4a_n + 3$ with $a_0 = 5$ is

   (a) $a_n = 4^n(2) + 3$
   (b) $a_n = 4^n(5) - 1$
   (c) $a_n = 4^n(5) + 3$
   (d) $a_n = 4^n(6) - 1$
   (e) None of the above

3. The solution to $a_{n+1} = 2a_n - 5$ with $a_3 = 9$ is

   (a) $a_n = 2^n(4) + 5$
   (b) $a_n = 2^n(0.5) + 5$
   (c) $a_n = 2^n(9) + 5$
   (d) $a_n = 2^n(1.75) - 5$
   (e) $a_n = 2^n(14) - 5$
   (f) None of the above

4. The solution to $a_{n+1} = 4a_n + 3$ with $a_0 = -1$ is

   (a) $a_n = -1$
   (b) $a_n = 4^n(-1) - 1$
   (c) $a_n = 4^n(-2) - 1$
   (d) $a_n = 4^n(2) - 1$

(e) None of the above

5. Which of the following is the solution to $a_{n+1} = a_n - 8$ with $a_0 = 5$?

   (a) $a_n = 5 - 8n$
   (b) $a_n = -8 + 5n$
   (c) $a_n = 1^n(5) - 8$
   (d) No solution can be found because there is no equilibrium value.

6. A solution to a difference equation is $a_n = 2^n(0.8) + 52$. What is the equilibrium value and is it stable or not?

   (a) $E = 0.8$ and it is stable
   (b) $E = 0.8$ and it is unstable
   (c) $E = 52$ and it is stable
   (d) $E = 52$ and it is unstable
   (e) Not enough information

7. Which of the following describes the long-term behavior of the solution $a_n = 2.7^n(4) - 3.8$?

   (a) It is unstable.
   (b) The solution will increase infinitely.
   (c) The solution will decrease infinitely.
   (d) The solution will converge to an equilibrium value of -3.8.

8. Discuss the long-term behavior of the solution $a_n = 0.7^n C + 2$ with $a_0 = 1$.

   (a) This solution will increase, converging to the equilibrium value of 2.
   (b) This solution will decrease forever.
   (c) This solution will increase forever.
   (d) None of the above

9. A difference equation was used to model monthly payments on a credit card with an outstanding balance. The solution to the difference equation is $a_n = 1.015^n(-1000) + 4000$.

   Which of the following is a true statement?

(a) The monthly payment is \$4000.

(b) The starting balance is \$1000.

(c) If the monthly payment is \$4000, the balance on the credit card will not change.

(d) All of the above

(e) None of the above

10. A difference equation was used to model monthly payments on a credit card with an outstanding balance. The solution to the difference equation is $a_n = 1.015^n(-1000) + 4000$.

    Which of the following is a true statement?

    (a) The starting balance is \$3000.

    (b) The interest rate is 1.5% per month.

    (c) With a starting balance of \$4000 and the same monthly payment, the balance on the credit card will not change.

    (d) All of the above

    (e) None of the above