MathQuest: Difference Equations

Solutions to Nonhomogeneous DEs with an Exponential Term

1. The difference equation $a_{n+1} = 1.04a_n + 1.05^n(1000)$ models the yearly balances in a savings account with annual deposits being made. Which of the following statements is true?

   (a) Each year the deposit increases by 4%.

   (b) Each year the deposit increases by 5%.

   (c) Each year the deposit increases by \$1000.

   (d) Each year the deposit increases $1.05 \times 1000 = \$1050$.

   (e) None of the above

2. A solution to a difference equation is $a_n = \frac{17}{2} \cdot 3^n + \frac{5}{2} \cdot 2^n$. What was the initial condition, a_0?

   (a) $\frac{17}{2}$

   (b) $\frac{5}{2}$

   (c) 2

   (d) 11

3. What is the best conjecture to use for the homogeneous solution to $a_{n+1} = 2a_n + 3 \cdot 5^n$?

   (a) $a_n = 2^n$

   (b) $a_n = 2^n C$

   (c) $a_n = 5^n$

   (d) $a_n = 5^n C$

   (e) None of the above

4. What is the best conjecture to use for the nonhomogeneous solution to $a_{n+1} = 2a_n + 3 \cdot 5^n$?

   (a) $a_n = 2^n C$

   (b) $a_n = 5^n C$

(c) $a_n = 3 \cdot 5^n$

(d) $a_n = 3 \cdot 5^n C$

(e) $a_n = 5^n C_1 + C_2$

(f) None of the above

5. We have $a_{n+1} = 3a_n - 5 \cdot 2^n$ and we have formed a conjecture of $a_n = 2^n C$ for the particular solution to the nonhomogeneous part. When we substitute our conjectured solution into the difference equation, what is the result?

   (a) $2^n C(n+1) = 2^n C - 5 \cdot 2^n$

   (b) $2^n C + 1 = 3 \cdot 2^n C - 5 \cdot 2^n$

   (c) $2^{n+1} C = 2^n C - 5 \cdot 2^n$

   (d) $2^{n+1} C = 3 \cdot 2^n C - 5 \cdot 2^n$

   (e) None of the above

6. We are trying to find a solution to $a_{n+1} = 9a_n + 5 \cdot 4^n$ where $a_0 = 1.5$. We have conjectured $a_n = 4^n C$ as the particular solution to the nonhomogeneous part and substituted our conjecture into the difference equation to obtain $4^{n+1} C = 9 \cdot 4^n C + 5 \cdot 4^n$. How do we proceed to find C?

   (a) Use the initial condition.

   (b) Divide both sides of the equation by C.

   (c) Divide both sides of the equation by 4.

   (d) Divide both sides of the equation by n.

   (e) Divide both sides of the equation by 4^n.

   (f) We don't have enough information to solve for C.

7. We are trying to find a solution to $a_{n+1} = 9a_n + 5 \cdot 4^n$ where $a_0 = 1.5$. We have conjectured $a_n = 4^n C$ as the particular solution to the nonhomogeneous part and substituted our conjecture into the difference equation to obtain $4^{n+1} C = 9 \cdot 4^n C + 5 \cdot 4^n$. What is the value of C?

   (a) $C = -1$

   (b) $C = 1$

   (c) $C = -5/8$

   (d) $C = 5/3$

   (e) None of the above

8. For which of the following difference equations will the nonhomogeneous conjecture need to be modified by multiplying by n?

   (a) $a_{n+1} = 2a_n + 2 \cdot 3^n$
   (b) $a_{n+1} = 3a_n + 2 \cdot 3^n$
   (c) $a_{n+1} = a_n + 4^n$
   (d) All of the above
   (e) None of the above

9. Which of the following is not a solution to $a_{n+1} = 3a_n + 5 \cdot 4^n$?

   (a) $a_n = 5 \cdot 4^n$
   (b) $a_n = 6 \cdot 3^n$
   (c) $a_n = 8 \cdot 3^n + 5 \cdot 4^n$
   (d) $a_n = 15 \cdot 3^n + 5 \cdot 4^n$
   (e) All are solutions