

MathQuest: Difference Equations

Solutions to Nonhomogeneous DEs with an Exponential Term

1. The difference equation $a_{n+1} = 1.04a_n + 1.05^n(1000)$ models the yearly balances in a savings account with annual deposits being made. Which of the following statements is true?
 - (a) Each year the deposit increases by 4%.
 - (b) Each year the deposit increases by 5%.
 - (c) Each year the deposit increases by \$1000.
 - (d) Each year the deposit increases $1.05 \times 1000 = \$1050$.
 - (e) None of the above

2. A solution to a difference equation is $a_n = \frac{17}{2} \cdot 3^n + \frac{5}{2} \cdot 2^n$. What was the initial condition, a_0 ?
 - (a) $\frac{17}{2}$
 - (b) $\frac{5}{2}$
 - (c) 2
 - (d) 11

3. What is the best conjecture to use for the homogeneous solution to $a_{n+1} = 2a_n + 3 \cdot 5^n$?
 - (a) $a_n = 2^n$
 - (b) $a_n = 2^n C$
 - (c) $a_n = 5^n$
 - (d) $a_n = 5^n C$
 - (e) None of the above

4. What is the best conjecture to use for the nonhomogeneous solution to $a_{n+1} = 2a_n + 3 \cdot 5^n$?
 - (a) $a_n = 2^n C$
 - (b) $a_n = 5^n C$

- (c) $a_n = 3 \cdot 5^n$
- (d) $a_n = 3 \cdot 5^n C$
- (e) $a_n = 5^n C_1 + C_2$
- (f) None of the above
5. We have $a_{n+1} = 3a_n - 5 \cdot 2^n$ and we have formed a conjecture of $a_n = 2^n C$ for the particular solution to the nonhomogeneous part. When we substitute our conjectured solution into the difference equation, what is the result?
- (a) $2^n C(n+1) = 2^n C - 5 \cdot 2^n$
- (b) $2^n C + 1 = 3 \cdot 2^n C - 5 \cdot 2^n$
- (c) $2^{n+1} C = 2^n C - 5 \cdot 2^n$
- (d) $2^{n+1} C = 3 \cdot 2^n C - 5 \cdot 2^n$
- (e) None of the above
6. We are trying to find a solution to $a_{n+1} = 9a_n + 5 \cdot 4^n$ where $a_0 = 1.5$. We have conjectured $a_n = 4^n C$ as the particular solution to the nonhomogeneous part and substituted our conjecture into the difference equation to obtain $4^{n+1} C = 9 \cdot 4^n C + 5 \cdot 4^n$. How do we proceed to find C ?
- (a) Use the initial condition.
- (b) Divide both sides of the equation by C .
- (c) Divide both sides of the equation by 4.
- (d) Divide both sides of the equation by n .
- (e) Divide both sides of the equation by 4^n .
- (f) We don't have enough information to solve for C .
7. We are trying to find a solution to $a_{n+1} = 9a_n + 5 \cdot 4^n$ where $a_0 = 1.5$. We have conjectured $a_n = 4^n C$ as the particular solution to the nonhomogeneous part and substituted our conjecture into the difference equation to obtain $4^{n+1} C = 9 \cdot 4^n C + 5 \cdot 4^n$. What is the value of C ?
- (a) $C = -1$
- (b) $C = 1$
- (c) $C = -5/8$
- (d) $C = 5/3$
- (e) None of the above

8. For which of the following difference equations will the nonhomogeneous conjecture need to be modified by multiplying by n ?

(a) $a_{n+1} = 2a_n + 2 \cdot 3^n$

(b) $a_{n+1} = 3a_n + 2 \cdot 3^n$

(c) $a_{n+1} = a_n + 4^n$

(d) All of the above

(e) None of the above

9. Which of the following is not a solution to $a_{n+1} = 3a_n + 5 \cdot 4^n$?

(a) $a_n = 5 \cdot 4^n$

(b) $a_n = 6 \cdot 3^n$

(c) $a_n = 8 \cdot 3^n + 5 \cdot 4^n$

(d) $a_n = 15 \cdot 3^n + 5 \cdot 4^n$

(e) All are solutions