MathQuest: Difference Equations

Solutions to Nonhomogeneous DEs with an Exponential Term

1. The difference equation \( a_{n+1} = 1.04a_n + 1.05^n(1000) \) models the yearly balances in a savings account with annual deposits being made. Which of the following statements is true?

   (a) Each year the deposit increases by 4%.
   (b) Each year the deposit increases by 5%.
   (c) Each year the deposit increases by $1000.
   (d) Each year the deposit increases \( 1.05 \times 1000 = 1050 \).
   (e) None of the above

2. A solution to a difference equation is \( a_n = \frac{17}{2} \cdot 3^n + \frac{5}{2} \cdot 2^n \). What was the initial condition, \( a_0 \)?

   (a) \( \frac{17}{2} \)
   (b) \( \frac{5}{2} \)
   (c) 2
   (d) 11

3. What is the best conjecture to use for the homogeneous solution to \( a_{n+1} = 2a_n + 3 \cdot 5^n \)?

   (a) \( a_n = 2^n \)
   (b) \( a_n = 2^nC \)
   (c) \( a_n = 5^n \)
   (d) \( a_n = 5^nC \)
   (e) None of the above

4. What is the best conjecture to use for the nonhomogeneous solution to \( a_{n+1} = 2a_n + 3 \cdot 5^n \)?

   (a) \( a_n = 2^nC \)
   (b) \( a_n = 5^nC \)
(c) \( a_n = 3 \cdot 5^n \)
(d) \( a_n = 3 \cdot 5^n C \)
(e) \( a_n = 5^n C_1 + C_2 \)
(f) None of the above

5. We have \( a_{n+1} = 3a_n - 5 \cdot 2^n \) and we have formed a conjecture of \( a_n = 2^n C \) for the particular solution to the nonhomogeneous part. When we substitute our conjectured solution into the difference equation, what is the result?

(a) \( 2^n C (n + 1) = 2^n C - 5 \cdot 2^n \)
(b) \( 2^n C + 1 = 3 \cdot 2^n C - 5 \cdot 2^n \)
(c) \( 2^{n+1} C = 2^n C - 5 \cdot 2^n \)
(d) \( 2^{n+1} C = 3 \cdot 2^n C - 5 \cdot 2^n \)
(e) None of the above

6. We are trying to find a solution to \( a_{n+1} = 9a_n + 5 \cdot 4^n \) where \( a_0 = 1.5 \). We have conjectured \( a_n = 4^n C \) as the particular solution to the nonhomogeneous part and substituted our conjecture into the difference equation to obtain \( 4^{n+1} C = 9 \cdot 4^n C + 5 \cdot 4^n \). How do we proceed to find \( C \)?

(a) Use the initial condition.
(b) Divide both sides of the equation by \( C \).
(c) Divide both sides of the equation by 4.
(d) Divide both sides of the equation by \( n \).
(e) Divide both sides of the equation by \( 4^n \).
(f) We don’t have enough information to solve for \( C \).

7. We are trying to find a solution to \( a_{n+1} = 9a_n + 5 \cdot 4^n \) where \( a_0 = 1.5 \). We have conjectured \( a_n = 4^n C \) as the particular solution to the nonhomogeneous part and substituted our conjecture into the difference equation to obtain \( 4^{n+1} C = 9 \cdot 4^n C + 5 \cdot 4^n \). What is the value of \( C \)?

(a) \( C = -1 \)
(b) \( C = 1 \)
(c) \( C = -5/8 \)
(d) \( C = 5/3 \)
(e) None of the above
8. For which of the following difference equations will the nonhomogeneous conjecture need to be modified by multiplying by \( n \)?

(a) \( a_{n+1} = 2a_n + 2 \cdot 3^n \)

(b) \( a_{n+1} = 3a_n + 2 \cdot 3^n \)

(c) \( a_{n+1} = a_n + 4^n \)

(d) All of the above

(e) None of the above

9. Which of the following is not a solution to \( a_{n+1} = 3a_n + 5 \cdot 4^n \)?

(a) \( a_n = 5 \cdot 4^n \)

(b) \( a_n = 6 \cdot 3^n \)

(c) \( a_n = 8 \cdot 3^n + 5 \cdot 4^n \)

(d) \( a_n = 15 \cdot 3^n + 5 \cdot 4^n \)

(e) All are solutions