

MathQuest: Difference Equations

Solutions to Nonhomogeneous DEs with a Polynomial Term

1. Upon graduation from college you land a job with a starting salary of \$35,000. You are told that as long as your performance is up to par, you can expect a 3% raise each year. Additionally, at the end of your first year you will receive a bonus of \$100, and at the end of each year after that you will receive a bonus equal to \$100 times the number of years you have completed. On the first day of your job, you open a bank account with \$200. This account will earn interest at a rate of 5% per year, and you decide that each year you will deposit your bonus into the account. If a_n represents the amount of money in your account at the end of n years, which of the following difference equations models your account balances?

- (a) $a_{n+1} = 1.03a_n + 100 + 35,000$
- (b) $a_{n+1} = 1.05a_n + 100$
- (c) $a_{n+1} = 1.05a_n + 100(n + 1)$
- (d) $a_{n+1} = 1.05a_n + 1.05(100)n$
- (e) $a_{n+1} = 1.03a_n + 1.05(100) + 200$
- (f) None of the above

2. What is the best conjecture to use for the nonhomogeneous solution to $a_{n+1} = 3a_n + 5n$?

- (a) $a_n = C_1n$
- (b) $a_n = C_1n + C_0$
- (c) $a_n = C_2n^2 + C_1n + C_0$
- (d) None of the above

3. We are trying to solve $a_{n+1} = 2a_n + 7n - 5$. For a particular solution to the nonhomogeneous part, we conjecture $a_n = C_1n + C_0$. When we substitute this into the difference equation, what is the result?

- (a) $C_1n + C_0 = 2(C_1n + C_0) + 7(C_1n + C_0) - 5$
- (b) $C_1(n + 1) + C_0 = 2(C_1n + C_0) + 7(C_1n + C_0) - 5$
- (c) $C_1(n + 1) + C_0 = 2(C_1n + C_0) + 7(C_1n + C_0) - 5$
- (d) $C_1(n + 1) + C_0 = 2(C_1n + C_0) + 7n - 5$

- (e) None of the above
4. We are trying to solve $a_{n+1} = 3a_n + 5n$ where $a_0 = 10$. We have conjectured $a_n = C_1n + C_0$ as a solution to the nonhomogeneous equation, and after substituting we have $C_1(n + 1) + C_0 = 3(C_1n + C_0) + 5n$. What are the values of C_1 and C_0 ?
- (a) $C_1 = -5/2$ and $C_0 = -5/4$
 (b) $C_1 = 5$ and $C_0 = 5/2$
 (c) $C_1 = 5$ and $C_0 = 10$
 (d) $C_1 = 3$ and $C_0 = 5$
 (e) Not enough information is given.
5. For which of the following difference equations will the nonhomogeneous conjecture need to be modified by multiplying by n ?
- (a) $a_{n+1} = 3a_n + 3n + 4$
 (b) $a_{n+1} = 5a_n + n + 5$
 (c) $a_{n+1} = a_n + 7n^2 + 3n$
 (d) All of the above
 (e) None of the above
6. If $a_n = 2^n C$ is the solution to the homogeneous part of a difference equation, which of the following could *not* be a particular solution to the nonhomogeneous equation?
- (a) $a_n = n \cdot 2^n$
 (b) $b_n = C_1n^2 + C_2n + C_3$
 (c) $c_n = 5 \cdot 2^n$
 (d) $d_n = 6$
 (e) All of the above answer the question correctly.
 (f) None of the above answer the question correctly.
7. Suppose we have a nonhomogeneous difference equation that we solve by finding the general solution to the homogeneous part and a particular solution to the nonhomogeneous equation. If $a_n = (0.8)^n C$ is the solution to the homogeneous part, and $b_n = 3n^2 - 4$ is the particular solution to the nonhomogeneous equation, then which of the following is a solution to the original difference equation?

- (a) $a_n = (0.8)^n C$
- (b) $b_n = 3n^2 - 4$
- (c) $c_n = (0.8)^n C + 2(3n^2 - 4)$
- (d) All of the above
- (e) None of the above

8. We have the difference equation $a_{n+1} = 2a_n + 3n + 4$. What is the solution to the associated homogeneous equation?

- (a) $a_n = C_0 2^n$
- (b) $a_n = C_0 2^n + C_1 n + C_2$
- (c) $a_n = C_0 2^n + C_2$
- (d) $a_n = C_0 3^n$
- (e) None of the above

9. Is the general term that was the correct answer to the previous question a solution to the difference equation $a_{n+1} = 2a_n + 3n + 4$?

- (a) Yes
- (b) No
- (c) It cannot be determined.

10. $a_{n+1} = 2a_n + 3n + 4$. What is the particular solution to the nonhomogenous equation?

- (a) $a_n = 2n - 6$
- (b) $a_n = -3n - 4$
- (c) $a_n = 2n - 2$
- (d) $a_n = -3n - 7$
- (e) It cannot be determined.

11. Is the particular solution that was the correct answer to the previous question a solution to the difference equation $a_{n+1} = 2a_n + 3n + 4$?

- (a) Yes
- (b) No
- (c) It cannot be determined.

12. For what initial condition is $a_n = -3n - 7$ a solution to the difference equation $a_{n+1} = 2a_n + 3n + 4$?
- (a) For all initial conditions, because we found this without using an initial condition.
 - (b) For $a_0 = -7$.
 - (c) For $a_0 = -10$.
 - (d) For $a_0 = -3$.
 - (e) Answers b, c, and d, are all correct.
 - (f) This corresponds to no initial conditions.