

# MathQuest: Difference Equations

## Solutions to Nonhomogeneous DEs with a Polynomial Term

1. Upon graduation from college you land a job with a starting salary of \$35,000. You are told that as long as your performance is up to par, you can expect a 3% raise each year. Additionally, at the end of your first year you will receive a bonus of \$100, and at the end of each year after that you will receive a bonus equal to \$100 times the number of years you have completed. On the first day of your job, you open a bank account with \$200. This account will earn interest at a rate of 5% per year, and you decide that each year you will deposit your bonus into the account. If  $a_n$  represents the amount of money in your account at the end of  $n$  years, which of the following difference equations models your account balances?
  - (a)  $a_{n+1} = 1.03a_n + 100 + 35,000$
  - (b)  $a_{n+1} = 1.05a_n + 100$
  - (c)  $a_{n+1} = 1.05a_n + 100(n + 1)$
  - (d)  $a_{n+1} = 1.05a_n + 1.05(100)n$
  - (e)  $a_{n+1} = 1.03a_n + 1.05(100) + 200$
  - (f) None of the above
  
2. What is the best conjecture to use for the nonhomogeneous solution to  $a_{n+1} = 3a_n + 5n$ ?
  - (a)  $a_n = C_1n$
  - (b)  $a_n = C_1n + C_0$
  - (c)  $a_n = C_2n^2 + C_1n + C_0$
  - (d) None of the above
  
3. We are trying to solve  $a_{n+1} = 2a_n + 7n - 5$ . For a particular solution to the nonhomogeneous part, we conjecture  $a_n = C_1n + C_0$ . When we substitute this into the difference equation, what is the result?
  - (a)  $C_1n + C_0 = 2(C_1n + C_0) + 7(C_1n + C_0) - 5$
  - (b)  $C_1(n + 1) + C_0 = 2(C_1n + C_0) + 7(C_1n + C_0) - 5$
  - (c)  $C_1(n + 1) + C_0 = 2(C_1n + C_0) + 7(C_1n + C_0) - 5$
  - (d)  $C_1(n + 1) + C_0 = 2(C_1n + C_0) + 7n - 5$

- (e) None of the above
4. We are trying to solve  $a_{n+1} = 3a_n + 5n$  where  $a_0 = 10$ . We have conjectured  $a_n = C_1n + C_0$  as a solution to the nonhomogeneous equation, and after substituting we have  $C_1(n+1) + C_0 = 3(C_1n + C_0) + 5n$ . What are the values of  $C_1$  and  $C_0$ ?
- (a)  $C_1 = -5/2$  and  $C_0 = -5/4$   
 (b)  $C_1 = 5$  and  $C_0 = 5/2$   
 (c)  $C_1 = 5$  and  $C_0 = 10$   
 (d)  $C_1 = 3$  and  $C_0 = 5$   
 (e) Not enough information is given.
5. For which of the following difference equations will the nonhomogeneous conjecture need to be modified by multiplying by  $n$ ?
- (a)  $a_{n+1} = 3a_n + 3n + 4$   
 (b)  $a_{n+1} = 5a_n + n + 5$   
 (c)  $a_{n+1} = a_n + 7n^2 + 3n$   
 (d) All of the above  
 (e) None of the above
6. If  $a_n = 2^n C$  is the solution to the homogeneous part of a difference equation, which of the following could *not* be a particular solution to the nonhomogeneous equation?
- (a)  $a_n = n \cdot 2^n$   
 (b)  $b_n = C_1n^2 + C_2n + C_3$   
 (c)  $c_n = 5 \cdot 2^n$   
 (d)  $d_n = 6$   
 (e) All of the above answer the question correctly.  
 (f) None of the above answer the question correctly.
7. Suppose we have a nonhomogeneous difference equation that we solve by finding the general solution to the homogeneous part and a particular solution to the nonhomogeneous equation. If  $a_n = (0.8)^n C$  is the solution to the homogeneous part, and  $b_n = 3n^2 - 4$  is the particular solution to the nonhomogeneous equation, then which of the following is a solution to the original difference equation?

- (a)  $a_n = (0.8)^n C$   
 (b)  $b_n = 3n^2 - 4$   
 (c)  $c_n = (0.8)^n C + 2(3n^2 - 4)$   
 (d) All of the above  
 (e) None of the above
8. We have the difference equation  $a_{n+1} = 2a_n + 3n + 4$ . What is the solution to the associated homogeneous equation?
- (a)  $a_n = C_0 2^n$   
 (b)  $a_n = C_0 2^n + C_1 n + C_2$   
 (c)  $a_n = C_0 2^n + C_2$   
 (d)  $a_n = C_0 3^n$   
 (e) None of the above
9. Is the general term that was the correct answer to the previous question a solution to the difference equation  $a_{n+1} = 2a_n + 3n + 4$ ?
- (a) Yes  
 (b) No  
 (c) It cannot be determined.
10.  $a_{n+1} = 2a_n + 3n + 4$ . What is the particular solution to the nonhomogenous equation?
- (a)  $a_n = 2n - 6$   
 (b)  $a_n = -3n - 4$   
 (c)  $a_n = 2n - 2$   
 (d)  $a_n = -3n - 7$   
 (e) It cannot be determined.
11. Is the particular solution that was the correct answer to the previous question a solution to the difference equation  $a_{n+1} = 2a_n + 3n + 4$ ?
- (a) Yes  
 (b) No  
 (c) It cannot be determined.

12. For what initial condition is  $a_n = -3n - 7$  a solution to the difference equation  $a_{n+1} = 2a_n + 3n + 4$ ?
- (a) For all initial conditions, because we found this without using an initial condition.
  - (b) For  $a_0 = -7$ .
  - (c) For  $a_0 = -10$ .
  - (d) For  $a_0 = -3$ .
  - (e) Answers b, c, and d, are all correct.
  - (f) This corresponds to no initial conditions.