MathQuest: Difference Equations

Systems of Difference Equations

1. Two cell phone service providers, Alpha and Beta, are constantly competing for the largest market share. Each month 5% of Alpha customers switch their service to Beta, and each month 7% of Beta customers switch their service to Alpha. The total number of customers served by the two companies stays fixed. Which system of difference equations models the monthly changes in customer base for each provider?

   (a)
   $$\begin{aligned} A_{n+1} &= 0.07B_n \\ B_{n+1} &= 0.05A_n \end{aligned}$$

   (b)
   $$\begin{aligned} A_{n+1} &= -0.05A_n \\ B_{n+1} &= -0.07B_n \end{aligned}$$

   (c)
   $$\begin{aligned} A_{n+1} &= A_n + 0.07B_n \\ B_{n+1} &= 0.05A_n + B_n \end{aligned}$$

   (d)
   $$\begin{aligned} A_{n+1} &= A_n - 0.05A_n + 0.07B_n \\ B_{n+1} &= 0.05A_n + B_n - 0.07B_n \end{aligned}$$

   (e) None of the above

2.
   $$\begin{aligned} A_{n+1} &= A_n - 0.2A_n + 0.3B_n \\ B_{n+1} &= B_n - 0.3B_n \end{aligned}$$

   These difference equations allow us to predict how the populations of two towns, A and B, change each year.

   Which of the following would be a true statement?

   (a) All of the people who leave town A move to town B.

(b) All of the people who leave town B move to town A.

(c) None of the people who leave town B move to town A.

(d) Some of the people who leave town A move to town B.

3.

$$\begin{aligned} A_{n+1} &= A_n - 0.2A_n + 0.3B_n \\ B_{n+1} &= B_n - 0.3B_n \end{aligned}$$

These difference equations allow us to predict how the populations of two towns, A and B, change each year.

Which of the following would be a true statement?

(a) 30% of the people in A move to B each year.

(b) 20% of the people in B move to A each year.

(c) Only 70% of the people in B stay in B each year.

(d) Only 20% of the people in A stay in A each year.

4. Scrabble or Boggle? Both are great word games. Each month, 10% of Scrabble players stop playing Scrabble and switch to Boggle, while 12% of Boggle players stop playing Boggle and switch to Scrabble. In addition, 1587 new people take up the game of Scrabble each month, while 1298 new people start playing Boggle each month. Each month 2% of players of each game stop playing word games entirely. Which system of difference equations models the month-to-month changes in the numbers of Scrabble and Boggle players?

(a)

$$\begin{aligned} S_{n+1} &= -0.10S_n - 0.02S_n + 1587 \\ B_{n+1} &= -0.12B_n - 0.02B_n + 1298 \end{aligned}$$

(b)

$$\begin{aligned} S_{n+1} &= S_n - 0.10S_n - 0.02S_n + 1298 \\ B_{n+1} &= B_n - 0.12B_n - 0.02B_n + 1587 \end{aligned}$$

(c)

$$\begin{aligned} S_{n+1} &= S_n - 0.02S_n + 0.12B_n \\ B_{n+1} &= 0.10S_n + B_n - 0.02B_n \end{aligned}$$

(d)

$$\begin{aligned} S_{n+1} &= S_n - 0.10S_n - 0.02S_n + 0.12B_n + 1587 \\ B_{n+1} &= 0.10S_n + B_n - 0.12B_n - 0.02B_n + 1298 \end{aligned}$$

(e) None of the above

5. Mr. Jones has a checking account and a savings account. He earns 1.2% annual interest (compounded monthly) on his checking account and 3.6% interest (compounded monthly) on his savings account. Each month Mr. Jones transfers 10% of the balance of his checking account to his savings account, but he does not make any other deposits or withdrawals from his savings account. He has a monthly paycheck deposit of \$2400 into his checking account, and he writes checks totalling \$2200 each month. Which system of difference equations models the monthly balances in Mr. Jones' accounts?

(a)

$$\begin{aligned} c_{n+1} &= c_n + 0.012c_n - 0.10c_n \\ s_{n+1} &= s_n + 0.036s_n \end{aligned}$$

(b)

$$\begin{aligned} c_{n+1} &= c_n + 0.012c_n - 0.10c_n + 200 \\ s_{n+1} &= s_n + 0.036s_n \end{aligned}$$

(c)

$$\begin{aligned} c_{n+1} &= c_n + 0.001c_n - 0.10c_n + 200 \\ s_{n+1} &= s_n + 0.003s_n + 0.10c_n \end{aligned}$$

(d)

$$\begin{aligned} c_{n+1} &= c_n + 0.001c_n - 0.10c_n + 200 \\ s_{n+1} &= s_n + 0.003s_n + 0.10s_n \end{aligned}$$

(e) None of the above

6.

$$\begin{aligned} E_{n+1} &= 0.75E_n + 0.10F_n + 500 \\ F_{n+1} &= 0.25E_n + 0.80F_n \end{aligned}$$

These difference equations allow us to predict the number of customers of companies E and F each year.

Which of the following would be a true statement?

(a) 20% of F's customers switch to E each year.

(b) 500 new customers sign up with F each year.

(c) 10% of E's customers switch to F each year.

(d) 10% of F's customers switch to E each year.

7.

$$\begin{aligned} E_{n+1} &= 0.75E_n + 0.10F_n + 500 \\ F_{n+1} &= 0.25E_n + 0.80F_n \end{aligned}$$

These difference equations allow us to predict the number of customers of companies E and F each year.

Which of the following would be a true statement?

(a) All the customers that F loses switch to E.

(b) All the customers that E loses switch to F.

(c) None of the customers that F loses switch to E.

(d) None of the customers that E loses switch to F.

8. We are given the following system of difference equations:

$$\begin{aligned} a_{n+1} &= 2a_n + 3b_n + 2 \\ b_{n+1} &= a_n + 2b_n \end{aligned}$$

with $a_0 = 5$ and $b_0 = 1$. What are a_1 and b_1?

(a) $a_1 = 27$ and $b_1 = 3$

(b) $a_1 = 15$ and $b_1 = 7$

(c) $a_1 = 19$ and $b_1 = 11$

(d) $a_1 = 16$ and $b_1 = 7$

(e) $a_1 = 15$ and $b_1 = 17$

(f) None of the above

9. We are given the following system of difference equations:

$$\begin{aligned} a_{n+1} &= .4a_n + .6b_n \\ b_{n+1} &= .6a_n + .4b_n \end{aligned}$$

with $a_0 = 10$ and $b_0 = 10$. What are a_5 and b_5?

(a) $a_5 = 10$ and $b_5 = 10$

(b) $a_5 = 4$ and $b_5 = 6$

(c) $a_5 = 6$ and $b_5 = 4$

(d) $a_5 = 1$ and $b_5 = 1$

(e) None of the above

10.

$$\begin{aligned} C_{n+1} &= 1.15C_n - (4 \times 10^{-5})C_nD_n \\ D_{n+1} &= 0.95D_n + (3 \times 10^{-5})C_nD_n \end{aligned}$$

These difference equations allow us to predict how two animal populations, C and D, change each year.

Which of the following would be a true statement?

(a) If $C_n = 0$ then D will grow by 95% each year.

(b) If $C_n = 0$ then D will grow by 15% each year.

(c) If $D_n = 0$ then C will grow by 95% each year.

(d) If $D_n = 0$ then C will grow by 15% each year.

11.

$$\begin{aligned} C_{n+1} &= 1.15C_n - (4 \times 10^{-5})C_nD_n \\ D_{n+1} &= 0.95D_n + (3 \times 10^{-5})C_nD_n \end{aligned}$$

These difference equations allow us to predict how two animal populations, C and D, change each year.

Which of the following would be a true statement?

(a) Larger populations of C cause the population of D to grow.

(b) Larger populations of C cause the population of D to shrink.

(c) Larger populations of D cause the population of C to grow.

(d) None of the above

12.

$$\begin{aligned} C_{n+1} &= 1.15C_n - (4 \times 10^{-5})C_nD_n \\ D_{n+1} &= 0.95D_n + (3 \times 10^{-5})C_nD_n \end{aligned}$$

These difference equations allow us to predict how two animal populations, C and D, change each year.

Which of the following would be a true statement?

(a) We could say that C is a predator and D is prey.
(b) We could say that D is a predator and C is prey.
(c) We could say that C and D are competitors.
(d) We could say that C and D benefit from each other.
(e) None of the above

13.

$$\begin{aligned} C_{n+1} &= 0.85C_n - (4 \times 10^{-5})C_nD_n \\ D_{n+1} &= 0.95D_n + (3 \times 10^{-5})C_nD_n \end{aligned}$$

These difference equations allow us to predict how two animal populations, C and D, change each year.

Which of the following would be a true statement?

(a) We could say that C eats D.
(b) We could say that D eats C.
(c) We could say that C and D are competitors.
(d) We could say that C and D benefit from each other.
(e) None of the above

14.

$$\begin{aligned} C_{n+1} &= 0.85C_n + (4 \times 10^{-5})C_nD_n \\ D_{n+1} &= 0.95D_n + (3 \times 10^{-5})C_nD_n \end{aligned}$$

These difference equations allow us to predict how two species, C and D, change each year.

Which of the following would be a true statement?

(a) We could say that C is a predator and D is prey.
(b) We could say that D is a predator and C is prey.
(c) We could say that C and D are competitors.
(d) We could say that C and D benefit from each other.
(e) None of the above