

MathQuest: Difference Equations

Solving Nonhomogeneous Systems of Difference Equations

1. Suppose we are solving a system of two difference equations. For which pair of eigenvalues would the system have a stable equilibrium?
 - (a) $\lambda_1 = 0.8$ and $\lambda_2 = 0.7$
 - (b) $\lambda_1 = 1$ and $\lambda_2 = 0.8$
 - (c) $\lambda_1 = 1$ and $\lambda_2 = 2.5$
 - (d) All of the above
 - (e) None of the above
 - (f) More than one of the above

2. If we wish to solve this system,

$$\begin{aligned}a_{n+1} &= 0.6a_n + 0.3b_n + 3 \\ b_{n+1} &= 0.7b_n + 0.2a_n + 1\end{aligned}$$

what matrix do we need to find eigenvalues and eigenvectors for?

- (a) $\begin{bmatrix} 0.6 & 0.3 & 3 \\ 0.7 & 0.2 & 1 \end{bmatrix}$
- (b) $\begin{bmatrix} 0.6 & 0.3 & 3 \\ 0.2 & 0.7 & 1 \end{bmatrix}$
- (c) $\begin{bmatrix} 0.6 & 0.3 \\ 0.7 & 0.2 \end{bmatrix}$
- (d) $\begin{bmatrix} 0.6 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$
- (e) None of the above

3. What is the equilibrium vector for this system when $a_0 = 5$ and $b_0 = 3$?

$$\begin{aligned} a_{n+1} &= 0.6a_n + 0.3b_n + 3 \\ b_{n+1} &= 0.7b_n + 0.2a_n + 1 \end{aligned}$$

(a) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} 20 \\ 50/3 \end{bmatrix}$

(c) $\begin{bmatrix} 40 \\ 110/3 \end{bmatrix}$

(d) $\begin{bmatrix} 24.5 \\ 22.7 \end{bmatrix}$

4. You are solving a system of difference equations, and you find that the eigenvalues of the coefficient matrix are 0.93723 and 0.36277 with corresponding eigenvectors of $\begin{bmatrix} 0.80818 \\ 0.58893 \end{bmatrix}$ and $\begin{bmatrix} -0.97483 \\ 0.22296 \end{bmatrix}$. If the initial conditions are $\begin{bmatrix} a_0 \\ b_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$ and the equilibrium vector has been determined to be $\begin{bmatrix} 35 \\ 27.5 \end{bmatrix}$, what is the particular solution for this system?

(a) $\begin{bmatrix} a_n \\ b_n \end{bmatrix} = (0.93723)^n \begin{bmatrix} 0.80818 \\ 0.58893 \end{bmatrix} + (0.36277)^n \begin{bmatrix} -0.97483 \\ 0.22296 \end{bmatrix}$

(b) $\begin{bmatrix} a_n \\ b_n \end{bmatrix} = 35(0.93723)^n \begin{bmatrix} 0.80818 \\ 0.58893 \end{bmatrix} + 27.5(0.36277)^n \begin{bmatrix} -0.97483 \\ 0.22296 \end{bmatrix}$

(c) $\begin{bmatrix} a_n \\ b_n \end{bmatrix} = 1(0.93723)^n \begin{bmatrix} 0.80818 \\ 0.58893 \end{bmatrix} + 7(0.36277)^n \begin{bmatrix} -0.97483 \\ 0.22296 \end{bmatrix} + \begin{bmatrix} 35 \\ 27.5 \end{bmatrix}$

(d) $\begin{bmatrix} a_n \\ b_n \end{bmatrix} = -36.543(0.93723)^n \begin{bmatrix} 0.80818 \\ 0.58893 \end{bmatrix} + 4.582(0.36277)^n \begin{bmatrix} -0.97483 \\ 0.22296 \end{bmatrix} + \begin{bmatrix} 35 \\ 27.5 \end{bmatrix}$

(e) None of the above