# MathQuest: Difference Equations

## Sequences and Difference Equations

1. If the general term for a sequence is $a_n = 4 + 6n$ then
   (a) $a_3 = 10$
   (b) $a_3 = 16$
   (c) $a_3 = 22$
   (d) $a_3 = 28$

2. Consider the difference equation $a_{n+1} = -3a_n + 5$ with the initial condition $a_0 = 1$. What is $a_3$?
   (a) $a_3 = 8$
   (b) $a_3 = -19$
   (c) $a_3 = -1$
   (d) $a_3 = 2$

3. Consider the difference equation $a_{n+1} = 2a_n + n$ with the initial condition $a_0 = 1$. What is $a_2$?
   (a) $a_2 = 4$
   (b) $a_2 = 5$
   (c) $a_2 = 8$

(d) $a_2 = 27$

4. The sequence $0, 1, 2, 3, 4, \ldots$ can be represented by which discrete dynamical system (difference equation and initial condition)?

   (a) $a_{n+1} = a_n + 1$ with $a_0 = 0$
   (b) $a_{n+3} = a_{n+2} + 1$ with $a_0 = 0$
   (c) $b_{k+2} = b_{k+1} + 1$ with $b_0 = 0$
   (d) $a_{n+1} = a_n + 1$ with $a_4 = 4$
   (e) All of the above
   (f) None of the above

5. The sequence $0, 2, 4, 6, 8, \ldots$ can be represented by which discrete dynamical system (difference equation and initial condition)?

   (a) $a_{n+1} = 2a_n$ with $a_0 = 0$
   (b) $a_{n+1} = a_n + 2$ with $a_0 = 0$
   (c) $a_n = 2a_n$ with $a_0 = 0$
   (d) $a_n = a_n + 2$ with $a_0 = 0$
   (e) None of the above

6. Given the sequence $0, 2, 4, 6, 8, \ldots$, what is the $a_n$, noting that $a_0 = 0$?

(a) $a_n = n + 2$

(b) $a_n = 2^n$

(c) $a_n = n^2$

(d) $a_n = 2n$

7. **True or False** The sequences $0, 2, 4, 6, 8, \ldots$ and $2, 4, 6, 8, \ldots$ are represented by the same difference equation, $a_{n+1} = a_n + 2$.

   (a) True, and I am very confident

   (b) True, but I am not very confident

   (c) False, but I am not very confident

   (d) False, and I am very confident

8. **True or False** The sequences $0, 2, 4, 6, 8, \ldots$ and $2, 4, 6, 8, \ldots$ have the same $n^{th}$ term.

   (a) True, and I am very confident

   (b) True, but I am not very confident

   (c) False, but I am not very confident

   (d) False, and I am very confident

9. Which difference equation represents the sequence $15, 30, 60,$

   (a) $a_{n+1} = a_n + 15$

(b) $a_{n+1} = 15a_n$
(c) $a_{n+1} = 2a_n$
(d) None of the above

10. Given the sequence $15, 30, 60, 120, 240, ...,$ what is the $a_n$, noting that $a_0 = 15$?

(a) $a_n = n + 15$
(b) $a_n = 2^n$
(c) $a_n = 2^n(15)$
(d) $a_n = n^2$
(e) $a_n = 2n$
(f) None of the above

11. Which difference equation represents the sequence $10, 7, 4, 1,$

(a) $a_{n+1} - a_n = -3$
(b) $a_{n+1} = 3 - a_n$
(c) $a_{n+1} = \frac{7}{10}a_n$
(d) None of the above

12. Given the sequence $10, 7, 4, 1, -2, ...,$ what is the $a_n$, noting that $a_0 = 10$?

(a) $a_n = n - 3$

(b) $a_n = 3 - n$

(c) $a_n = 10 - n$

(d) $a_n = 10 - 3n$

(e) None of the above

13. Which of the following represents the general term for the sequence $7, 5, 7, 5, 7 \cdots$?

(a) $a_n = 6 - (-1)^n$

(b) $a_n = 6 + (-1)^n$

(c) $a_n = 7 - 2n$

(d) $a_n = 5 + (-2)^n$

14. Which difference equation represents the sequence $1, 3, 6, 10,$

(a) $a_{n+1} = a_n + 2$

(b) $a_{n+1} = a_n + 3$

(c) $a_{n+1} = a_n + n$

(d) None of the above

15. Which of the following difference equations describes the sequence $a_0 = 10, a_1 = 9, a_2 = 7, a_3 = 4, a_4 = 0, a_5 = -5, \cdots$.

(a) $a_{n+1} = a_n - n$

(b) $a_{n+1} = a_n - 1$

(c) $a_{n+1} = a_n - n - 1$

(d) $a_{n+1} = a_n - n^2 - 1$

16. Which difference equation represents the sequence $1, 1, 2, 3, 5$

(a) $a_{n+1} = a_n + 1$

(b) $a_{n+2} = a_n + 3$

(c) $a_{n+2} = a_{n+1} + a_n$

(d) None of the above

17. If the general term of a sequence is $a_n = 2n - 4$ which of the following difference equations work?

(a) $a_{n+1} = a_n + 2$

(b) $a_{n+1} = 2a_n - 4$

(c) $a_{n+1} = a_n + 2n$

(d) $a_{n+1} = 4a_n - 2$

18. If a sequence follows the difference equation $a_{n+1} = a_n + 4$ with the initial condition $a_0 = -3$, which of the following general terms describes the sequence?

(a) $a_n = 4 - 3n$

(b) $a_n = 4 + n$

(c) $a_n = -3 \cdot (4n)$

(d) $a_n = -3 + 4n$

19. Which of the following sequences follows the difference equation $a_{n+1} = 2a_n - n^2$?

(a) $a_0 = 2,\ a_1 = 4,\ a_2 = 7,\ a_3 = 10,\ \cdots$

(b) $a_0 = 1,\ a_1 = 2,\ a_2 = 3,\ a_3 = 2,\ \cdots$

(c) $a_0 = 0,\ a_1 = 0,\ a_2 = -1,\ a_3 = -6,\ \cdots$

(d) $a_0 = -1,\ a_1 = -2,\ a_2 = -5,\ a_3 = -14,\ \cdots$

(e) All of the above

20. Which of the following is *not* a difference equation?

(a) $a_{n+2} = 3a_n + 5$

(b) $b_n - b_{n-1} = 3$

(c) $c_n = 5n + 2$

(d) $d_n = 3n + d_{n+1}$

(e) All of these are difference equations.

(f) None of these are difference equations.

**Differences and Derivatives**

21. The first difference, $f(n+1) - f(n)$,

(a) is exactly the same as the first derivative.
(b) is a discrete approximation of the first derivative.
(c) is not very helpful for learning about functions.
(d) None of the above

22. The first difference can
    (a) pinpoint exactly where a function has a critical point.
    (b) pinpoint exactly where a function changes concavity.
    (c) approximate the location of a function's critical points.
    (d) approximate the location of a function's inflection point.
    (e) None of the above

23. The second difference can
    (a) pinpoint exactly where a function has a critical point.
    (b) pinpoint exactly where a function changes concavity.
    (c) approximate the location of a function's critical points.

(d) approximate the location of a function's inflection point.
(e) None of the above

24. If we sample $f(x) = 3x + b$ using step sizes of 1, what are the first differences?

    (a) 0
    (b) 1
    (c) 1.5
    (d) 3
    (e) Cannot determine without knowing $b$.

25. If we sample $f(x) = 3x + b$ using step sizes of 0.5, what are the first differences?

    (a) 0
    (b) 1
    (c) 1.5
    (d) 3
    (e) Cannot determine without knowing $b$.

26. If we sample $f(x) = 3x + b$ using step sizes of 0.5, what are the second differences?

    (a) 0

(b) 0.5

(c) 1

(d) 3

(e) Cannot determine without knowing $b$.

27. If we sampled a second degree polynomial in step sizes of 1, what would the second differences be?

    (a) 0

    (b) 0.5

    (c) 1

    (d) 2

    (e) Cannot determine without knowing the polynomial.

28. If we sampled the function $f(x) = x^2 + bx + c$ in step sizes of 0.5, what would the second differences be?

    (a) 0

    (b) 0.5

    (c) 1

    (d) 2

    (e) 4

    (f) Cannot determine without knowing the polynomial.

29. The phrase "$y$ is proportional to $x$" means
   (a) $y = kx$
   (b) if $x$ doubles, then $y$ doubles.
   (c) a graph of $y$ versus $x$ would always go through the origin.
   (d) All of the above
   (e) None of the above

30. If the change in population is proportional to the population size, with proportionality constant $k$, we can say
   (a) $a_{n+1} = ka_n$
   (b) $\Delta a_n = ka_n$
   (c) $a_n = k$
   (d) All of the above
   (e) None of the above

31. In the year 2000 the population of the US was 281 million, and our population grows by about ten percent every decade. Formulate a difference equation to model the population of the US.
   (a) $\Delta a_n = 0.1a_n$
   (b) $a_{n+1} = 1.1a_n$

(c) $\frac{\Delta a_n}{a_n} = 0.1$

(d) All of the above

(e) None of the above

32. A difference equation to model the population of frogs is $\Delta f_n = 0.2 f_n$, where $n$ is in years. What is a verbal description of this scenario?

    (a) The frog population is increasing at a rate of 20 percent per year.

    (b) The size of the frog population next year will be 120% of this year's population size.

    (c) The change in the frog population is proportional to the current population, with constant of proportionality equal to 0.2.

    (d) All of the above

    (e) None of the above

**Discrete Dynamical System Models**

33. You open a bank account with \$500. You add \$25 to your account each month, and the bank pays you 0.2% interest each month. Which discrete dynamical system describes your account balances?

(a) $a_{n+1} = 0.2a_n + 25 + 500$ with $a_0 = 0$

(b) $a_{n+1} = 0.002a_n + 25$ with $a_0 = 500$

(c) $a_{n+1} = 1.002a_n + 500$ with $a_0 = 25$

(d) $a_{n+1} = 1.002a_n + 25$ with $a_0 = 500$

(e) None of the above

34. You've been spending too much money lately, and your credit card balance has risen to \$1200. You tear up the card and start paying \$40 per month to try to pay off the balance. Meanwhile, your credit card company charges you 1.5% interest on the balance each month. Which discrete dynamical system describes your account balances?

(a) $a_{n+1} = 1.015a_n - 40$ with $a_0 = 1200$

(b) $a_{n+1} = 1.015a_n - 40n + 1200$ with $a_0 = 0$

(c) $a_{n+1} = 1.5a_n + 40$ with $a_0 = 1200$

(d) $a_{n+1} = 0.015a_n - 40$ with $a_0 = 1200$

(e) None of the above

35. You open a bank account that pays 3% annual interest, compounded monthly. Which difference equation describes your account balances, assuming that you do not make any further deposits?

(a) $b_{n+1} = 3b_n$

(b) $b_{n+1} = 1.3b_n$

(c) $b_{n+1} = 1.03b_n$

(d) $b_{n+1} = b_n + 0.03$

(e) None of the above

36. The following difference equation describes the value of a car, where $n$ is in years.

$$V_{n+1} = 0.86V_n$$

Which of the following is a true statement?

(a) The value of the car increases 86% each year.

(b) The value of the car decreases 86% each year.

(c) The value of the car decreases by 14% each year.

(d) The value of the car will eventually level out at \$8600.

(e) None of the above

37. The following difference equation describes the value of a car, where $n$ is in years.

$$V_{n+1} = 0.86V_n$$

If $V_0 = \$20,000$, what is $V_1$?

(a) \$17,200

(b) \$19,914

(c) \$20,086

(d) Need more information.

38. The following difference equation describes the value of a car, where $n$ is in years.

$$V_{n+1} = 0.86V_n$$

Which of the following is the correct interpretation of $V_1$?

(a) $V_1$ is the change in the value of the car after one year.

(b) $V_1$ is the value of the car after one year.

(c) $V_0 - V_1$ gives the value of the car after one year.

(d) None of the above

39. Which scenario below could be modeled with this difference equation?

$$a_{n+1} = 1.01a_n + 180$$

(a) A bank account earns 1% annual interest and receives deposits of \$180 per month.

(b) A population is increasing at a rate of 101% per year plus a yearly increase of 180.

(c) There are initially 180 cows on a ranch, and each year the population increases by 1%.

(d) The number of deer at a wildlife sanctuary naturally increases by 1% per year. In addition, 180 new deer are brought to the sanctuary each year.

(e) All of the above

(f) None of the above

40. If $a_n = 1.01a_n + 180$ with $a_3 = 100$, what is $a_4$?

(a) 101

(b) 281

(c) 282.8

(d) You can't figure out $a_4$ without knowing $a_0$.

(e) None of the above

41. A growing bookstore's inventory changes according to a regular pattern. Each week the bookstore sells 250 books, and each week it receives 265 new books. In addition, the price of books goes up about 2% each year. Customers prefer comic books to serious literature, buying 3 times as many comic books as classics. Which of the following difference equations describes how the bookstore's inventory changes from week to week?

(a) $a_{n+1} = a_n - 250 + 265$

(b) $a_{n+1} = 1.02a_n - 250 + 265 - 3a_n$

(c) $a_{n+1} = a_n + (0.02/52)a_n - 250 + 265$

(d) $a_{n+1} = 3a_n + 15 + 1.02$

(e) None of the above

42.

$$E_{n+1} \ = \ E_n + 0.3E_n\left(1 - \frac{E_n}{1000}\right)$$

This difference equation allows us to predict the growth of an elephant population in an African preserve, where $n$ is in years.

Which of the following would be a true statement?

(a) The elephant population will grow at a rate of 30%.

(b) The preserve can support 1000 elephants.

(c) The elephant population will grow at a rate of 0.3%.

(d) The maximum number of elephants the preserve can support is 0.3 $E(n)$.

43.

$$E_{n+1} \ = \ E_n + 0.3E_n\left(1 - \frac{E_n}{1000}\right)$$

This difference equation allows us to predict the growth of an elephant population in an African preserve, where $n$ is in years.

Which of the following would be a true statement?

(a) If $E_0 < 1000$ then the elephant population will shrink.

(b) If $E_0 > 1000$ then the elephant population will grow.

(c) If $E_0 = 1000$ then the elephant population will hold steady.

(d) None of the above

44.

$$S_{n+1} = S_n + k \times S_n \left(1 - \frac{S_n}{P}\right)$$

This difference equation allows us to predict how a disease will spread through a town of population $P$, where $S_n$ is the number of people who are sick after $n$ weeks.

A larger value of $k$ means that...

(a) the town has a larger population.

(b) more people start out being sick.

(c) the disease will spread more quickly.

(d) the disease is more deadly.

45.

$$S_{n+1} \;=\; S_n + k \times S_n \left(1 - \frac{S_n}{P}\right)$$

This difference equation allows us to predict how a disease will spread through a town of population $P$, where $S_n$ is the number of people who are sick after $n$ weeks.

Suppose we calculate that $S_{10} > P$. This means that...

(a) people are getting sick more quickly.

(b) the town population has grown.

(c) no more people will get sick.

(d) our model has failed.

46.

$$B_{n+1} \;=\; B_n + 0.3B_n \left(1 - \frac{B_n}{100}\right)$$

This equation allows us to predict the number of Carroll faculty who have seen the movie “Harry Potter.”

Suppose that $B_{11} = B_{10}$. This means that...

(a) $B_{10} = 0.3$

(b) $B_{10} < 0$

(c) $B_{10} > 100$

(d) $B_{10} = 100$

47.

$$B_{n+1} \;=\; B_n + 0.3B_n\left(1 - \frac{B_n}{100}\right)$$

This equation allows us to predict the number of Carroll faculty who have seen the movie "Harry Potter." Suppose that $B_6 = 0$. This means that...

(a) $B_7 = 100$

(b) $B_7 = 0.3$

(c) $B_7 = 0$

(d) This is not possible.

48.

$$T_{n+1} \;=\; T_n + 0.005T_n\,(15{,}000 - T_n)$$

Suppose this equation allows us to predict the spread of a new computer technology, where $T$ is the number of companies that have this technology during year $n$.

Suppose that $T_{10} = 5{,}000$. How many more companies will get the technology during the next year?

(a) $0.005 \times 5,000$

(b) $0.005 \times 10,000$

(c) $0.005 \times 5000 \times 10,000$

(d) $0.005 \times 5000 \times 15,000$

49.

$$T_{n+1} = T_n + 0.005T_n\,(15,000 - T_n)$$

Suppose this equation allows us to predict the spread of a new computer technology, where $T$ is the number of companies that have this technology during year $n$. Suppose that $T_5 = 10,000$. If the total number of companies who will eventually get the technology was increased from 15,000 to 20,000 this would...

(a) speed up the spread of the technology.

(b) slow down the spread of the technology.

(c) not change how the technology spreads.

(d) stop the spread of the technology.

**Equilibrium and Long-Term Behavior**

50. Suppose we have $a_{n+1} = 3a_n$ with $a_0 = 0$. What is $a_{400}$?

(a) 0

(b) $3^{400}$

(c) 1200

(d) It would take way too long to figure this out.

51. Suppose we have $a_{n+1} = 3a_n$ with $a_0 = 1$. What is $a_3$?

(a) 0

(b) 1

(c) 9

(d) 27

(e) None of the above

52. Suppose we have $a_{n+1} = 0.9a_n$. What is the equilibrium value?

(a) 0

(b) 0.9

(c) There is no equilibrium value.

(d) The equilibrium value cannot be determined without knowing the initial value.

(e) None of the above

53. Suppose we have $a_{n+1} = 0.9a_n$ with $a_0 = 1$. What is $a_3$?

   (a) 0
   (b) 1
   (c) 0.9
   (d) 0.729
   (e) None of the above

54. Suppose we have $f_{n+1} = 3f_n - 10$ with $f_0 = 5$. What is $f_3$?

   (a) 0
   (b) 5
   (c) 15
   (d) 125

55. Suppose we have $f_{n+1} = 3f_n - 10$ with $f_0 = 4$. What is $f_3$?

   (a) 0
   (b) 2
   (c) 4
   (d) 12
   (e) None of the above

56. Suppose $d_{n+1} = 0.9d_n + 2$. What is the equilibrium value?

   (a) 9
   (b) 2
   (c) 20
   (d) 20/9
   (e) None of the above

57. Suppose $g_{n+1} = -2g_n + 3$. Which statement describes the long-term behavior of the solution with $g_0 = 0$.

   (a) The solution stays at 0.
   (b) The solution grows without bound.
   (c) The solution grows and approaches the equilibrium value.
   (d) The solution oscillates farther and farther from the equilibrium value.
   (e) None of the above

58. **True or False** An equilibrium value can never be negative.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident

(d) False, and I am very confident

59. The following difference equation describes the population of a small town, where $n$ is in years.

$$p_{n+1} = 0.9p_n$$

Which of the following is a true statement?

(a) Eventually this town will die out.
(b) Eventually this town will have 1,000,000 people.
(c) This town's population will fluctuate, but the town will never grow substantially or die out.
(d) A big city modeled with this difference equation would have more people in the long run than this town will have.

60. The following difference equation describes the population of a small town, where $n$ is in years.

$$p_{n+1} = 1.17p_n$$

Which of the following is a true statement?

(a) Eventually this town will die out.
(b) Eventually this town will have 1,000,000 people.
(c) This town's population will fluctuate, but the town will never grow substantially or die out.

(d) This difference equation does not have an equilibrium value.

61. The difference equation $a_{n+1} = 1.08a_n$ might model the population of some species. If $a_0 = 5000$, which of the following statements is true?

(a) $a_{10} > 5000$

(b) $a_{40} < 5000$

(c) It is possible that $a_{30} = 5000$

(d) More than one of these statements could be true.

(e) None of these statements has to be true.

62. The difference equation $a_{n+1} = 0.93a_n$ might model the population of some species. If $a_0 = 5000$, which of the following statements is true?

(a) $a_{10} > 5000$

(b) $a_{40} < 5000$

(c) It is possible that $a_{30} = 5000$

(d) More than one of these statements could be true.

(e) None of these statements has to be true.

63. When we are looking for an equilibrium value, why can we change both $a_n$ and $a_{n+1}$ to $E$ and then solve the resulting equation?

(a) $a_n$ and $a_{n+1}$ both represent amounts, so they're the same thing anyway.

(b) $a_n$ and $a_{n+1}$ are just symbols, so we can use a different symbol to represent them.

(c) At equilibrium, each term is the same as the one before.

(d) None of the above

64. A polluted lake has a percentage of its contaminants washed away each year, while factories on the lake dump in a constant amount of pollutants each year. The equilibrium value for this lake is 500 pounds. This means that

(a) if there are initially 500 pounds of contaminants, there will always be 500 pounds of contaminants (assuming the conditions remain the same).

(b) when there are 500 pounds of contaminants in the lake, the amount of pollutants being washed out each year is exactly equal to the amount being dumped in.

(c) if there are currently 750 pounds of pollutants in the lake, next year there will be fewer pollutants in the lake.

(d) All of the above are correct.

(e) Exactly two of the above are correct.

65. Suppose the only equilibrium value of a difference equation is 10, and this is a stable equilibrium. **True or False** If $a_5 = 7$, then $7 < a_8 < 10$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

66. **True or False** A difference equation can only have one equilibrium value.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

67. Consider the difference equation $a_{n+1} = ra_n$. If $r$ is increased, what is the effect on the equilibrium value?

(a) The equilibrium value increases.
(b) The equilibrium value decreases.
(c) The equilibrium value doesn't change.

(d) The effect depends on how much $r$ is increased by and what it started at.

68. Consider the difference equation $a_{n+1} = ra_n + b$ where $r = 3$ and $b = -2$. If $r$ is increased, what is the effect on the equilibrium value?

(a) The equilibrium value increases.

(b) The equilibrium value decreases.

(c) The equilibrium value doesn't change.

(d) The effect depends on how much $r$ is increased by.

**Testing Analytical Solutions**

69. We want to test the solution $a_n = 5 \cdot 4^n$ in the difference equation $a_{n+1} = 4a_n$. What equation results from substituting the solution into the difference equation?

(a) $5 \cdot 4^n = 4 \cdot 5 \cdot 4^n$

(b) $5 \cdot 4^{n+1} = 4 \cdot 5 \cdot 4^n$

(c) $5 \cdot 4^n(n+1) = 4 \cdot 5 \cdot 4^n \cdot n$

(d) $20^{n+1} = 4 \cdot 20^n$

(e) None of the above

70. We want to test the solution $a_n = 6n + C$ in the difference equation $a_{n+1} = a_n + 6$. What equation results from substituting the solution into the difference equation?

(a) $6n + 1 + C = 6n + C + 6$

(b) $6(n + 1) + C = 6n + C + 6$

(c) $6(n + 1) + C(n + 1) = 6n + Cn + 6$

(d) $a_n + 6 = 6(a_n + 6) + C$

(e) None of the above

71. After substituting a proposed solution into the difference equation, we arrive at

$$2^{n+1} \cdot 8 - 5 = 2(2^n \cdot 8 - 5) + 5.$$

Do we have a solution to the difference equation?

(a) Yes

(b) No

72. After substituting a proposed solution into the difference equation, we arrive at

$$3^{n+1} \cdot 4 - 7 = 2(3^n \cdot 4 - 7) + 7.$$

Do we have a solution to the difference equation?

(a) Yes

(b) No

73. After substituting a proposed solution into the difference equation, we arrive at

$$6(\sqrt{5})^{n+2} = 5 \cdot 6(\sqrt{5})^n.$$

Do we have a solution to the difference equation?

(a) Yes
(b) No

74. **True or False** $b_n = 2^n \times 5$ is a solution to $b_{n+1} = 2b_n$ with $b_0 = 5$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

75. **True or False** $a_n = 3^n \cdot 2$ is a solution to $a_{n+1} = 4a_n$ with $a_0 = 2$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

76. **True or False** $a_n = 3^n C$ is a solution to $a_{n+1} = 3a_n$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

77. **True or False** $a_n = 2n$ is a solution to $a_{n+1} = a_n - 5$ with $a_0 = 0$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

78. **True or False** $a_n = 6n + 3$ is a solution to $a_{n+1} = a_n + 6$ with $a_0 = 3$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

79. **True or False** $a_n = 3^n C + 2$ is a solution to $a_{n+1} = 3a_n - 4$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

80. **True or False** $a_n = 3^n \cdot 5 + 2$ is a solution to $a_{n+1} = 3a_n - 4$ with $a_0 = 2$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

81. **True or False** $a_n = 3^n \times 4 + 5$ is a solution to $a_{n+1} = 3a_n + 5$ with $a_0 = 9$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

82. A numerical solution to a difference equation is

(a) another difference equation.
(b) a number.

(c) a sequence.
(d) a function that describes a sequence.
(e) None of the above

83. An analytic solution to a difference equation is

(a) another difference equation.
(b) a number.
(c) a sequence.
(d) a function that describes a sequence.
(e) None of the above

**Classifying Difference Equations**

84. The equation $a_{n+1} = na_n + a_n a_{n-1} + n^3$ is nonlinear. Which term makes it nonlinear?

(a) $na_n$
(b) $a_n a_{n-1}$
(c) $n^3$
(d) All of the above

85. What is the order of the difference equation $a_{n+5} = 3a_{n+1} + a_{n+2} + 5$?

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5
(f) 6

86. How many initial conditions are needed to fully specify the sequence described by a 4th order difference equation?

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5

87. Which of the following difference equations is nonhomogeneous?

(a) $a_{n+1} = 3a_n$
(b) $b_{n+2} = 5b_n b_{n-1}$
(c) $c_{n+1} = 4c_n + 5n$
(d) $d_{n+1} = 3d_n + d_{n-2}$
(e) More than one of the above is nonhomogeneous.
(f) None of the above are nonhomogeneous.

**Solving Difference Equations**

88. The solution to $a_{n+1} = 5.2a_n$ with $a_0 = 6$ is

(a) $a_n = 6^n(5.2)$

(b) $a_n = 6^n C$

(c) $a_n = 5.2^n(6)$

(d) $a_n = 5.2n + 6$

(e) $a_n = 5.2^n(3)$

89. The solution to $a_{n+1} = 3a_n$ with $a_2 = 5$ is

(a) $a_n = 3^n(5)$

(b) $a_n = 5^n(3)$

(c) $a_n = 3^n\left(\frac{5}{9}\right)$

(d) None of the above

90. The solution to $c_{n+1} = c_n - 7$ with $c_0 = 2$ is

(a) $c_n = 2 - 7n$

(b) $c_n = 2 + 7n$

(c) $c_n = 7 - 2n$

(d) $c_n = 7 + 2n$

91. The solution to $a_{n+3} = a_{n+2} + 9$ with $a_3 = 5$ is

(a) $a_n = 9n + 5$

(b) $a_n = 5n + 9$

(c) $a_n = 9n - 22$

(d) We don't know how to solve difference equations that are in this form.

92. Which best describes the long-term behavior of the solution $a_n = 1.13^n(5)$?

(a) This solution will increase to infinity.

(b) This solution will increase, approaching equilibrium at infinity.

(c) This solution will decrease, approaching equilibrium at zero.

(d) None of the above

93. Which best describes the equilibrium of the difference equation with solution $a_n = 0.87^n(2.45)$?

(a) There is a stable equilibrium at zero.

(b) There is an unstable equilibrium at zero.

(c) There is a stable equilibrium at 2.45.

(d) There is an unstable equilibrium at 2.45.

(e) There is no equilibrium.

(f) We can't answer this without knowing the difference equation.

94. Which best describes the equilibrium of the difference equation with solution $a_n = 1.54^n C$ ?

(a) There is a stable equilibrium at zero.
(b) There is an unstable equilibrium at zero.
(c) There is a stable equilibrium somewhere other than zero.
(d) There is an unstable equilibrium somewhere other than zero.
(e) There is no equilibrium.
(f) We can't answer this without knowing $C$.

95. Which best describes the equilibrium of the difference equation with solution $a_n = 3n + 5$?

(a) There is a stable equilibrium at zero.
(b) There is an unstable equilibrium at zero.
(c) There is a stable equilibrium at 5.
(d) There is an unstable equilibrium at 5.
(e) There is no equilibrium.
(f) We can't answer this without knowing the difference equation.

96. Which best describes the long-term behavior of the solution $a_n = (-2)^n(12)$?

(a) As $n$ gets large, the values of $a_n$ grow without bound.

(b) As $n$ gets large, the values of $a_n$ decrease without bound.

(c) As $n$ gets large, the values of $a_n$ oscillate, getting farther and farther from zero.

(d) As $n$ gets large, the values of $a_n$ oscillate, getting closer and closer to zero.

(e) None of the above

97. What difference equation has the solution $a_n = 0.3^n(0.7)$?

(a) $a_{n+1} = 0.7a_n$

(b) $a_{n+1} = 0.3a_n$

(c) $a_{n+1} = 0.7a_n + 0.3$

(d) $a_{n+1} = 0.3a_n + 0.7$

(e) None of the above

98. What discrete dynamical system has the solution $a_n = 1.5^n(2)$?

(a) $a_{n+1} = 2a_n$ with $a_0 = 1.5$

(b) $a_{n+1} = 1.5a_n$ with $a_0 = 2$

(c) $a_{n+1} = 2a_n + 1.5$ with $a_0$ unknown

(d) $a_{n+1} = 1.5a_n + 2$ with $a_0$ unknown

99. What discrete dynamical system has the solution $a_n = 5 - 4n$?

(a) $a_{n+1} = a_n - 4$ with $a_0 = 5$

(b) $a_{n+1} = a_n + 5$ with $a_0 = -4$

(c) $a_{n+1} = 5a_n$ with $a_0 = -4$

(d) $a_{n+1} = -4a_n$ with $a_0 = 5$

100. Suppose $a_n = 3 \cdot 2^n$ solves a certain linear homogeneous difference equation. What other function also must solve the difference equation?

(a) $a_n = 3 \cdot 2^n + 1$

(b) $b_n = \frac{4}{5} \cdot 3 \cdot 2^n$

(c) $c_n = 2^n - 3^n$

(d) $d_n = 3$

(e) All of the above

(f) None of the above

**Solutions to Nonhomogeneous DEs with a Constant Term**

101. **True or False** The function $a_n = r^n C$ solves the difference equation $a_{n+1} = ra_n + b$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

102. The solution to $a_{n+1} = 4a_n + 3$ with $a_0 = 5$ is

(a) $a_n = 4^n(2) + 3$
(b) $a_n = 4^n(5) - 1$
(c) $a_n = 4^n(5) + 3$
(d) $a_n = 4^n(6) - 1$
(e) None of the above

103. The solution to $a_{n+1} = 2a_n - 5$ with $a_3 = 9$ is

(a) $a_n = 2^n(4) + 5$
(b) $a_n = 2^n(0.5) + 5$
(c) $a_n = 2^n(9) + 5$
(d) $a_n = 2^n(1.75) - 5$

(e) $a_n = 2^n(14) - 5$
(f) None of the above

104. The solution to $a_{n+1} = 4a_n + 3$ with $a_0 = -1$ is

(a) $a_n = -1$
(b) $a_n = 4^n(-1) - 1$
(c) $a_n = 4^n(-2) - 1$
(d) $a_n = 4^n(2) - 1$
(e) None of the above

105. Which of the following is the solution to $a_{n+1} = a_n - 8$ with $a_0 = 5$?

(a) $a_n = 5 - 8n$
(b) $a_n = -8 + 5n$
(c) $a_n = 1^n(5) - 8$
(d) No solution can be found because there is no equilibrium value.

106. A solution to a difference equation is $a_n = 2^n(0.8) + 52$. What is the equilibrium value and is it stable or not?

(a) $E = 0.8$ and it is stable
(b) $E = 0.8$ and it is unstable

(c) $E = 52$ and it is stable

(d) $E = 52$ and it is unstable

(e) Not enough information

107. Which of the following describes the long-term behavior of the solution $a_n = 2.7^n(4) - 3.8$?

   (a) It is unstable.

   (b) The solution will increase infinitely.

   (c) The solution will decrease infinitely.

   (d) The solution will converge to an equilibrium value of -3.8.

108. Discuss the long-term behavior of the solution $a_n = 0.7^n C + 2$ with $a_0 = 1$.

   (a) This solution will increase, converging to the equilibrium value of 2.

   (b) This solution will decrease forever.

   (c) This solution will increase forever.

   (d) None of the above

109. A difference equation was used to model monthly payments on a credit card with an outstanding balance. The solution to the difference equation is $a_n = 1.015^n(-1000$ 4000.

Which of the following is a true statement?

(a) The monthly payment is \$4000.
(b) The starting balance is \$1000.
(c) If the monthly payment is \$4000, the balance on the credit card will not change.
(d) All of the above
(e) None of the above

110. A difference equation was used to model monthly payments on a credit card with an outstanding balance. The solution to the difference equation is $a_n = 1.015^n(-1000)$ 4000.

Which of the following is a true statement?

(a) The starting balance is \$3000.
(b) The interest rate is 1.5% per month.
(c) With a starting balance of \$4000 and the same monthly payment, the balance on the credit card will not change.
(d) All of the above
(e) None of the above

**Solutions to Nonhomogeneous DEs with an Exponential Term**

111. The difference equation $a_{n+1} = 1.04a_n + 1.05^n(1000)$ models the yearly balances in a savings account with annual deposits being made. Which of the following statements is true?

(a) Each year the deposit increases by 4%.
(b) Each year the deposit increases by 5%.
(c) Each year the deposit increases by \$1000.
(d) Each year the deposit increases $1.05 \times 1000 =$ \$1050.
(e) None of the above

112. A solution to a difference equation is $a_n = \frac{17}{2} \cdot 3^n + \frac{5}{2} \cdot 2^n$. What was the initial condition, $a_0$?

(a) $\frac{17}{2}$
(b) $\frac{5}{2}$
(c) 2
(d) 11

113. What is the best conjecture to use for the homogeneous solution to $a_{n+1} = 2a_n + 3 \cdot 5^n$?

(a) $a_n = 2^n$
(b) $a_n = 2^n C$
(c) $a_n = 5^n$
(d) $a_n = 5^n C$
(e) None of the above

114. What is the best conjecture to use for the nonhomogeneous solution to $a_{n+1} = 2a_n + 3 \cdot 5^n$?

(a) $a_n = 2^n C$
(b) $a_n = 5^n C$
(c) $a_n = 3 \cdot 5^n$
(d) $a_n = 3 \cdot 5^n C$
(e) $a_n = 5^n C_1 + C_2$
(f) None of the above

115. We have $a_{n+1} = 3a_n - 5 \cdot 2^n$ and we have formed a conjecture of $a_n = 2^n C$ for the particular solution to the nonhomogeneous part. When we substitute our conjectured solution into the difference equation, what is the result?

(a) $2^n C(n+1) = 2^n C - 5 \cdot 2^n$
(b) $2^n C + 1 = 3 \cdot 2^n C - 5 \cdot 2^n$
(c) $2^{n+1} C = 2^n C - 5 \cdot 2^n$

(d) $2^{n+1}C = 3 \cdot 2^n C - 5 \cdot 2^n$

(e) None of the above

116. We are trying to find a solution to $a_{n+1} = 9a_n + 5 \cdot 4^n$ where $a_0 = 1.5$. We have conjectured $a_n = 4^n C$ as the particular solution to the nonhomogeneous part and substituted our conjecture into the difference equation to obtain $4^{n+1}C = 9 \cdot 4^n C + 5 \cdot 4^n$. How do we proceed to find $C$?

    (a) Use the initial condition.

    (b) Divide both sides of the equation by $C$.

    (c) Divide both sides of the equation by 4.

    (d) Divide both sides of the equation by $n$.

    (e) Divide both sides of the equation by $4^n$.

    (f) We don't have enough information to solve for $C$.

117. We are trying to find a solution to $a_{n+1} = 9a_n + 5 \cdot 4^n$ where $a_0 = 1.5$. We have conjectured $a_n = 4^n C$ as the particular solution to the nonhomogeneous part and substituted our conjecture into the difference equation to obtain $4^{n+1}C = 9 \cdot 4^n C + 5 \cdot 4^n$. What is the value of $C$?

    (a) $C = -1$

    (b) $C = 1$

(c) $C = -5/8$
(d) $C = 5/3$
(e) None of the above

118. For which of the following difference equations will the nonhomogeneous conjecture need to be modified by multiplying by $n$?

(a) $a_{n+1} = 2a_n + 2 \cdot 3^n$
(b) $a_{n+1} = 3a_n + 2 \cdot 3^n$
(c) $a_{n+1} = a_n + 4^n$
(d) All of the above
(e) None of the above

119. Which of the following is not a solution to $a_{n+1} = 3a_n + 5 \cdot 4^n$?

(a) $a_n = 5 \cdot 4^n$
(b) $a_n = 6 \cdot 3^n$
(c) $a_n = 8 \cdot 3^n + 5 \cdot 4^n$
(d) $a_n = 15 \cdot 3^n + 5 \cdot 4^n$
(e) All are solutions

## Solutions to Nonhomogeneous DEs with a Polynomial Term

120. Upon graduation from college you land a job with a starting salary of \$35,000. You are told that as long as your peformance is up to par, you can expect a 3% raise each year. Additionally, at the end of your first year you will receive a bonus of \$100, and at the end of each year after that you will receive a bonus equal to \$100 times the number of years you have completed. On the first day of your job, you open a bank account with \$200. This account will earn interest at a rate of 5% per year, and you decide that each year you will deposit your bonus into the account. If $a_n$ represents the amount of money in your account at the end of $n$ years, which of the following difference equations models your account balances?

    (a) $a_{n+1} = 1.03a_n + 100 + 35,000$

    (b) $a_{n+1} = 1.05a_n + 100$

    (c) $a_{n+1} = 1.05a_n + 100(n + 1)$

    (d) $a_{n+1} = 1.05a_n + 1.05(100)n$

    (e) $a_{n+1} = 1.03a_n + 1.05(100) + 200$

    (f) None of the above

121. What is the best conjecture to use for the nonhomogeneous solution to $a_{n+1} = 3a_n + 5n$?

(a) $a_n = C_1 n$
(b) $a_n = C_1 n + C_0$
(c) $a_n = C_2 n^2 + C_1 n + C_0$
(d) None of the above

122. We are trying to solve $a_{n+1} = 2a_n + 7n - 5$. For a particular solution to the nonhomogeneous part, we conjecture $a_n = C_1 n + C_0$. When we substitute this into the difference equation, what is the result?

(a) $C_1 n + C_0 = 2(C_1 n + C_0) + 7(C_1 n + C_0) - 5$
(b) $C_1(n+1) + C_0 = 2(C_1 n + C_0) + 7(C_1 n + C_0) - 5$
(c) $C_1(n+1) + C_0 = 2(C_1 n + C_0) + 7(C_1 n + C_0) - 5$
(d) $C_1(n+1) + C_0 = 2(C_1 n + C_0) + 7n - 5$
(e) None of the above

123. We are trying to solve $a_{n+1} = 3a_n + 5n$ where $a_0 = 10$. We have conjectured $a_n = C_1 n + C_0$ as a solution to the nonhomogeneous equation, and after substituting we have $C_1(n+1) + C_0 = 3(C_1 n + C_0) + 5n$. What are the values of $C_1$ and $C_0$?

(a) $C_1 = -5/2$ and $C_0 = -5/4$

(b) $C_1 = 5$ and $C_0 = 5/2$
(c) $C_1 = 5$ and $C_0 = 10$
(d) $C_1 = 3$ and $C_0 = 5$
(e) Not enough information is given.

124. For which of the following difference equations will the nonhomogeneous conjecture need to be modified by multiplying by $n$?

(a) $a_{n+1} = 3a_n + 3n + 4$
(b) $a_{n+1} = 5a_n + n + 5$
(c) $a_{n+1} = a_n + 7n^2 + 3n$
(d) All of the above
(e) None of the above

125. If $a_n = 2^n C$ is the solution to the homogeneous part of a difference equation, which of the following could *not* be a particular solution to the nonhomogeneous equation?

(a) $a_n = n \cdot 2^n$
(b) $b_n = C_1 n^2 + C_2 n + C_3$
(c) $c_n = 5 \cdot 2^n$
(d) $d_n = 6$
(e) All of the above answer the question correctly.

(f) None of the above answer the question correctly.

126. Suppose we have a nonhomogeneous difference equation that we solve by finding the general solution to the homogeneous part and a particular solution to the nonhomogeneous equation. If $a_n = (0.8)^n C$ is the solution to the homogeneous part, and $b_n = 3n^2 - 4$ is the particular solution to the nonhomogeneous equation, then which of the following is a solution to the original difference equation?

(a) $a_n = (0.8)^n C$
(b) $b_n = 3n^2 - 4$
(c) $c_n = (0.8)^n C + 2(3n^2 - 4)$
(d) All of the above
(e) None of the above

127. We have the difference equation $a_{n+1} = 2a_n + 3n + 4$. What is the solution to the associated homogeneous equation?

(a) $a_n = C_0 2^n$
(b) $a_n = C_0 2^n + C_1 n + C_2$
(c) $a_n = C_0 2^n + C_2$
(d) $a_n = C_0 3^n$

(e) None of the above

128. Is the general term that was the correct answer to the previous question a solution to the difference equation $a_{n+1} = 2a_n + 3n + 4$?

(a) Yes
(b) No
(c) It cannot be determined.

129. $a_{n+1} = 2a_n + 3n + 4$. What is the particular solution to the nonhomogenous equation?

(a) $a_n = 2n - 6$
(b) $a_n = -3n - 4$
(c) $a_n = 2n - 2$
(d) $a_n = -3n - 7$
(e) It cannot be determined.

130. Is the particular solution that was the correct answer to the previous question a solution to the difference equation $a_{n+1} = 2a_n + 3n + 4$?

(a) Yes
(b) No
(c) It cannot be determined.

131. For what initial condition is $a_n = -3n - 7$ a solution to the difference equation $a_{n+1} = 2a_n + 3n + 4$?

(a) For all initial conditions, because we found this without using an initial condition.

(b) For $a_0 = -7$.

(c) For $a_0 = -10$.

(d) For $a_0 = -3$.

(e) Answers b, c, and d, are all correct.

(f) This corresponds to no initial conditions.

**Systems of Difference Equations**

132. Two cell phone service providers, Alpha and Beta, are constantly competing for the largest market share. Each month 5% of Alpha customers switch their service to Beta, and each month 7% of Beta customers switch their service to Alpha. The total number of customers served by the two companies stays fixed. Which system of difference equations models the monthly changes in customer base for each provider?

(a)

$$\begin{aligned} A_{n+1} &= 0.07B_n \\ B_{n+1} &= 0.05A_n \end{aligned}$$

(b)

$$\begin{aligned} A_{n+1} &= -0.05A_n \\ B_{n+1} &= -0.07B_n \end{aligned}$$

(c)

$$\begin{aligned} A_{n+1} &= A_n + 0.07B_n \\ B_{n+1} &= 0.05A_n + B_n \end{aligned}$$

(d)

$$\begin{aligned} A_{n+1} &= A_n - 0.05A_n + 0.07B_n \\ B_{n+1} &= 0.05A_n + B_n - 0.07B_n \end{aligned}$$

(e) None of the above

133.

$$\begin{aligned} A_{n+1} &= A_n - 0.2A_n + 0.3B_n \\ B_{n+1} &= B_n - 0.3B_n \end{aligned}$$

These difference equations allow us to predict how the populations of two towns, A and B, change each year. Which of the following would be a true statement?

(a) All of the people who leave town A move to town B.

(b) All of the people who leave town B move to town A.

(c) None of the people who leave town B move to town A.

(d) Some of the people who leave town A move to town B.

134.

$$\begin{aligned} A_{n+1} &= A_n - 0.2A_n + 0.3B_n \\ B_{n+1} &= B_n - 0.3B_n \end{aligned}$$

These difference equations allow us to predict how the populations of two towns, A and B, change each year. Which of the following would be a true statement?

(a) 30% of the people in A move to B each year.

(b) 20% of the people in B move to A each year.

(c) Only 70% of the people in B stay in B each year.

(d) Only 20% of the people in A stay in A each year.

135. Scrabble or Boggle? Both are great word games. Each month, 10% of Scrabble players stop playing Scrabble and switch to Boggle, while 12% of Boggle players stop playing Boggle and switch to Scrabble. In addition, 1587 new people take up the game of Scrabble each month, while 1298 new people start playing Boggle each month. Each month 2% of players of

each game stop playing word games entirely. Which system of difference equations models the month-to-month changes in the numbers of Scrabble and Boggle players?

(a)

$$\begin{aligned} S_{n+1} &= -0.10S_n - 0.02S_n + 1587 \\ B_{n+1} &= -0.12B_n - 0.02B_n + 1298 \end{aligned}$$

(b)

$$\begin{aligned} S_{n+1} &= S_n - 0.10S_n - 0.02S_n + 1298 \\ B_{n+1} &= B_n - 0.12B_n - 0.02B_n + 1587 \end{aligned}$$

(c)

$$\begin{aligned} S_{n+1} &= S_n - 0.02S_n + 0.12B_n \\ B_{n+1} &= 0.10S_n + B_n - 0.02B_n \end{aligned}$$

(d)

$$\begin{aligned} S_{n+1} &= S_n - 0.10S_n - 0.02S_n + 0.12B_n + 1587 \\ B_{n+1} &= 0.10S_n + B_n - 0.12B_n - 0.02B_n + 1298 \end{aligned}$$

(e) None of the above

136. Mr. Jones has a checking account and a savings account. He earns 1.2% annual interest (compounded monthly) on his checking account and 3.6% interest

(compounded monthly) on his savings account. Each month Mr. Jones transfers 10% of the balance of his checking account to his savings account, but he does not make any other deposits or withdrawals from his savings account. He has a monthly paycheck deposit of $2400 into his checking account, and he writes checks totalling $2200 each month. Which system of difference equations models the monthly balances in Mr. Jones' accounts?

(a)

$$\begin{aligned} c_{n+1} &= c_n + 0.012c_n - 0.10c_n \\ s_{n+1} &= s_n + 0.036s_n \end{aligned}$$

(b)

$$\begin{aligned} c_{n+1} &= c_n + 0.012c_n - 0.10c_n + 200 \\ s_{n+1} &= s_n + 0.036s_n \end{aligned}$$

(c)

$$\begin{aligned} c_{n+1} &= c_n + 0.001c_n - 0.10c_n + 200 \\ s_{n+1} &= s_n + 0.003s_n + 0.10c_n \end{aligned}$$

(d)

$$\begin{aligned} c_{n+1} &= c_n + 0.001c_n - 0.10c_n + 200 \\ s_{n+1} &= s_n + 0.003s_n + 0.10s_n \end{aligned}$$

(e) None of the above

137.

$$\begin{aligned} E_{n+1} &= 0.75E_n + 0.10F_n + 500 \\ F_{n+1} &= 0.25E_n + 0.80F_n \end{aligned}$$

These difference equations allow us to predict the number of customers of companies $E$ and $F$ each year. Which of the following would be a true statement?

(a) 20% of $F$'s customers switch to $E$ each year.
(b) 500 new customers sign up with $F$ each year.
(c) 10% of $E$'s customers switch to $F$ each year.
(d) 10% of $F$'s customers switch to $E$ each year.

138.

$$\begin{aligned} E_{n+1} &= 0.75E_n + 0.10F_n + 500 \\ F_{n+1} &= 0.25E_n + 0.80F_n \end{aligned}$$

These difference equations allow us to predict the number of customers of companies $E$ and $F$ each year. Which of the following would be a true statement?

(a) All the customers that $F$ loses switch to $E$.
(b) All the customers that $E$ loses switch to $F$.
(c) None of the customers that $F$ loses switch to $E$.
(d) None of the customers that $E$ loses switch to $F$.

139. We are given the following system of difference equations:

$$
\begin{aligned}
a_{n+1} &= 2a_n + 3b_n + 2 \\
b_{n+1} &= a_n + 2b_n
\end{aligned}
$$

with $a_0 = 5$ and $b_0 = 1$. What are $a_1$ and $b_1$?

(a) $a_1 = 27$ and $b_1 = 3$
(b) $a_1 = 15$ and $b_1 = 7$
(c) $a_1 = 19$ and $b_1 = 11$
(d) $a_1 = 16$ and $b_1 = 7$
(e) $a_1 = 15$ and $b_1 = 17$
(f) None of the above

140. We are given the following system of difference equations:

$$
\begin{aligned}
a_{n+1} &= .4a_n + .6b_n \\
b_{n+1} &= .6a_n + .4b_n
\end{aligned}
$$

with $a_0 = 10$ and $b_0 = 10$. What are $a_5$ and $b_5$?

(a) $a_5 = 10$ and $b_5 = 10$
(b) $a_5 = 4$ and $b_5 = 6$
(c) $a_5 = 6$ and $b_5 = 4$
(d) $a_5 = 1$ and $b_5 = 1$

(e) None of the above

141.

$$\begin{aligned} C_{n+1} &= 1.15C_n - (4 \times 10^{-5})C_nD_n \\ D_{n+1} &= 0.95D_n + (3 \times 10^{-5})C_nD_n \end{aligned}$$

These difference equations allow us to predict how two animal populations, C and D, change each year. Which of the following would be a true statement?

(a) If $C_n = 0$ then $D$ will grow by 95% each year.
(b) If $C_n = 0$ then $D$ will grow by 15% each year.
(c) If $D_n = 0$ then $C$ will grow by 95% each year.
(d) If $D_n = 0$ then $C$ will grow by 15% each year.

142.

$$\begin{aligned} C_{n+1} &= 1.15C_n - (4 \times 10^{-5})C_nD_n \\ D_{n+1} &= 0.95D_n + (3 \times 10^{-5})C_nD_n \end{aligned}$$

These difference equations allow us to predict how two animal populations, $C$ and $D$, change each year. Which of the following would be a true statement?

(a) Larger populations of $C$ cause the population of $D$ to grow.

(b) Larger populations of $C$ cause the population of $D$ to shrink.
(c) Larger populations of $D$ cause the population of $C$ to grow.
(d) None of the above

143.

$$\begin{aligned} C_{n+1} &= 1.15C_n - (4 \times 10^{-5})C_nD_n \\ D_{n+1} &= 0.95D_n + (3 \times 10^{-5})C_nD_n \end{aligned}$$

These difference equations allow us to predict how two animal populations, $C$ and $D$, change each year. Which of the following would be a true statement?

(a) We could say that $C$ is a predator and $D$ is prey.
(b) We could say that $D$ is a predator and $C$ is prey.
(c) We could say that $C$ and $D$ are competitors.
(d) We could say that $C$ and $D$ benefit from each other.
(e) None of the above

144.

$$\begin{aligned} C_{n+1} &= 0.85C_n - (4 \times 10^{-5})C_nD_n \\ D_{n+1} &= 0.95D_n + (3 \times 10^{-5})C_nD_n \end{aligned}$$

These difference equations allow us to predict how two animal populations, $C$ and $D$, change each year.

Which of the following would be a true statement?

(a) We could say that $C$ eats $D$.
(b) We could say that $D$ eats $C$.
(c) We could say that $C$ and $D$ are competitors.
(d) We could say that $C$ and $D$ benefit from each other.
(e) None of the above

145.

$$\begin{aligned} C_{n+1} &= 0.85C_n + (4 \times 10^{-5})C_nD_n \\ D_{n+1} &= 0.95D_n + (3 \times 10^{-5})C_nD_n \end{aligned}$$

These difference equations allow us to predict how two species, $C$ and $D$, change each year.

Which of the following would be a true statement?

(a) We could say that $C$ is a predator and $D$ is prey.
(b) We could say that $D$ is a predator and $C$ is prey.
(c) We could say that $C$ and $D$ are competitors.
(d) We could say that $C$ and $D$ benefit from each other.
(e) None of the above

**Solving Homogeneous Systems of Difference Equations**

146. If we are told that the general solution to a system of difference equations is

$$A_n = \begin{bmatrix} a_n \\ b_n \end{bmatrix} = c_1 \cdot (0.9)^n \begin{bmatrix} 1 \\ \frac{7}{8} \end{bmatrix} + c_2(-0.5)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix},$$

then which is an equivalent form of the solution?

(a) $a_n = c_1(0.9)^n + \frac{7}{8}c_1(0.9)^n$ and $b_n = -c_2(-0.5)^n + c_2(-0.5)^n$

(b) $a_n = c_1(0.9)^n - c_2(-0.5)^n$ and $b_n = \frac{7}{8}c_1(0.9)^n + c_2(-0.5)^n$

(c) $a_n = c_1(0.9)^n - c_1(-0.5)^n$ and $b_n = \frac{7}{8}c_2(0.9)^n + c_2(-0.5)^n$

(d) All of the above

(e) None of the above

147. The solution to a system of difference equations is

$$A_n = \begin{bmatrix} a_n \\ b_n \end{bmatrix} = c_1 \cdot (0.9)^n \begin{bmatrix} 1 \\ \frac{7}{8} \end{bmatrix} + c_2(-0.5)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

Which of the following is a true statement?

(a) This system has an unstable equilibrium.

(b) In the long-run, $b$ will hold 7/8 of the population.

(c) The equilibrium value of this system is $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

(d) All of the above

(e) None of the above

148. If we wish to solve this system,

$$\begin{aligned} a_{n+1} &= a_n - 0.2a_n + 0.3b_n \\ b_{n+1} &= b_n - 0.3b_n \end{aligned}$$

which matrix do we need to find eigenvalues and eigenvectors for?

(a)

$$\begin{bmatrix} 1 & -0.2 & 0.3 \\ 1 & -0.3 & 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & -0.2 & 0.3 \\ 0 & 1 & -0.3 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 0.8 & 0.3 \\ 0.7 & 0 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 0.8 & 0.3 \\ 0 & 0.7 \end{bmatrix}$$

(e) None of the above

149. In solving the system

$$\begin{aligned} a_{n+1} &= a_n - 0.2a_n + 0.3b_n \\ b_{n+1} &= b_n - 0.3b_n \end{aligned}$$

we find that the eigenvalues of the coefficient matrix are 0.8 and 0.7 with corresponding eigenvectors of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$. What is the solution to this system?

(a) $A_n = c_1(0.8)^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2(0.7)^n \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

(b) $A_n = c_1(0.8)^n \begin{bmatrix} -3 \\ 1 \end{bmatrix} + c_2(0.7)^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(c) $A_n = c_1(0.8) \begin{bmatrix} 1 \\ 0 \end{bmatrix}^n + c_2(0.7) \begin{bmatrix} -3 \\ 1 \end{bmatrix}^n$

(d) $A_n = c_1(0.8) \begin{bmatrix} -3 \\ 1 \end{bmatrix}^n + c_2(0.7) \begin{bmatrix} 1 \\ 0 \end{bmatrix}^n$

(e) None of the above

150. The solution to a system of difference equations is $A_n = c_1(0.8)^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2(0.7)^n \begin{bmatrix} -3 \\ 1 \end{bmatrix}$. If $A_0 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, what are $c_1$ and $c_2$?

(a) $c_1 = 2$ and $c_2 = 3$

(b) $c_1 = 55/4$ and $c_2 = 110/8$

(c) $c_1 = 11$ and $c_2 = 3$

(d) $c_1 = -7$ and $c_2 = 3$

(e) None of the above

151. The following system of difference equations allows us to predict how the populations of two towns, A and B, change each year.

$$\begin{aligned} a_{n+1} &= a_n - 0.2a_n + 0.3b_n \\ b_{n+1} &= b_n - 0.3b_n \end{aligned}$$

The solution to this system is

$$A_n = c_1(0.8)^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2(0.7)^n \begin{bmatrix} -3 \\ 1 \end{bmatrix}.$$

Which of the following is a true statement?

(a) This system has a stable equilibrium.

(b) In the long-run, both of these towns will be ghost towns.

(c) If there are initially 10,000 people in town B, then $b_{10} = 282$ people.

(d) All of the above

(e) None of the above

152. If $A_n = (2)^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \left(\frac{1}{3}\right)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is a solution to the system of difference equations $A_{n+1} = RA_n$, which of the following is also a solution?

(a) $(2^n) \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(b) $3 \cdot (2)^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 4 \cdot \left(\frac{1}{3}\right)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(c) $8 \cdot \left(\frac{1}{3}\right)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(d) All of the above

(e) None of the above

153. **True or False** If either column of the coefficient matrix of a system of homogeneous difference equations sums to a value greater than one, then the system has an unstable equilibrium.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

154. **True or False** When solving a system of two homogeneous difference equations, if one eigenvalue is

greater than one and one is between 0 and 1, then one population will grow without bound while the other declines.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

**Solving Nonhomogeneous Systems of Difference Equations**

155. Suppose we are solving a system of two difference equations. For which pair of eigenvalues would the system have a stable equilibrium?

(a) $\lambda_1 = 0.8$ and $\lambda_2 = 0.7$
(b) $\lambda_1 = 1$ and $\lambda_2 = 0.8$
(c) $\lambda_1 = 1$ and $\lambda_2 = 2.5$
(d) All of the above
(e) None of the above
(f) More than one of the above

156. If we wish to solve this system,

$$\begin{aligned} a_{n+1} &= 0.6a_n + 0.3b_n + 3 \\ b_{n+1} &= 0.7b_n + 0.2a_n + 1 \end{aligned}$$

what matrix do we need to find eigenvalues and eigenvectors for?

(a)
$$\begin{bmatrix} 0.6 & 0.3 & 3 \\ 0.7 & 0.2 & 1 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 0.6 & 0.3 & 3 \\ 0.2 & 0.7 & 1 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 0.6 & 0.3 \\ 0.7 & 0.2 \end{bmatrix}$$

(d)
$$\begin{bmatrix} 0.6 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$$

(e) None of the above

157. What is the equilibrium vector for this system when $a_0 = 5$ and $b_0 = 3$?

$$\begin{aligned} a_{n+1} &= 0.6a_n + 0.3b_n + 3 \\ b_{n+1} &= 0.7b_n + 0.2a_n + 1 \end{aligned}$$

(a) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} 20 \\ 50/3 \end{bmatrix}$

(c) $\begin{bmatrix} 40 \\ 110/3 \end{bmatrix}$

(d) $\begin{bmatrix} 24.5 \\ 22.7 \end{bmatrix}$

158. You are solving a system of difference equations, and you find that the eigenvalues of the coefficient matrix are 0.93723 and 0.36277 with corresponding eigenvectors of $\begin{bmatrix} 0.80818 \\ 0.58893 \end{bmatrix}$ and $\begin{bmatrix} -0.97483 \\ 0.22296 \end{bmatrix}$. If the initial conditions are $\begin{bmatrix} a_0 \\ b_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$ and the equilibrium vector has been determined to be $\begin{bmatrix} 35 \\ 27.5 \end{bmatrix}$, what is the particular solution for this system?

(a) $\begin{bmatrix} a_n \\ b_n \end{bmatrix} = (0.93723)^n \begin{bmatrix} 0.80818 \\ 0.58893 \end{bmatrix} + (0.36277)^n \begin{bmatrix} -0.97483 \\ 0.22296 \end{bmatrix}$

(b) $\begin{bmatrix} a_n \\ b_n \end{bmatrix} = 35(0.93723)^n \begin{bmatrix} 0.80818 \\ 0.58893 \end{bmatrix} + 27.5(0.36277)^n \begin{bmatrix} -0. \\ 0.2 \end{bmatrix}$

(c) $\begin{bmatrix} a_n \\ b_n \end{bmatrix} = 1(0.93723)^n \begin{bmatrix} 0.80818 \\ 0.58893 \end{bmatrix} + 7(0.36277)^n \begin{bmatrix} -0.9748 \\ 0.2229 \end{bmatrix}$

$$\begin{bmatrix} 35 \\ 27.5 \end{bmatrix}$$

(d) $\begin{bmatrix} a_n \\ b_n \end{bmatrix} = -36.543(0.93723)^n \begin{bmatrix} 0.80818 \\ 0.58893 \end{bmatrix} + 4.582(0.36277)^{n}$

$$\begin{bmatrix} 35 \\ 27.5 \end{bmatrix}$$

(e) None of the above