MathQuest: Differential Equations

What is a Differential Equation?

1. Which of the following is not a differential equation?

   (a) $y' = 3y$
   (b) $2x^2y + y^2 = 6$
   (c) $tx\frac{dx}{dt} = 2$
   (d) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 7y + 8x = 0$
   (e) All are differential equations.

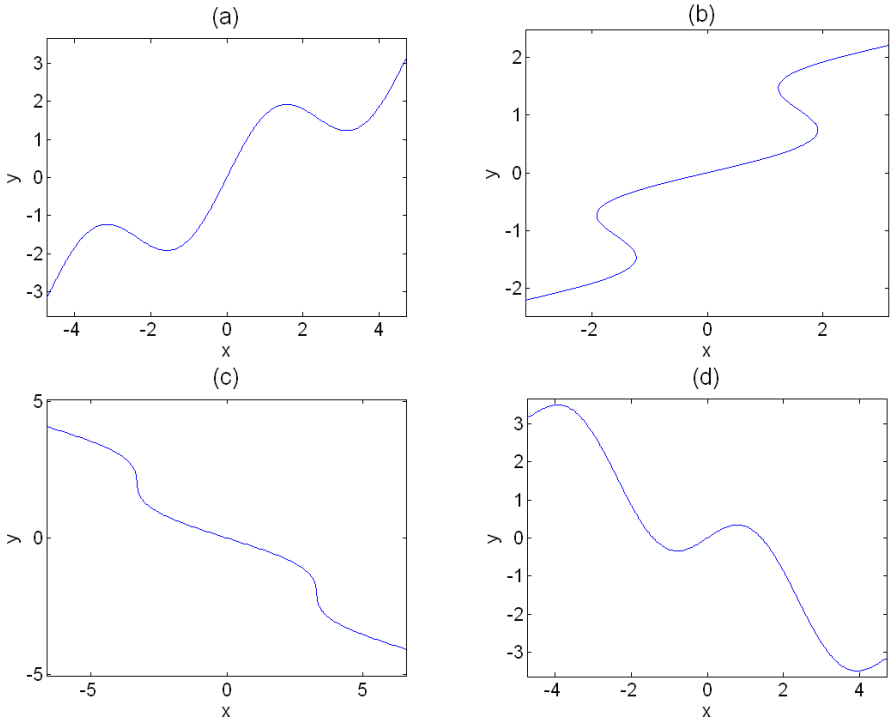
2. Which of the following is not a differential equation?

   (a) $6\frac{dy}{dx} + 3xy$
   (b) $8 = \frac{y'}{y}$
   (c) $2\frac{d^2f}{dt^2} + 7\frac{df}{dt} = f$
   (d) $h(x) + 2h'(x) = g(x)$
   (e) All are differential equations.

3. Which of the following couldn't be the solution of a differential equation?

   (a) $z(t) = 6$
   (b) $y = 3x^2 + 7$
   (c) $x = 0$
   (d) $y = 3x + y'$
   (e) All could be solutions of a differential equation.

4. Which of the following could not be a solution of a differential equation?



5. Which of the following could not be a solution of a differential equation?

   (a) $f = 2y + 7$
   (b) $q(d) = 2d^2 - 6e^d$
   (c) $6y^2 + 2yx = \sqrt{x}$
   (d) $y = 4 \sin 8\pi z$
   (e) All could be a solution of a differential equation.

6. True or False? A differential equation is a type of function.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

7. Suppose $\dfrac{dx}{dt} = 0.5x$ and $x(0) = 8$. Then the value of $x(2)$ is approximately

   (a) 4
   (b) 8

(c) 9

(d) 12

(e) 16

8. Which of the following is a solution to the differential equation $\dfrac{dy}{dt} = 72 - y$?

   (a) $y(t) = 72t - \frac{1}{2}t^2$

   (b) $y(t) = 72 + e^{-t}$

   (c) $y(t) = e^{-72t}$

   (d) $y(t) = e^{-t}$

9. The amount of a chemical in a lake is decreasing at a rate of 30% per year. If $p(t)$ is the total amount of the chemical in the lake as a function of time t (in years), which differential equation models this situation?

   (a) $p'(t) = -30$

   (b) $p'(t) = -0.30$

   (c) $p'(t) = p - 30$

   (d) $p'(t) = -0.3p$

   (e) $p'(t) = 0.7p$

10. The evolution of the temperature of a hot cup of coffee cooling off in a room is described by $\frac{dT}{dt} = -0.01T + 0.6$, where T is in °F and t is in hours. What are the units of the numbers -0.01 and 0.6?

    (a) -0.01 °F, and 0.6 °F

    (b) -0.01 per hour, and 0.6 °F per hour

    (c) -0.01 °F per hour, and 0.6 °F

    (d) neither number has units

11. We want to test the function $z(x) = 4\sin 3x$ to see if it solves $z'' + 2z' + 4z = 0$, by substituting the function into the differential equation. What is the resulting equation before simplification?

    (a) $-36\sin 3x + 24\cos 3x + 16\sin 3x = 0$

    (b) $4\sin 3x + 8\sin 3x + 16\sin 3x = 0$

    (c) $-36\sin 3x + 12\cos 3x + 4\sin 3x = 0.$

(d) $4\sin 3x + 8\cos 3x + 4\sin 3x = 0$

(e) none of the above

12. If we test the function $f(x) = ae^{bx}$ to see if it could solve $\frac{df}{dx} = cf^2$, which equation is the result?

    (a) $\frac{df}{dx} = ca^2e^{2bx}$

    (b) $abe^{bx} = cf^2$

    (c) $ae^{bx} = ca^2e^{(bx)^2}$

    (d) $abe^{bx} = ca^2e^{2bx}$

    (e) $abe^{bx} = cae^{bx}$

    (f) None of the above

13. We want to test the function $f(x) = 3e^{2x} + 6x$ to see if it solves the differential equation $\frac{df}{dx} = 2f + 3x$, so we insert the function and its derivative, getting $6e^{2x} + 6 = 2(3e^{2x} + 6x) + 3x$. This means that:

    (a) This function is a solution.

    (b) This function is a solution if $x = 2/5$.

    (c) This function is not a solution.

    (d) Not enough information is given.

14. A bookstore is constantly discarding a certain percentage of its unsold inventory and also receiving new books from its supplier so that the rate of change of the number of books in inventory is $B'(t) = -0.02B + 400 + 0.05t$, where B is the number of books and t is in months. If the store begins with 10,000 books in inventory, at what rate is it receiving books from its supplier at $t = 0$?

    (a) 200 books per month

    (b) 400 books per month

    (c) -200 books per month

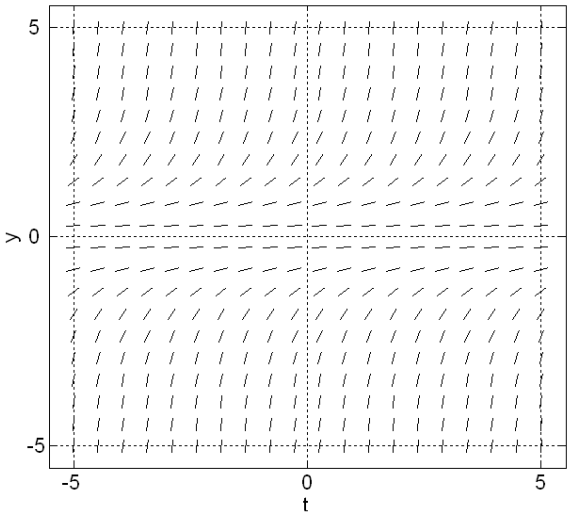
    (d) 900 books per month

Slope Fields and Euler's Method

15. What does the differential equation $\frac{dy}{dx} = 2y$ tell us about the slope of the solution curves at at any point?

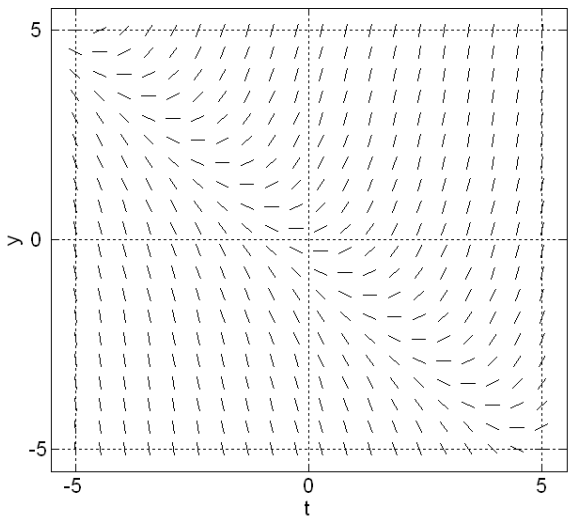
    (a) The slope is always 2.
    (b) The slope is equal to the x-coordinate.
    (c) The slope is equal to the y-coordinate.
    (d) The slope is equal to two times the x-coordinate.
    (e) The slope is equal to two times the y-coordinate.
    (f) None of the above.

16. The slopefield below indicates that the differential equation has which form?



    (a) $\frac{dy}{dt} = f(y)$
    (b) $\frac{dy}{dt} = f(t)$
    (c) $\frac{dy}{dt} = f(y, t)$

17. The slopefield below indicates that the differential equation has which form?

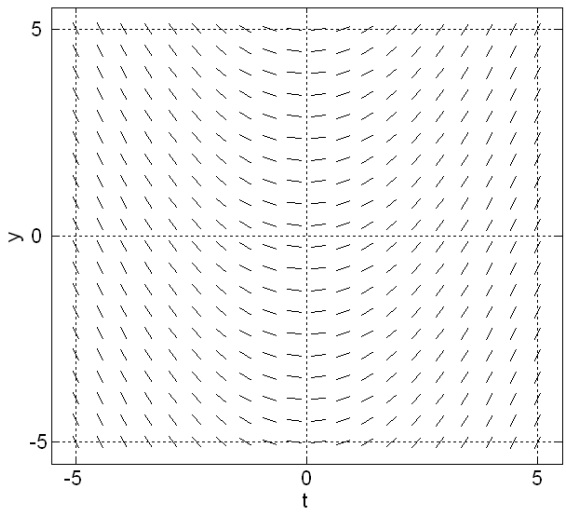


(a) $\frac{dy}{dt} = f(y)$

(b) $\frac{dy}{dt} = f(t)$

(c) $\frac{dy}{dt} = f(y, t)$

18. The slopefield below indicates that the differential equation has which form?

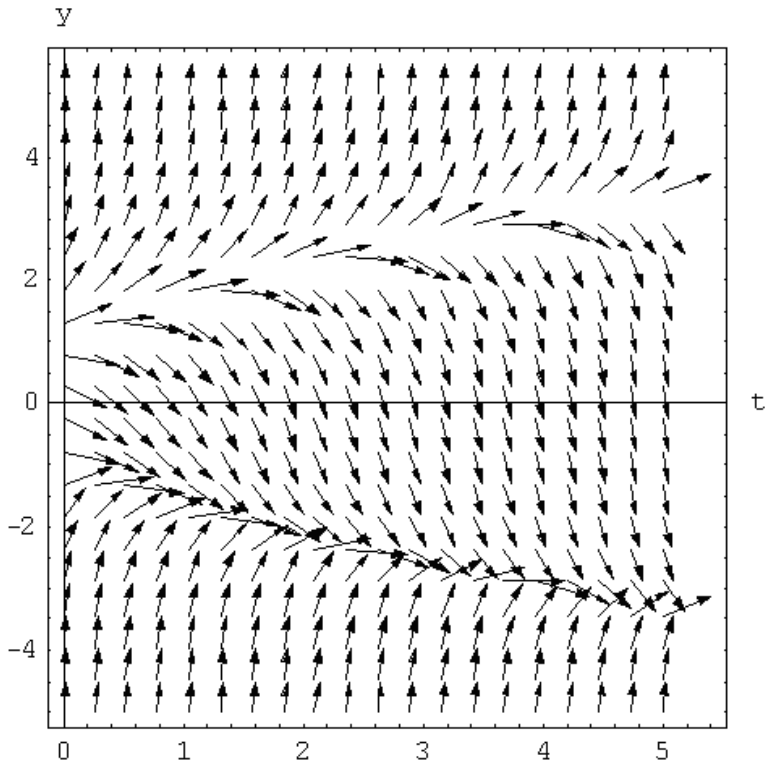


(a) $\frac{dy}{dt} = f(y)$

(b) $\frac{dy}{dt} = f(t)$

(c) $\frac{dy}{dt} = f(y, t)$

19. The arrows in the slope field below have slopes that match the derivative y' for a range of values of the function y and the independent variable t. Suppose that $y(0) = 0$. What would you predict for $y(5)$?



(a) $y(5) \approx -3$

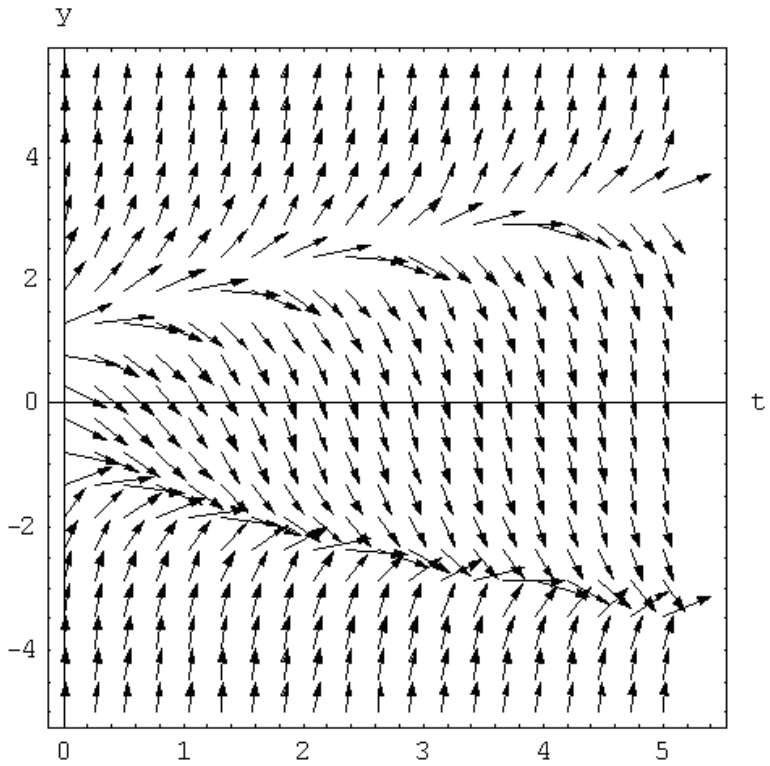
(b) $y(5) \approx +3$

(c) $y(5) \approx 0$

(d) $y(5) < -5$

(e) None of the above

20. The arrows in the slope field below give the derivative y' for a range of values of the function y and the independent variable t. Suppose that $y(0) = -4$. What would you predict for $y(5)$?



(a) $y(5) \approx -3$

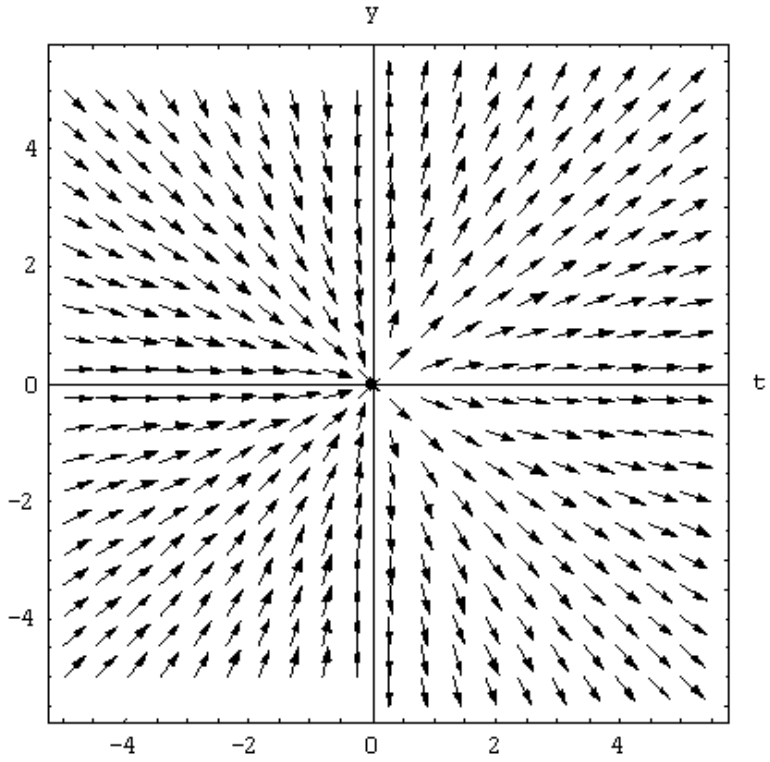
(b) $y(5) \approx +3$

(c) $y(5) \approx 0$

(d) $y(5) < -5$

(e) None of the above

21. The slope field below represents which of the following differential equations?



(a) $y' = yt$

(b) $y' = \frac{y}{t}$

(c) $y' = -yt$

(d) $y' = -\frac{y}{t}$

22. Consider the differential equation $y' = ay + b$ with parameters a and b. To approximate this function using Euler's method, what difference equation would we use?

(a) $y_{n+1} = ay_n + b$

(b) $y_{n+1} = y_n + ay_n\Delta t + b\Delta t$

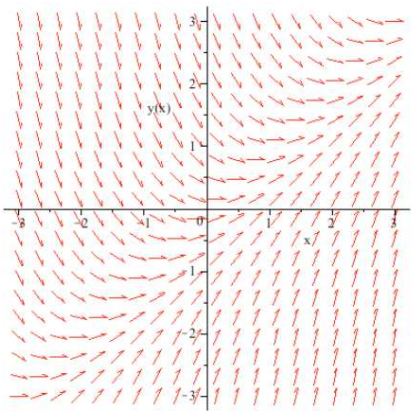
(c) $y_{n+1} = ay_n\Delta t + b\Delta t$

(d) $y_{n+1} = y_n\Delta t + ay_n\Delta t + b\Delta t$

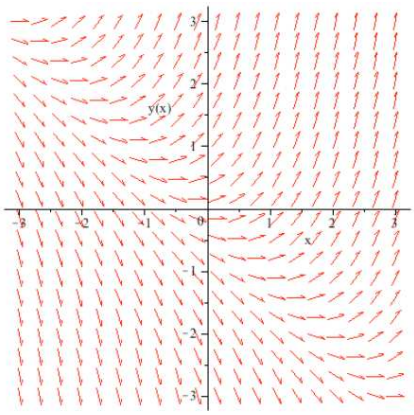
(e) None of the above

23. Which of the following is the slope field for $dy/dx = x + y$?

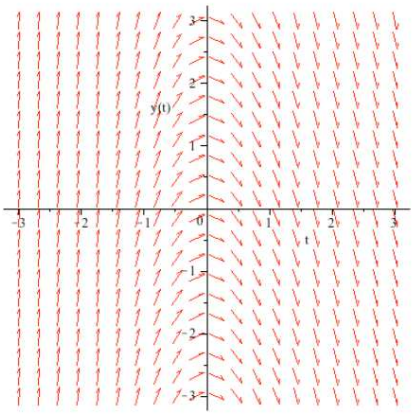
(A)



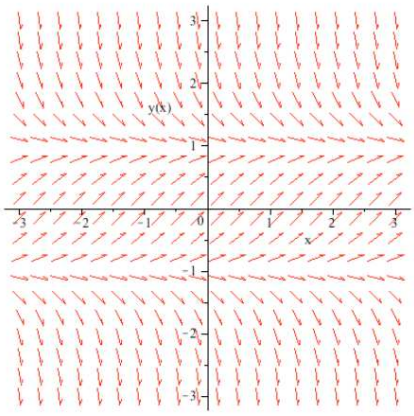
(B)



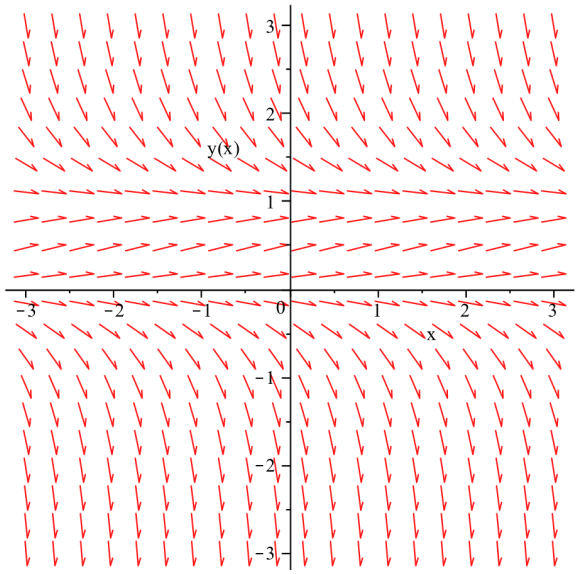
(C)



(D)



24. Below is the slope field for $dy/dx = y(1-y)$:



As $x \to \infty$, the solution to this differential equation that satisfies the initial condition $y(0) = 2$ will

(a) Increase asymptotically to $y = 1$

(b) Decrease asymptotically to $y = 1$

(c) Increase without bound

(d) Decrease without bound

(e) Start and remain horizontal

25. Using Euler's method, we set up the difference equation $y_{n+1} = y_n + c\Delta t$ to approximate a differential equation. What is the differential equation?

(a) $y' = cy$

(b) $y' = c$

(c) $y' = y + c$

(d) $y' = y + c\Delta t$

(e) None of the above

26. We know that $f(2) = -3$ and we use Euler's method to estimate that $f(2.5) \approx -3.6$, when in reality $f(2.5) = -3.3$. This means that between $x = 2$ and $x = 2.5$,

(a) $f(x) > 0$.

(b) $f'(x) > 0$.

(c) $f''(x) > 0$.

(d) $f'''(x) > 0$.

(e) None of the above

27. We have used Euler's method and $\Delta t = 0.5$ to approximate the solution to a differential equation with the difference equation $y_{n+1} = y_n + 0.2$. We know that the function $y = 7$ when $t = 2$. What is our approximate value of y when $t = 3$?

(a) $y(3) \approx 7.2$

(b) $y(3) \approx 7.4$

(c) $y(3) \approx 7.6$

(d) $y(3) \approx 7.8$

(e) None of the above

28. We have used Euler's method to approximate the solution to a differential equation with the difference equation $z_{n+1} = 1.2z_n$. We know that the function $z(0) = 3$. What is the approximate value of $z(2)$?

(a) $z(2) \approx 3.6$

(b) $z(2) \approx 4.32$

(c) $z(2) \approx 5.184$

(d) Not enough information is given.

29. We have used Euler's method and $\Delta t = 0.5$ to approximate the solution to a differential equation with the difference equation $y_{n+1} = y_n + t + 0.2$. We know that the function $y = 7$ when $t = 2$. What is our approximate value of y when $t = 3$?

    (a) $y(3) \approx 7.4$

    (b) $y(3) \approx 11.4$

    (c) $y(3) \approx 11.9$

    (d) $y(3) \approx 12.9$

    (e) None of the above

30. We have a differential equation for $\frac{dx}{dt}$, we know that $x(0) = 5$, and we want to know $x(10)$. Using Euler's method and $\Delta t = 1$ we get the result that $x(10) \approx 25.2$. Next, we use Euler's method again with $\Delta t = 0.5$ and find that $x(10) \approx 14.7$. Finally we use Euler's method and $\Delta t = 0.25$, finding that $x(10) \approx 65.7$. What does this mean?

    (a) These may all be correct. We need to be told which stepsize to use, otherwise we have no way to know which is the right approximation in this context.

    (b) Fewer steps means fewer opportunities for error, so $x(10) \approx 25.2$.

    (c) Smaller stepsize means smaller errors, so $x(10) \approx 65.7$.

    (d) We have no way of knowing whether any of these estimates is anywhere close to the true value of $x(10)$.

    (e) Results like this are impossible: We must have made an error in our calculations.

31. We have a differential equation for $f'(x)$, we know that $f(12) = 0.833$, and we want to know $f(15)$. Using Euler's method and $\Delta t = 0.1$ we get the result that $f(15) \approx 0.275$. Next, we use $\Delta t = 0.2$ and find that $f(15) \approx 0.468$. When we use $\Delta t = 0.3$, we get $f(15) \approx 0.464$. Finally, we use $\Delta t = 0.4$ and we get $f(15) \approx 0.462$. What does this mean?

    (a) These results appear to be converging to $f(15) \approx 0.46$.

    (b) Our best estimate is $f(15) \approx 0.275$.

    (c) This data does not allow us to make a good estimate of $f(15)$.

Separation of Variables

32. Which of the following DE's is/are separable?

    (a) $dy/dx = xy$
    (b) $dy/dx = x + y$
    (c) $dy/dx = \cos(xy)$
    (d) Both (a) and (b)
    (e) Both (a) and (c)
    (f) All of the above

33. Which of the following differential equations is not separable?

    (a) $y' = 3 \sin x \cos y$
    (b) $y' = x^2 + 3y$
    (c) $y' = e^{2x+y}$
    (d) $y' = 4x + 7$
    (e) More than one of the above

34. Which of the following differential equations is not separable?

    (a) $\frac{dx}{dt} = xt^2 - 4x$
    (b) $\frac{dx}{dt} = 3x^2t^3$
    (c) $\frac{dx}{dt} = \sin(2xt)$
    (d) $\frac{dx}{dt} = t^4 \ln(5x)$

35. Which of the following differential equations is separable?

    (a) $uu' = 2x + u$
    (b) $3ux = \sin(u')$
    (c) $\frac{2x^3}{5u'+u} = 1$
    (d) $e^{2u'x^2} = e^{u^3}$

36. If we separate the variables in the differential equation $3z't = z^2$, what do we get?

    (a) $3z^{-2}dz = t^{-1}dt$

(b) $3tdt = z^2dt$

(c) $3z'tdz = z^2dt$

(d) $z = \sqrt{3z't}$

(e) This equation cannot be separated.

37. If we separate the variables in the differential equation $y' = 2y + 3$, what do we get?

(a) $\frac{dy}{2y} = 3dx$

(b) $dy = 2y = 3dx$

(c) $\frac{dy}{y} = 5dx$

(d) $\frac{dy}{2y+3} = dx$

(e) This equation cannot be separated.

38. What is the solution to the differential equation: $\frac{dy}{dx} = 2xy$.

(a) $y = e^{x^2} + C$

(b) $y = Ce^{x^2}$

(c) $y = e^{2x} + C$

(d) $y = Ce^{2x}$

39. The general solution to the equation $dy/dt = ty$ is

(a) $y = t^2/2 + C$

(b) $y = \sqrt{t^2 + C}$

(c) $y = e^{t^2/2} + C$

(d) $y = Ce^{t^2/2}$

(e) Trick question, equation is not separable

40. The general solution to the equation $\dfrac{dR}{dy} + R = 1$ is

(a) $R = 1 - \sqrt{\dfrac{1}{C - y}}$

(b) $R = 1 - Ce^{y}$

(c) $R = 1 - Ce^{-y}$

(d) Trick question, equation is not separable

41. A plant grows at a rate that is proportional to the square root of its height $h(t)$ – use k as the constant of proportionality. If we separate the variables in the differential equation for its growth, what do we get?

    (a) $kh^{1/2}dt = dh$
    (b) $\sqrt{hdh} = kdt$
    (c) $h^{1/2}dh = kdt$
    (d) $h^{-1/2}dh = kdt$
    (e) None of the above

Exponential Solutions, Growth and Decay

42. A star's brightness is decreasing at a rate equal to 10% of its current brightness per million years. If B_0 is a constant with units of brightness and t is in millions of years, what function could describe the brightness of the star?

    (a) $B'(t) = -0.1B(t)$
    (b) $B(t) = B_0e^t$
    (c) $B(t) = B_0e^{-0.1t}$
    (d) $B(t) = B_0e^{0.1t}$
    (e) $B(t) = B_0e^{0.9t}$
    (f) $B(t) = -0.1B_0t$

43. A small company grows at a rate proportional to its size, so that $c'(t) = kc(t)$. We set $t = 0$ in 1990 when there were 50 employees. In 2005 there were 250 employees. What equation must we solve in order to find the growth constant k?

    (a) $50e^{2005k} = 250$
    (b) $50e^{15k} = 250$
    (c) $250e^{15k} = 50$
    (d) $50e^{tk} = 250$
    (e) Not enough information is given.

44. What differential equation is solved by the function $f(x) = 0.4e^{2x}$?

    (a) $\frac{df}{dx} = 0.4f$

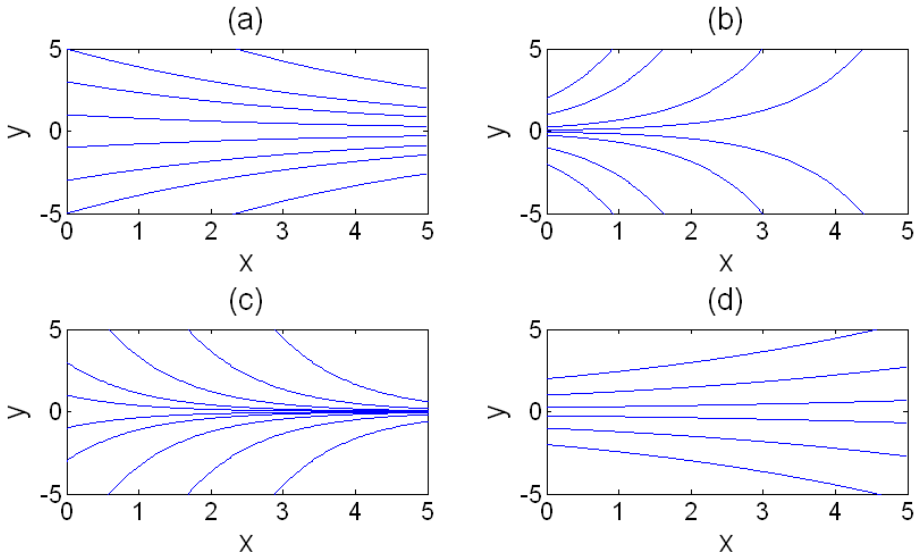
(b) $\frac{df}{dx} = 2f$

(c) $\frac{df}{dx} = 2f + 0.4$

(d) $\frac{df}{dx} = 0.4f + 2$

(e) None of the above.

45. Each of the graphs below show solutions of $y' = k_i y$ for a different k_i. Rank these constants from smallest to largest.



(a) $k_b < k_d < k_a < k_c$

(b) $k_d < k_c < k_b < k_a$

(c) $k_c < k_a < k_d < k_b$

(d) $k_a < k_b < k_c < k_d$

46. The function $f(y)$ solves the differential equation $f' = -0.1f$ and we know that $f(0) > 0$. This means that:

(a) When y increases by 1, f decreases by exactly 10%.

(b) When y increases by 1, f decreases by a little more than 10%.

(c) When y increases by 1, f decreases by a little less than 10%.

(d) Not enough information is given.

47. The function $g(z)$ solves the differential equation $\frac{dg}{dz} = 0.03g$. This means that:

(a) g is an increasing function that changes by 3% every time z increases by 1.

(b) g is an increasing function that changes by more than 3% every time z increases by 1.

(c) g is an increasing function that changes by less than 3% every time z increases by 1.

(d) g is a decreasing function that changes by more than 3% every time z increases by 1.

(e) g is a decreasing function that changes by less than 3% every time z increases by 1.

(f) Not enough information is given.

48. 40 grams of a radioactive element with a half-life of 35 days are put into storage. We solve $y' = -ky$ with $k = 0.0198$ to find a function that describes how the amount of this element will decrease over time. Another facility stores 80 grams of the element and we want to derive a similar function. When solving the differential equation, what value of k should we use?

(a) $k = 0.0099$

(b) $k = 0.0198$

(c) $k = 0.0396$

(d) None of the above

49. A star's brightness is decreasing at a rate equal to 10% of its current brightness per million years, so $B'(t) = -0.1B(t)$, where t is measured in millions of years. If we want t to be measured in years, how would the differential equation change?

(a) $B'(t) = -0.1B(t)$

(b) $B'(t) = -10^5 B(t)$

(c) $B'(t) = -10^{-6} B(t)$

(d) $B'(t) = -10^{-7} B(t)$

(e) None of the above

50. The solution to which of the following will approach $+\infty$ as x becomes very large?

(a) $y' = -2y$, $y(0) = 2$

(b) $y' = 0.1y$, $y(0) = 1$

(c) $y' = 6y$, $y(0) = 0$

(d) $y' = 3y$, $y(0) = -3$

(e) None of the above

51. $y' = -\frac{1}{3}y$ with $y(0) = 2$. As x becomes large, the solution will

(a) diverge to $+\infty$.
(b) diverge to $-\infty$.
(c) approach 0 from above.
(d) approach 0 from below.
(e) do none of the above.

52. Suppose H is the temperature of a hot object placed into a room whose temperature is 70 degrees, and t represents time. Suppose k is a positive number. Which of the following differential equations best corresponds to Newton's Law of Cooling?

(a) $dH/dt = -kH$
(b) $dH/dt = k(H - 70)$
(c) $dH/dt = -k(H - 70)$
(d) $dH/dt = -k(70 - H)$
(e) $dH/dt = -kH(H - 70)$

53. Suppose H is the temperature of a hot object placed into a room whose temperature is 70 degrees.The function H giving the object's temperature as a function of time is most likely

(a) Increasing, concave up
(b) Increasing, concave down
(c) Decreasing, concave up
(d) Decreasing, concave down

54. Suppose H is the temperature of a hot object placed into a room whose temperature is 70 degrees, and t represents time. Then $\lim_{t\to\infty} H$ should equal approximately

(a) $-\infty$
(b) 0
(c) 32
(d) 70
(e) Whatever the difference is between the object's initial temperature and 70

Equilibria and Stability

55. The differential equation $\frac{dy}{dt} = (t-3)(y-2)$ has equilibrium values of

    (a) $y = 2$ only
    (b) $t = 3$ only
    (c) $y = 2$ and $t = 3$
    (d) No equilibrium values

56. Suppose that 3 is an equilibrium value of a differential equation. This means that

    (a) the values will approach 3.
    (b) if the initial value is below 3, the values will decrease.
    (c) if the initial value is 3, then all of the values will be 3.
    (d) all of the above.

57. We know that a given differential equation is in the form $y' = f(y)$, where f is a differentiable function of y. Suppose that $f(5) = 2$ and $f(-1) = -6$.

    (a) y must have an equilibrium value between $y = 5$ and $y = -1$.
    (b) y must have an equilibrium value between $y = 2$ and $y = -6$.
    (c) This does not necessarily indicate that any equilibrium value exists.

58. We know that a given differential equation is in the form $y' = f(y)$, where f is a differentiable function of y. Suppose that $f(10) = 0$, $f(9) = 3$, and $f(11) = -3$.

    (a) This means that $y = 10$ is a stable equilibrium.
    (b) $y = 10$ is an equilibrium, but it might not be stable.
    (c) This does not tell us for certain that $y = 10$ is an equilibrium.

59. We know that a given differential equation is in the form $y' = f(y)$, where f is a differentiable function of y. Suppose that $f(6) = 0$, $f(14) = 0$, and $y(10) = 10$.

    (a) This means that $y(0)$ must have been between 6 and 14.
    (b) This means that $y(20) = 0$ is impossible.
    (c) This means that $y(20) = 20$ is impossible.
    (d) All of the above.

(e) None of the above.

60. We know that a given differential equation is in the form $y' = f(y)$, where f is a differentiable function of y. Suppose that $f(2) = 3$ and that $y(0) = 0$. Which of the following is impossible?

    (a) $y(10) = 6$
    (b) $y(10) = -6$
    (c) $y(-10) = 6$
    (d) $y(-10) = -6$
    (e) All of these are possible

61. We know that a given differential equation is in the form $y' = f(y)$, where f is a differentiable function of y. Suppose that $f(5) = -2$, $f(10) = 4$, and that $y(10) = 3$.

    (a) $y(0)$ must be below 5.
    (b) $y(20)$ must be below 5.
    (c) $y(5)$ could be above 10.
    (d) $y(15)$ must be less than 3.

62. A differential equation has a stable equilibrium value of $T = 6$. Which of the following functions is definitely not a solution?

    (a) $T(t) = 5e^{-3t} + 6$
    (b) $T(t) = -4e^{-2t} + 6$
    (c) $T(t) = 4e^{2t} + 10$
    (d) They could all be solutions

63. Consider the differential equation $\frac{df}{dx} = \sin(f)$

    (a) $f = 0$ is a stable equilibrium.
    (b) $f = 0$ is an unstable equilibrium.
    (c) $f = 0$ is not an equilibrium.

64. Consider the differential equation $\frac{df}{dx} = af + b$, where a and b are positive parameters. If we increase b, what will happen to the equilibrium value?

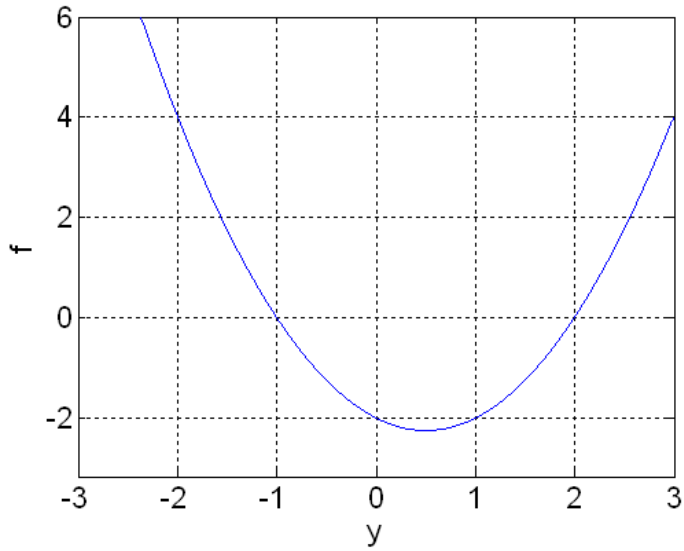
    (a) it increases

(b) is decreases

(c) it stays the same

(d) not enough information is given

65. Suppose that $\frac{dy}{dt} = f(y)$, which is plotted below. What are the equilibrium values of the system?

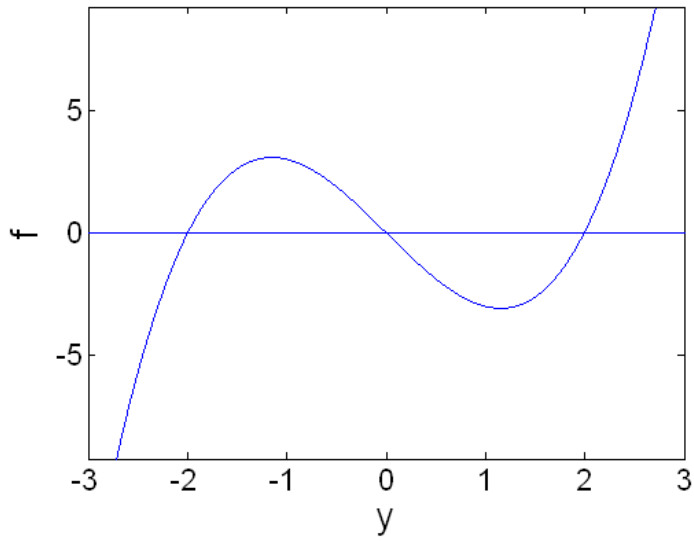


(a) $y = \frac{1}{2}$ is the only equilibrium.

(b) $y = -1$ and $y = 2$ are both equilibria.

(c) Not enough information is given.

66. Suppose that $\frac{dy}{dt} = f(y)$, which is plotted below. What can we say about the equilibria of this system?



(a) $y = 0$ is stable, $y = \pm 2$ are unstable.

(b) $y = 0$ is unstable, $y = \pm 2$ are stable.

(c) $y = -2, 0$ are stable, $y = 2$ is unstable.

(d) $y = -2$ is unstable, $y = 0, 2$ are unstable

(e) None of the above

67. **True or False** A differential equation could have infinitely many equilibria.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

68. **True or False** A differential equation could have infinitely many equilibria over a finite range.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

69. Consider the differential equation $\frac{df}{dx} = af + b$, where a and b are non-negative parameters. This equation would have no equilibrium if

(a) $a = 0$

(b) $b = 0$

(c) $a = 1$

(d) More than one of the above

70. What is the equilibrium value of $\frac{dg}{dz} = -\frac{1}{2}g + 3e^z$?

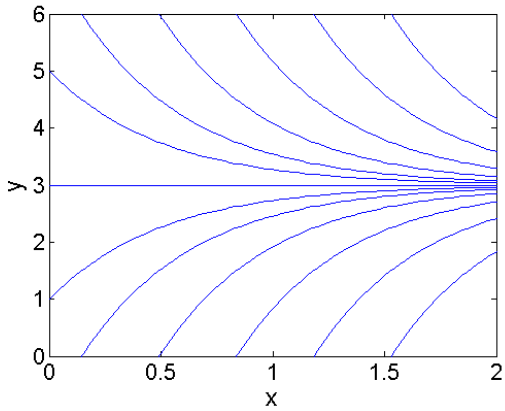
(a) This system is at equilibrium when $g = 6e^z$.

(b) This system is at equlibrium when $z = \ln\left(\frac{g}{6}\right)$.

(c) Both a and b are true.

(d) This equation has no equlibrium.

71. The figure below plots several functions which all solve the differential equation $y' = ay + b$. What could be the values of a and b?



(a) $a = 1$, $b = 3$

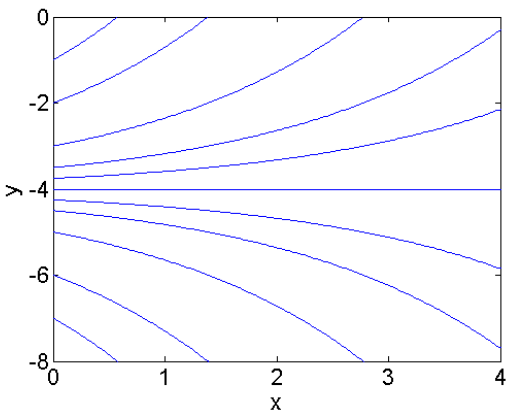
(b) $a = 2$, $b = -6$

(c) $a = -1$, $b = -3$

(d) $a = -2$, $b = 6$

(e) $b = 3$ but a is not easy to tell

72. The figure below plots several functions which all solve the differential equation $\frac{dy}{dx} = ay + b$. What could be the values of a and b?



(a) $a = 0.5$, $b = 2$

(b) $a = 0.5$, $b = -2$

(c) $a = -0.5$, $b = 2$

(d) $a = -0.5$, $b = -2$

(e) None of the above are possible.

First Order Linear Models

73. Water from a thunderstorm flows into a reservoir at a rate given by the function $g(t) = 250e^{-0.1t}$, where g is in gallons per day, and t is in days. The water in the reservoir evaporates at a rate of 2.25% per day. What equation could describe this scenario?

(a) $f'(t) = -0.0225f + 250e^{-0.1t}$

(b) $f'(t) = -0.0225(250e^{-0.1t})$

(c) $f'(t) = 0.9775f + 250e^{-0.1t}$

(d) None of the above

74. The state of ripeness of a banana is described by the differential equation $R'(t) = 0.05(2 - R)$ with $R = 0$ corresponding to a completely green banana and $R = 1$ a perfectly ripe banana. If all bananas start completely green, what value of R describes the state of a completely black, overripe banana?

(a) $R = 0.05$

(b) $R = \frac{1}{2}$

(c) $R = 1$

(d) $R = 2$

(e) $R = 4$

(f) None of the above.

75. The evolution of the temperature T of a hot cup of coffee cooling off in a room is described by $\frac{dT}{dt} = -0.01T + 0.6$, where T is in °F and t is in hours. What is the temperature of the room?

(a) 0.6

(b) -0.01

(c) 60

(d) 0.006

(e) 30

(f) none of the above

76. The evolution of the temperature of a hot cup of coffee cooling off in a room is described by $\frac{dT}{dt} = -0.01(T-60)$, where T is in °F and t is in hours. Next, we add a small heater to the coffee which adds heat at a rate of 0.1 °F per hour. What happens?

(a) There is no equilibrium, so the coffee gets hotter and hotter.

(b) The coffee reaches an equilibrium temperature of 60°F.

(c) The coffee reaches an equilibrium temperature of 70°F.

(d) The equilibrium temperature becomes unstable.

(e) None of the above

77. A drug is being administered intravenously into a patient at a certain rate d and is breaking down at a certain fractional rate $k > 0$. If $c(t)$ represents the concentration of the drug in the bloodstream, which differential equation represents this scenario?

(a) $\frac{dc}{dt} = -k + d$

(b) $\frac{dc}{dt} = -kc + d$

(c) $\frac{dc}{dt} = kc + d$

(d) $\frac{dc}{dt} = c(d - k)$

(e) None of the above

78. A drug is being administered intravenously into a patient. The drug is flowing into the bloodstream at a rate of 50 mg/hr. The rate at which the drug breaks down is proportional to the total amount of the drug, and when there is a total of 1000 mg of the drug in the patient, the drug breaks down at a rate of 300 mg/hr. If y is the number of milligrams of drug in the bloodstream at time t, what differential equation would describe the evolution of the amount of the drug in the patient?

    (a) $y' = -0.3y + 50$
    (b) $y' = -0.3t + 50$
    (c) $y' = 0.7y + 50$
    (d) None of the above

79. The amount of a drug in the bloodstream follows the differential equation $c' = -kc + d$, where d is the rate it is being added intravenously and k is the fractional rate at which it breaks down. If the initial concentration is given by a value $c(0) > d/k$, then what will happen?

    (a) This equation predicts that the concentration of the drug will be negative, which is impossible.
    (b) The concentration of the drug will decrease until there is none left.
    (c) This means that the concentration of the drug will get smaller, until it reaches the level $c = d/k$, where it will stay.
    (d) This concentration of the drug will approach but never reach the level d/k.
    (e) Because $c(0) > d/k$ this means that the concentration of the drug will increase, so the dose d should be reduced.

80. The amount of a drug in the bloodstream follows the differential equation $c' = -kc + d$, where d is the rate it is being added intravenously and k is the fractional rate at which is breaks down. If we double the rate at which the drug flows in, how will this change the equilibrium value?

    (a) It will be double the old value.
    (b) It will be greater than the old, but not quite doubled.
    (c) It will be more than doubled.
    (d) It will be the same.
    (e) Not enough information is given.

81. If we construct an electric circuit with a battery, a resistor, and a capacitor all in series, then the voltage is described by the equation $V_{bat} = \frac{Q}{C} + IR$. Here V_{bat} is the voltage produced by the battery, and the constants C and R give the capacitance and resistance respectively. $Q(t)$ is the charge on the capacitor and $I(t) = \frac{dQ}{dt}$ is the current flowing through the circuit. What is the equilibrium charge on the capacitor?

(a) $Q_e = V_{bat}C$
(b) $Q_e = V_{bat}/R$
(c) $Q_e = 0$
(d) Not enough information is given.

82. If we construct an electric circuit with a battery, a resistor, and a capacitor all in series, then the voltage is described by the equation $V_{bat} = \frac{Q}{C} + IR$. Here V_{bat} is the voltage produced by the battery, and the constants C and R give the capacitance and resistance respectively. $Q(t)$ is the charge on the capacitor and $I(t) = \frac{dQ}{dt}$ is the current flowing through the circuit. Which of the following functions could describe the charge on the capacitor $Q(t)$?

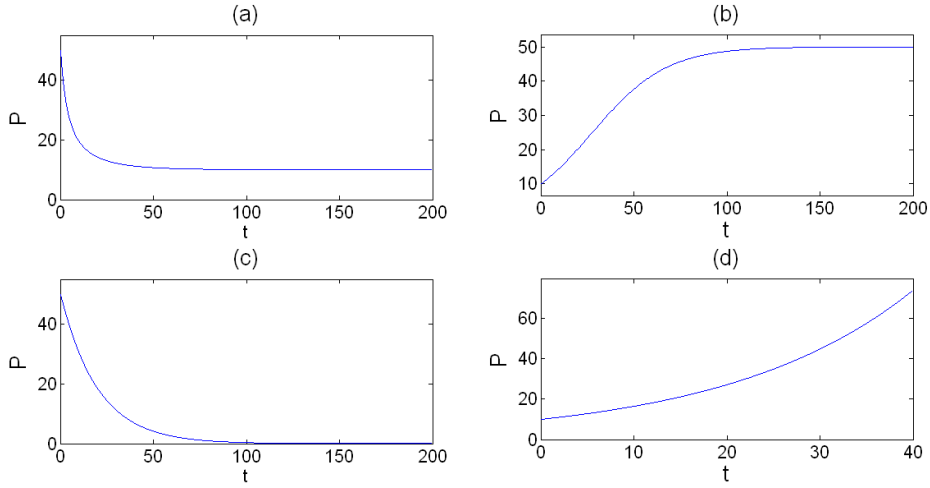
(a) $Q(t) = 5e^{-t/RC}$
(b) $Q(t) = 4e^{-RCt} + V_{bat}C$
(c) $Q(t) = 3e^{-t/RC} - V_{bat}C$
(d) $Q(t) = -6e^{-t/RC} + V_{bat}C$
(e) None of the above

83. If we construct an electric circuit with a battery, a resistor, and a capacitor all in series, then the voltage is described by the equation $V_{bat} = \frac{Q}{C} + IR$. Here V_{bat} is the voltage produced by the battery, and the constants C and R give the capacitance and resistance respectively. $Q(t)$ is the charge on the capacitor and $I(t) = \frac{dQ}{dt}$ is the current flowing through the circuit. Which of the following functions could describe the current flowing through the circuit $I(t)$?

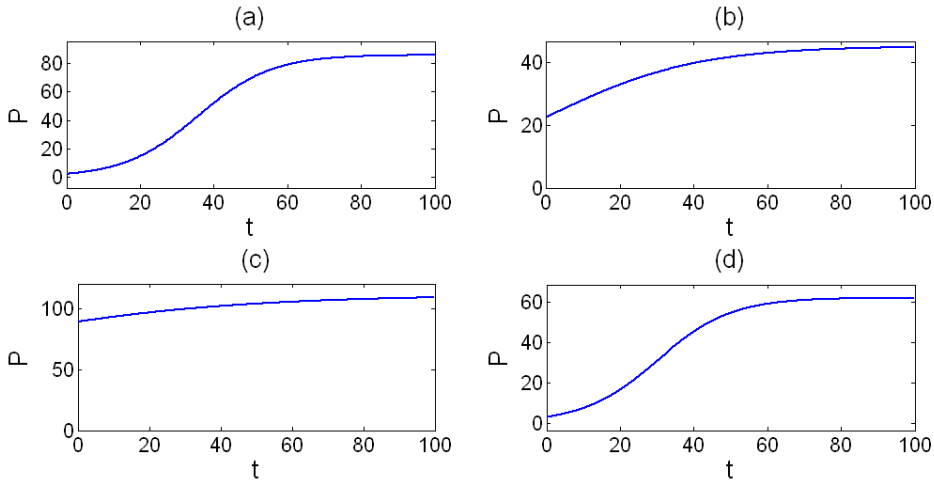
(a) $I(t) = 5e^{-t/RC}$
(b) $I(t) = 4e^{-RCt} + V_{bat}C$
(c) $I(t) = 3e^{-t/RC} - V_{bat}C$
(d) $I(t) = -6e^{-t/RC} + V_{bat}C$
(e) None of the above

Logistic Models

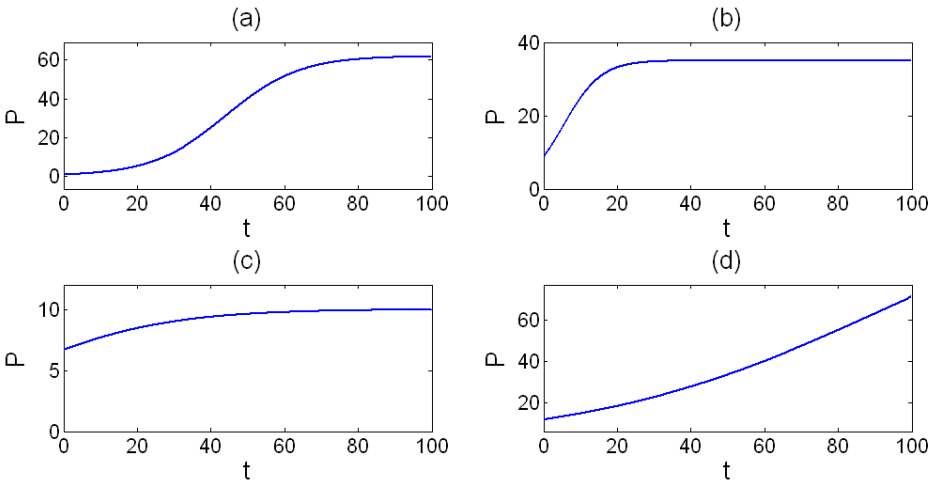
84. Consider the function $P = \frac{L}{1+Ae^{-kt}}$, where $A = (L - P_0)/P_0$. Suppose that $P_0 = 10$, $L = 50$, and $k = 0.05$, which of the following could be a graph of this function?



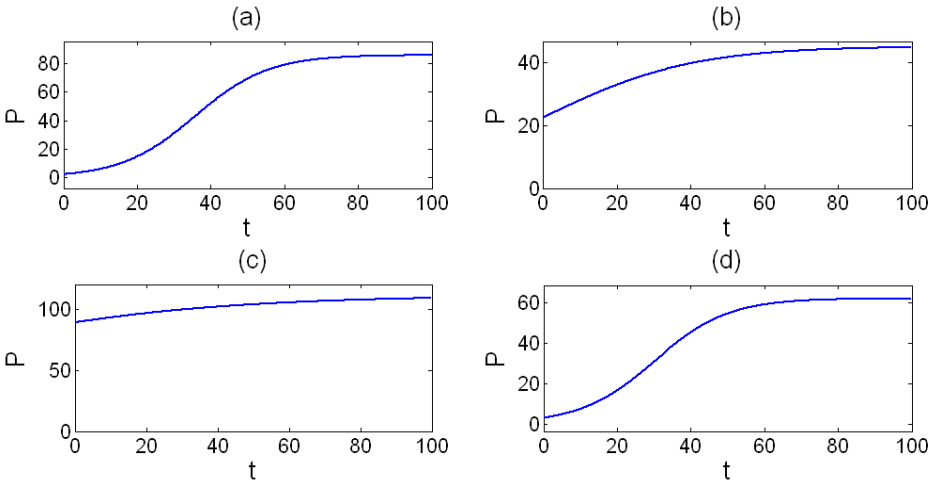
85. The following graphs all plot the function $P = \frac{L}{1+Ae^{-kt}}$. The function plotted in which graph has the largest value of L?



86. The following graphs all plot the function $P = \frac{L}{1+Ae^{-kt}}$. The function plotted in which graph has the largest value of k?



87. The following graphs all plot the function $P = \frac{L}{1+Ae^{-kt}}$. The function plotted in which graph has the largest value of A?



88. Consider the differential equation $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$, called the logistic equation. What are the equilibria of this system?

i. $k = 0$ is a stable equilibrium.
ii. $L = 0$ is an unstable equilibrium.
iii. $P = L$ is a stable equilibrium.
iv. $P = 0$ is an unstable equilibrium.
v. $P = L$ is an unstable equilibrium.
vi $P = 0$ is a stable equilibrium.

(a) i

(b) ii

(c) Both iii and v

(d) Both v and vi

(e) Both iv and v

(f) Both iii and iv

89. The population of rainbow trout in a river system is modeled by the differential equation $P' = 0.2P - 4 \times 10^{-5}P^2$. What is the maximum number of trout that the river system could support?

(a) 4×10^5 trout

(b) 4,000 trout

(c) 5,000 trout

(d) 25,000 trout

(e) Not enough information is given

90. The solution to the logistic equation $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$ is $P = \frac{L}{1+Ae^{-kt}}$, where $A = (L - P_0)/P_0$. If we are modeling a herd of elk, with an initial population of 50, in a region with a carrying capacity of 300, and knowing that the exponential growth rate of an elk population is 0.07, which function would describe our elk population as a function of time?

(a) $P(t) = \frac{300}{1+5e^{-0.07t}}$

(b) $P(t) = \frac{50}{1+\frac{5}{6}e^{0.07t}}$

(c) $P(t) = \frac{300}{1+\frac{6}{5}e^{0.07t}}$

(d) $P(t) = \frac{300}{1+\frac{1}{6}e^{-0.07t}}$

91. The population of mice on a farm is modeled by the differential equation $\frac{3000}{P}\frac{dP}{dt} = 200 - P$. If we know that today there are 60 mice on the farm, what function will describe how the mouse population will develop in the future?

(a) $P = \frac{200}{1+\frac{7}{3}e^{-t/15}}$

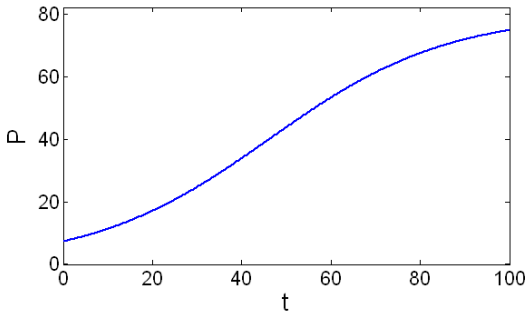
(b) $P = \frac{200}{1+\frac{7}{3}e^{-200t}}$

(c) $P = \frac{3000}{1+49e^{-t/15}}$

(d) $P = \frac{3000}{1+49e^{-t/20}}$

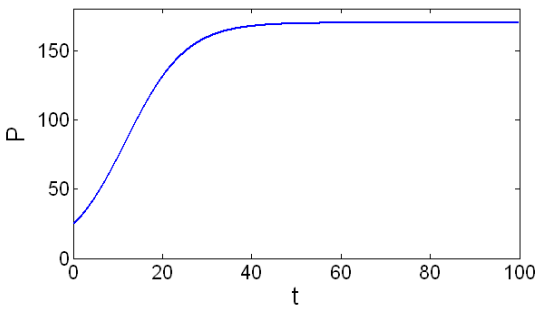
(e) None of the above

92. The function plotted below could be a solution to which of the following differential equations?



(a) $\frac{dP}{dt} = -0.05P\left(1 - \frac{P}{80}\right)$
(b) $P = \frac{P'}{\frac{1}{20} - 6.25\times 10^{-4}P}$
(c) $40\frac{P'}{P^2} + \frac{1}{40} = \frac{2}{P}$
(d) $-20\frac{dP}{dt} + P = \frac{P^2}{80}$
(e) All of the above

93. The function plotted below could be a solution of which of the following?



(a) $\frac{dP}{dt} = 0.15P\left(1 - \frac{P}{170}\right)$
(b) $\frac{dP}{dt} = 0.15P\left(1 - \frac{P}{240}\right)$
(c) $\frac{dP}{dt} = 0.05P\left(1 - \frac{P}{170}\right)$
(d) $\frac{dP}{dt} = 0.05P\left(1 - \frac{P}{240}\right)$
(e) None of the above

Nonhomogeneous Differential Equations & Undetermined Coefficients

94. Consider the equation $\frac{df}{dx} = 2f + e^{3x}$. When we separate the variables, this equation becomes:

(a) $\frac{1}{2f}df = e^{3x}dx$

(b) $df = 2f + e^{3x}dx$

(c) $\frac{1}{f}df = (2 + e^{3x})dx$

(d) $-2fdf = e^{3x}dx$

(e) This equation is not separable

95. $x'(t) + 4x(t) = e^t$ and we want to test the function $x(t) = C_0e^{-4t} + C_1e^t$ to see if it is a solution. What equation is the result?

(a) $(-4C_0e^{-4t} + C_1e^t) + 4C_0e^{-4t} + 4C_1e^t = e^t$

(b) $(C_0e^{-4t} + C_1e^t) + 4C_0e^{-4t} + 4C_1e^t = e^t$

(c) $-4C_0e^{-4t} + 4C_1e^t = e^t$

(d) None of the above

96. We are testing the function $f(x) = C_0e^{3x}$ as a possible solution to a differential equation. After we substitute the function and its derivative into the differential equation we get: $3C_0e^{3x} = -2C_0e^{3x} + 4e^{3x}$. What was the differential equation?

(a) $f' = -2f + \frac{4}{C_0}f$

(b) $f' = -2f + 4e^{3x}$

(c) $3f = -2f + 4e^{3x}$

(d) $3C_0e^{3x} = -2f + 4e^{3x}$

(e) None of the above.

97. We are testing the function $f(x) = C_0e^{2x} + C_1e^{-2x}$ as a possible solution to a differential equation. After we substitute the function and its derivative into the differential equation we get: $2C_0e^{2x} - 2C_1e^{-2x} = -2(C_0e^{2x} + C_1e^{-2x}) + 3e^{2x}$. What value of C_0 will allow this function to work?

(a) $C_0 = \frac{3}{4}$

(b) $C_0 = \frac{3}{2}$

(c) $C_0 = 3$

(d) $C_0 = 2$

(e) Any value of C_0 will work.

(f) No value of C_0 will work.

98. We are testing the function $f(x) = C_0e^{2x} + C_1e^{-2x}$ as a possible solution to a differential equation. After we substitute the function and its derivative into the differential equation we get: $2C_0e^{2x} - 2C_1e^{-2x} = -2(C_0e^{2x} + C_1e^{-2x}) + 3e^{2x}$. What value of C_1 will allow this function to work?

(a) $C_1 = \frac{3}{4}$
(b) $C_1 = \frac{3}{2}$
(c) $C_1 = 3$
(d) $C_1 = 2$
(e) Any value of C_1 will work.
(f) No value of C_1 will work.

99. When we have $y' = 7y + 2x$ we should conjecture $y = C_0e^{7x} + C_1x + C_2$. Why do we add the C_2?

(a) Because the $7y$ becomes a constant 7 when we take the derivative and we need a term to cancel this out.
(b) Because when we take the derivative of C_1x we get a constant C_1 and we need a term to cancel this out.
(c) Because this will allow us to match different initial conditions.
(d) This does not affect the equation because it goes away when we take the derivative.

100. We have the equation $y' = 2y + \sin 3t$. What should be our conjecture?

(a) $y = C_0e^{2t} + \sin 3t$
(b) $y = C_0e^{2t} + \sin 3t + \cos 3t$
(c) $y = C_0e^{2t} + C_1 \sin 3t$
(d) $y = C_0e^{2t} + C_1 \sin 3t + C_2 \cos 3t$
(e) $y = C_0e^{2t} + C_1e^{-2t} + C_2 \sin 3t + C_3 \cos 3t$
(f) None of the above

101. Consider $\frac{dg}{dz} = ag + b \cos cz$, where a, b, and c are all positive parameters. What will be the long term behavior of this system?

(a) It will grow exponentially.
(b) It will converge to an equilibrium.
(c) It will oscillate.

(d) Different behaviors are possible depending on the values of a, b, and c.

102. A bookstore is constantly discarding a certain percentage of its unsold inventory and also receiving new books from its supplier so that the rate of change of the number of books in inventory is $B'(t) = -0.02B + 400 + 0.05t$, where B is the number of books and t is in months. In the long run, what will happen to the number of books in inventory, according to this model?

(a) The number of books will approach zero.

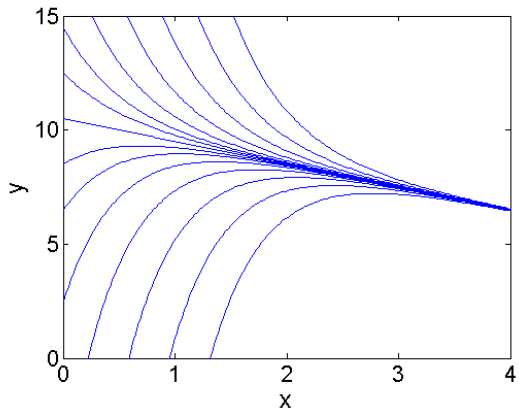
(b) The number of books will approach a stable equilibrium.

(c) The number of books will exponentially diverge from an unstable equilibrium.

(d) The number of books will grow linearly.

(e) None of the above

103. The figure below shows several functions that solve the differential equation $y' = ay + bx + c$. What could be the values of a, b, and c?



(a) $a = 2$, $b = 2$, $c = 20$

(b) $a = -2$, $b = -2$, $c = -20$

(c) $a = 2$, $b = -2$, $c = -20$

(d) $a = -2$, $b = -2$, $c = 20$

(e) $a = -2$, $b = 2$, $c = 20$

(f) Not enough information is given.

104. It is currently 10 degrees outside and your furnace goes out, so the temperature of your house will follow $\frac{dT}{dt} = 0.1(10 - T)$. You find an old heater which will add heat to your house at a rate of $h(t) = 3 + 2\sin 0.1t$ degrees per hour. What should you conjecture as a function to describe the temperature of your house?

(a) $T(t) = Ae^{-0.1t} + B$
(b) $T(t) = A\sin 0.1t + B$
(c) $T(t) = Ae^{-0.1t} + B\sin 0.1t + C$
(d) $T(t) = Ae^{-0.1t} + B\sin 0.1t + C\cos 0.1t + D$
(e) $T(t) = Ae^{0.1t} + Be^{-0.1t} + C\sin 0.1t + D\cos 0.1t + E$
(f) None of the above.

105. Which of the following is not a solution to $y'(t) = 5y + 3t$?

(a) $y = 8e^{5t}$
(b) $y = -\frac{3}{5}t - \frac{3}{25}$
(c) $y = 8e^{5t} - \frac{3}{5}t - \frac{3}{25}$
(d) All are solutions.
(e) More than one of (a) - (c) are not solutions.

Second Order Differential Equations: Oscillations

106. A branch sways back and forth with position $f(t)$. Studying its motion you find that its acceleration is proportional to its position, so that when it is 8 cm to the right, it will accelerate to the left at a rate of 2 cm/s^2. Which differential equation describes the motion of the branch?

(a) $\frac{d^2f}{dt^2} = 8f$
(b) $\frac{d^2f}{dt^2} = -4f$
(c) $\frac{d^2f}{dt^2} = -2$
(d) $\frac{d^2f}{dt^2} = \frac{f}{4}$
(e) $\frac{d^2f}{dt^2} = -\frac{f}{4}$

107. The differential equation $\frac{d^2f}{dt^2} = -0.1f + 3$ is solved by a function $f(t)$ where f is in feet and t is in minutes. What units does the number 3 have?

(a) feet
(b) minutes
(c) per minute
(d) per minute^2

(e) feet per minute^2

(f) no units

108. The differential equation $y'' = 7y$ is solved by a function $y(t)$ where y is in meters and t is in seconds. What units does the number 7 have?

(a) meters

(b) seconds

(c) per second

(d) per second^2

(e) meters per second^2

(f) no units

109. A differential equation is solved by the function $y(t) = 3\sin 2t$ where y is in meters and t is in seconds. What units do the numbers 3 and 2 have?

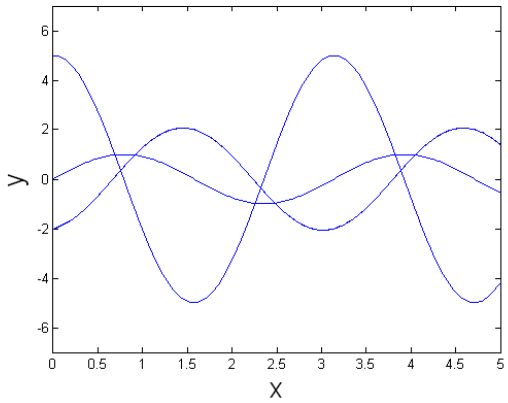
(a) 3 is in meters, 2 is in seconds

(b) 3 is in meters, 2 is in per second

(c) 3 is in meters per second, 2 has no units

(d) 3 is in meters per second, 2 is in seconds

110. Three different functions are plotted below. Could these all be solutions of the same second order differential equation?



(a) Yes

(b) No

(c) Not enough information is given.

111. Which of the following is not a solution of $y'' + ay = 0$ for some value of a?

(a) $y = 4\sin 2t$
(b) $y = 8\cos 3t$
(c) $y = 2e^{2t}$
(d) all are solutions

112. The functions below are solutions of $y'' + ay = 0$ for different values of a. Which represents the largest value of a?

(a) $y(t) = 100\sin 2\pi t$
(b) $y(t) = 25\cos 6\pi t$
(c) $y(t) = 0.1\sin 50t$
(d) $y(t) = 3\sin 2t + 8\cos 2t$

113. Each of the differential equations below represents the motion of a mass on a spring. If the mass is the same in each case, which spring is stiffer?

(a) $s'' + 8s = 0$
(b) $s'' + 2s = 0$
(c) $2s'' + s = 0$
(d) $8s'' + s = 0$

114. The motion of a mass on a spring follows the equation $mx'' = -kx$ where the displacement of the mass is given by $x(t)$. Which of the following would result in the highest frequency motion?

(a) $k = 6$, $m = 2$
(b) $k = 4$, $m = 4$
(c) $k = 2$, $m = 6$
(d) $k = 8$, $m = 6$
(e) All frequencies are equal

115. Each of the differential equations below represents the motion of a mass on a spring. Which system has the largest maximum velocity?

(a) $2s'' + 8s = 0, s(0) = 5, s'(0) = 0$

(b) $2s'' + 4s = 0, s(0) = 7, s'(0) = 0$
(c) $s'' + 4s = 0, s(0) = 10, s'(0) = 0$
(d) $8s'' + s = 0, s(0) = 20, s'(0) = 0$

116. Which of the following is not a solution of $\frac{d^2y}{dt^2} = -ay$ for some positive value of a?

(a) $y = 2\sin 6t$
(b) $y = 4\cos 5t$
(c) $y = 3\sin 2t + 8\cos 2t$
(d) $y = 2\sin 3t + 2\cos 5t$

117. Which function does not solve both $z' = 3z$ and $z'' = 9z$?

(a) $z = 7e^{3t}$
(b) $z = 0$
(c) $z = 12e^{-3t}$
(d) $z = -6e^{3t}$
(e) all are solutions to both

118. How are the solutions of $y'' = \frac{1}{4}y$ different from solutions of $y' = \frac{1}{2}y$?

(a) The solutions of $y'' = \frac{1}{4}y$ grow half as fast as solutions of $y' = \frac{1}{2}y$.
(b) The solutions of $y'' = \frac{1}{4}y$ include decaying exponentials.
(c) The solutions of $y'' = \frac{1}{4}y$ include sines and cosines.
(d) None of the above

119. How are the solutions of $y'' = -\frac{1}{4}y$ different from solutions of $y'' = -\frac{1}{2}y$?

(a) The solutions of $y'' = -\frac{1}{4}y$ oscillate twice as fast as the solutions of $y'' = -\frac{1}{2}y$.
(b) The solutions of $y'' = -\frac{1}{4}y$ have a period which is twice as long as the solutions of $y'' = -\frac{1}{2}y$.
(c) The solutions of $y'' = -\frac{1}{4}y$ have a smaller maximum value than the solutions of $y'' = -\frac{1}{2}y$.
(d) More than one of the above is true.
(e) None of the above are true.

120. What function solves the equation $y'' + 10y = 0$?

(a) $y = 10 \sin 10t$
(b) $y = 60 \cos \sqrt{10}t$
(c) $y = \sqrt{10}e^{-10t}$
(d) $y = 20e^{\sqrt{10t}}$
(e) More than one of the above

121. We know that the solution of a differential equation is of the form $y = A \sin 3x + B \cos 3x$. Which of the following would tell us that $A = 0$?

(a) $y(0) = 0$
(b) $y'(0) = 0$
(c) $y(1) = 0$
(d) none of the above

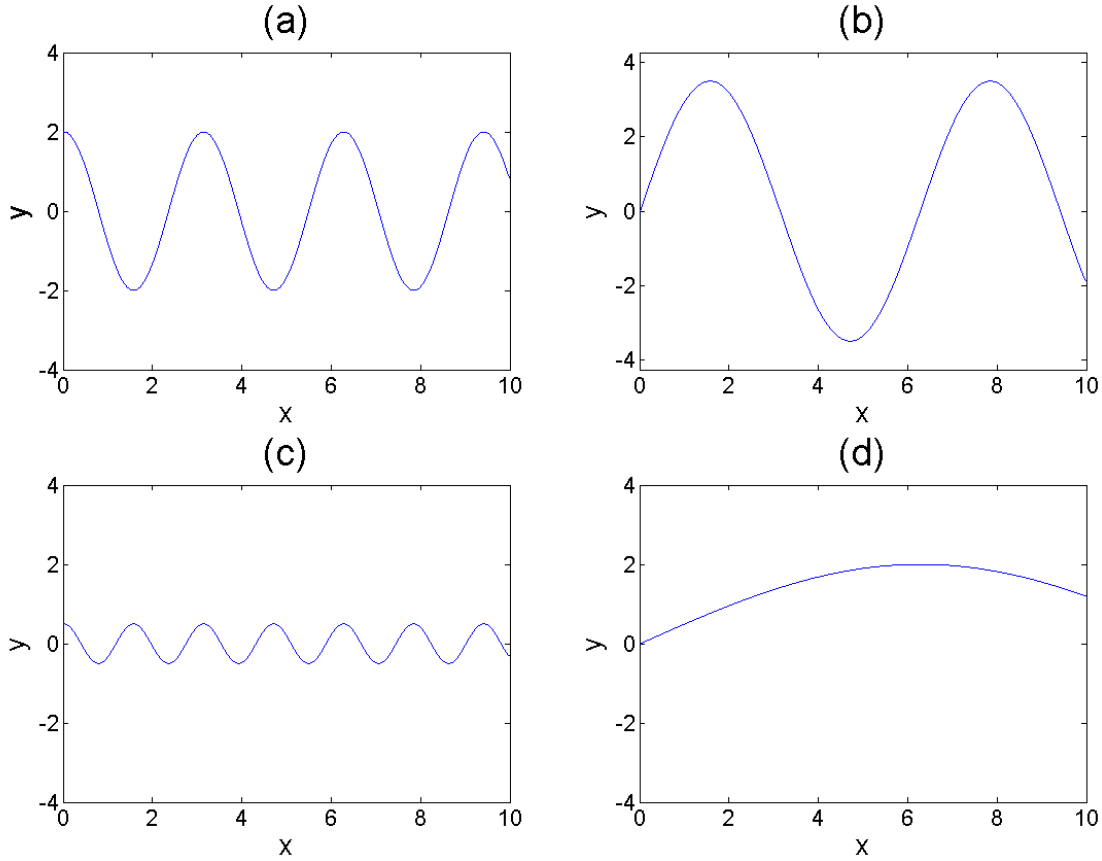
122. We know that the solutions to a differential equation are of the form $y = Ae^{3x} + Be^{-3x}$. If we know that $y = 0$ when $x = 0$, this means that

(a) $A = 0$
(b) $B = 0$
(c) $A = -B$
(d) $A = B$
(e) none of the above

123. An ideal spring produces an acceleration that is proportional to the displacement, so $my'' = -ky$ for some positive constant k. In the lab, we find that a mass is held on an imperfect spring: As the mass gets farther from equilibrium, the spring produces a force stronger than an ideal spring. Which of the following equations could model this scenario?

(a) $my'' = ky^2$
(b) $my'' = -k\sqrt{y}$
(c) $my'' = -k|y|$
(d) $my'' = -ky^3$
(e) $my'' = -ke^{-y}$
(f) None of the above

124. The functions plotted below are solutions of $y'' = -ay$ for different positive values of a. Which case corresponds to the largest value of a?



125. The motion of a child bouncing on a trampoline is modeled by the equation $p''(t) + 3p(t) = 6$ where p is in inches and t is in seconds. Suppose we want the position function to be in feet instead of inches. How does this change the differential equation?

(a) There is no change

(b) $p''(t) + 3p(t) = 0.5$

(c) $p''(t) + 3p(t) = 72$

(d) $144p''(t) + 3p(t) = 3$

(e) $p''(t) + 36p(t) = 3$

(f) $144p''(t) + 36p(t) = 3$

126. A float is bobbing up and down on a lake, and the distance of the float from the lake floor follows the equation $2d'' + 5d - 30 = 0$, where $d(t)$ is in feet and t is in seconds. At what distance from the lake floor could the float reach equilibrium?

(a) 2 feet

(b) 5 feet
(c) 30 feet
(d) 6 feet
(e) 15 feet
(f) No equilibrium exists.

Mixing Models

127. The differential equation for a mixing problem is $x' + 0.08x = 4$, where x is the amount of dissolved substance, in pounds, and time is measured in minutes. What are the units of 4?

(a) pounds
(b) minutes
(c) pounds/minute
(d) minutes/pound
(e) None of the above

128. The differential equation for a mixing problem is $x' + 0.08x = 4$, where x is the amount of dissolved substance, in pounds, and time is measured in minutes. What are the units of 0.08?

(a) pounds
(b) minutes
(c) per pound
(d) per minute
(e) None of the above

129. The differential equation for a mixing problem is $x' + 0.08x = 4$, where x is the amount of dissolved substance, in pounds, and time is measured in minutes. What is the equilibrium value for this model?

(a) 0.08
(b) 50
(c) 0
(d) 0.02

(e) 4

(f) None of the above

130. In a mixing model where x' has units of pounds per minute, the equilibirum value is 80. Which of the following is a correct interpretation of the equilibrium?

(a) In the long-run, there will be 80 pounds of contaminant in the system.

(b) After 80 minutes, the mixture will stabilize.

(c) The rate in is equal to the rate out when the concentration of contaminant is 80 pounds per gallon.

(d) The rate in is equal to the rate out when the amount of contaminant is 80 pounds.

(e) None of the above

131. A tank initially contains 60 gallons of pure water. A solution containing 3 pounds/gallon of salt is pumped into the tank at a rate of 2 gallons/minute. The mixture is stirred constantly and flows out at a rate of 2 gallons per minute. If $x(t)$ is the amount of salt in the tank at time t, which initial value problem represents this scenario?

(a) $x'(t) = 2 - 2$, with $x(0) = 60$

(b) $x'(t) = 6 - \frac{x}{30}$, with $x(0) = 0$

(c) $x'(t) = 3x - 2$, with $x(0) = 60$

(d) $x'(t) = 6 - \frac{x}{60}$, with $x(0) = 0$

(e) None of the above

132. The solution to a mixing problem is

$$x(t) = -0.01(100 - t)^2 + (100 - t),$$

where $x(t)$ is the amount of a contaminant in a tank of water. What is the long-term behavior of this solution?

(a) The amount of contaminant will reach a steady-state of 100 pounds.

(b) The amount of contaminant will increase forever.

(c) The amount of contaminant will approach zero.

(d) The tank will run out of water.

133. Tank A initially contains 30 gallons of pure water, and tank B initially contains 40 gallons of pure water. A solution containing 2 pounds/gallon of salt is pumped into tank A at a rate of 1.5 gallons/minute. The mixture in tank A is stirred constantly and flows into tank B at a rate of 1.5 gallons/minute. The mixture in tank B is also stirred constantly, and tank B drains at a rate of 1.5 gallons/minute. If $A(t)$ is the amount of salt in the tank A at time t and $B(t)$ is the amount of salt in tank B at time t, which initial value problem represents this scenario?

(a)

$$\begin{aligned} A'(t) &= 1.5 - \frac{A}{30} \qquad A(0) = 0 \\ B'(t) &= 1.5 - \frac{B}{40} \qquad B(0) = 0 \end{aligned}$$

(b)

$$\begin{aligned} A'(t) &= 1.5 - \frac{A}{30} \qquad A(0) = 0 \\ B'(t) &= \frac{A}{30} - \frac{B}{40} \qquad B(0) = 0 \end{aligned}$$

(c)

$$\begin{aligned} A'(t) &= 3 - \frac{A}{30} \qquad A(0) = 0 \\ B'(t) &= \frac{A}{30} - \frac{B}{40} \qquad B(0) = 0 \end{aligned}$$

(d)

$$\begin{aligned} A'(t) &= 3 - \frac{A}{20} \qquad A(0) = 0 \\ B'(t) &= \frac{A}{20} - \frac{3B}{80} \qquad B(0) = 0 \end{aligned}$$

(e) None of the above

134. Medication flows from the GI tract into the bloodstream. Suppose that A units of an antihistamine are present in the GI tract at time 0 and that the medication moves from the GI tract into the blood at a rate proportional to the amount in the GI tract, x. Assume that no further medication enters the GI tract, and that the kidneys and liver clear the medication from the blood at a rate proportional to the amount currently in the blood, y. If k_1 and k_2 are positive constants, which initial value problem models this scenario?

(a)

$$\begin{aligned} \frac{dx}{dt} &= k_1 x - k_2 y \qquad x(0) = A \\ \frac{dy}{dt} &= k_2 y \qquad y(0) = 0 \end{aligned}$$

(b)

$$\begin{aligned} \frac{dx}{dt} &= k_1 x - k_2 y \qquad x(0) = 0 \\ \frac{dy}{dt} &= k_2 y \qquad y(0) = 0 \end{aligned}$$

(c)

$$\begin{aligned} \frac{dx}{dt} &= -k_1 x \qquad x(0) = A \\ \frac{dy}{dt} &= k_1 x - k_2 y \qquad y(0) = 0 \end{aligned}$$

(d)

$$\begin{aligned} \frac{dx}{dt} &= k_1 x - k_2 y \qquad x(0) = 0 \\ \frac{dy}{dt} &= k_1 x + k_2 y \qquad y(0) = 0 \end{aligned}$$

135. Referring to the antihistamine model developed in the previous question, if time is measured in hours and the quantity of antihistamine is measured in milligrams, what are the units of k_1?

(a) hours
(b) hours per milligram
(c) milligrams per hour
(d) 1/hours
(e) milligrams
(f) None of the above

136. Referring to the antihistamine model discussed in the previous questions, what is the effect of increasing k_2?

(a) The blood will be cleaned faster.
(b) Antihistamine will be removed from the GI tract at a faster rate.

(c) It will take longer for the antihistamine to move from the GI tract into the blood.

(d) Antihistamine will accumulate in the blood at a faster rate and the patient will end up with an overdose.

(e) None of the above

137. Referring to the antihistamine model discussed in the previous questions, describe how the amount of antihistamine in the blood changes with time.

(a) It decreases and asymptotically approaches zero.

(b) It levels off at some nonzero value.

(c) It increases indefinitely.

(d) It increases, reaches a peak, and then decreases, asymptotically approaching zero.

(e) None of the above

Existence & Uniqueness

138. Based upon observations, Kate developed the differential equation $\frac{dT}{dt} = -0.08(T - 72)$ to predict the temperature in her vanilla chai tea. In the equation, T represents the temperature of the chai in °F and t is time. Kate has a cup of chai whose initial temperature is 110°F and her friend Nate has a cup of chai whose initial temperature is 120°F. According Kate's model, will there be a point in time when the two cups of chai have exactly the same temperature?

(a) Yes

(b) No

(c) Can't tell with the information given

139. A bucket of water has a hole in the bottom, and so the water is slowly leaking out. The height of the water in the bucket is thus a decreasing function of time $h(t)$ which changes according to the differential equation $h' = -kh^{1/2}$, where k is a positive constant that depends on the size of the hole and the bucket. If we start out a bucket with 25 cm of water in it, then according to this model, will the bucket ever be empty?

(a) Yes

(b) No

(c) Can't tell with the information given

140. A scientist develops the logistic population model $P' = 0.2P(1 - \frac{P}{8.2})$ to describe her research data. This model has an equilibrium value of $P = 8.2$. In this model, from the initial condition $P_0 = 4$, the population never reaches the equilibrium value because:

(a) you can't have a population of 8.2.

(b) the population is asymptotically approaching a value of 8.2.

(c) the population will grow towards infinity.

(d) the population will drop to 0.

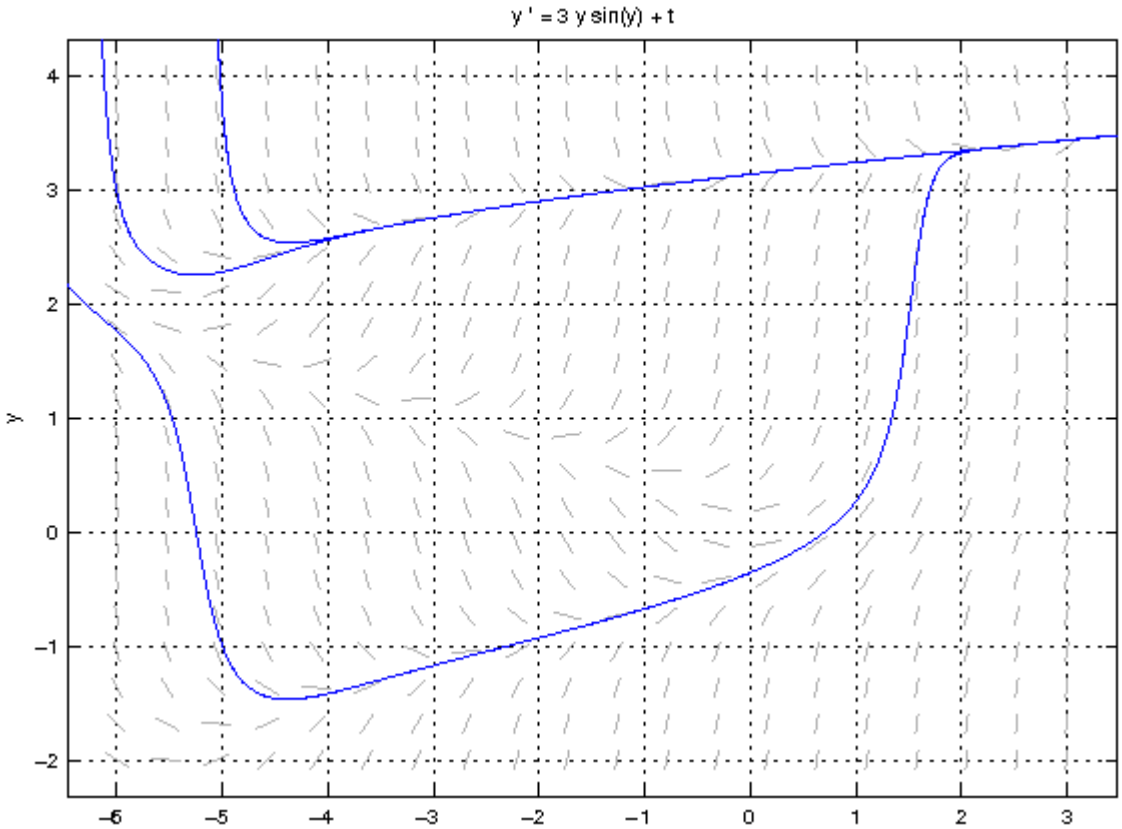
141. A scientist develops the logistic population model $P' = 0.2P(1 - \frac{P}{8})$ to describe her research data. This model has an equilibrium value of $P = 8$. In this model, from the initial condition $P_0 = 4$, the population will never reach the equilibrium value because:

(a) the population will grow toward infinity.

(b) the population is asymptotically approaching a value of 8.

(c) the population will drop to 0.

142. Do the solution trajectories shown below for the for the differential equation $y' = 3y\sin(y) + t$ ever simultaneously reach the same value?



(a) Yes
(b) No
(c) Can't tell with the information given

Bifurcations

143. How many equilibria does the differential equation $y' = y^2 + a$ have?

(a) Zero
(b) One
(c) Two
(d) Three
(e) Not enough information is given.

144. A *bifurcation* occurs if the number of equilibria of a system changes when we change the value of a parameter. For the differential equation $\frac{df}{dx} = bf^2 - 2$, a bifurcation occurs at what value of b?

(a) $b = 0$
(b) $b = 2$
(c) $b = -2$
(d) $b = 2/f^2$
(e) Not enough information is given.

145. $x'(t) = \frac{1}{2}x^2 + bx + 8$. If $b = 5$ what are the equilibria of the system?

(a) $x = -5$
(b) $x = -3, 3$
(c) $x = -8, -2$
(d) No equilibria exist and all solutions are increasing.
(e) No equilibria exist and all solutions are decreasing.

146. $x'(t) = \frac{1}{2}x^2 + bx + 8$. If $b = 2$ what are the equilibria of the system?

(a) $x = -8$

(b) $x = -2 \pm \sqrt{12}$
(c) $x = 2$
(d) No equilibria exist and all solutions are increasing.
(e) No equilibria exist and all solutions are decreasing.

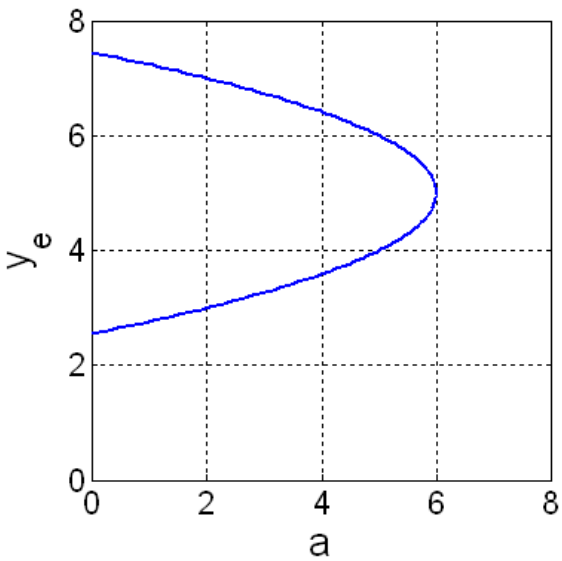
147. $x'(t) = \frac{1}{2}x^2 + bx + 8$. A bifurcation occurs where?

(a) $x = -b \pm \sqrt{b^2 - 16}$
(b) $b = 0$
(c) $b = \frac{1}{2}$
(d) $b = 4$
(e) $b = 8$
(f) Not enough information is given.

148. $\frac{dg}{dt} = g^3 + cg$. How many equilibria does this system have?

(a) Two
(b) One or two
(c) One or three
(d) Two or three
(e) Three
(f) Not enough information is given

149. A *bifurcation diagram* plots a system's equilibria on the y axis and the value of a parameter on the x axis. Consider the bifurcation diagram below. When our parameter is $a = 5$, what are the equilibria of the system?



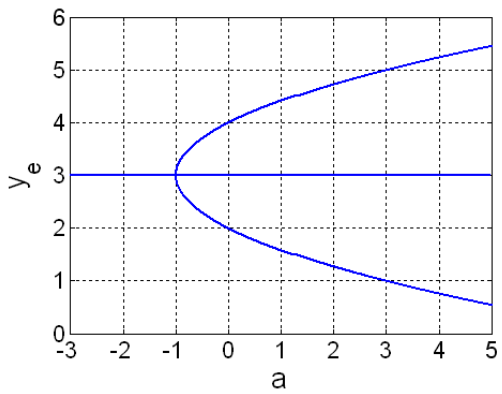
(a) $y = 0$ and $y = 3$

(b) $y = 5$

(c) $y = 4$ and $y = 6$

(d) Not enough information is given.

150. Consider the bifurcation diagram below. If our system has equilibria at $y = 1$, $y = 3$ and $y = 5$ what is the value of the parameter a?



(a) $a = -1$

(b) $a = 0$

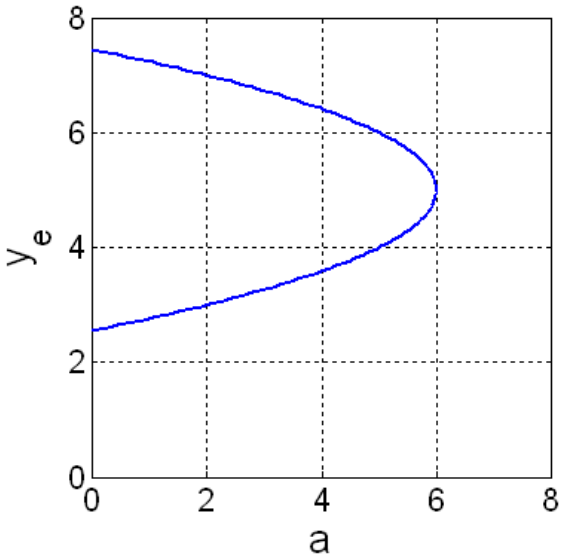
(c) $a = 1$

(d) $a = 3$

(e) $a = 5$

(f) Not enough information is given.

151. Consider the bifurcation diagram below. At what value of a does the system have a bifurcation?



(a) $a = 0$

(b) $a = 2$

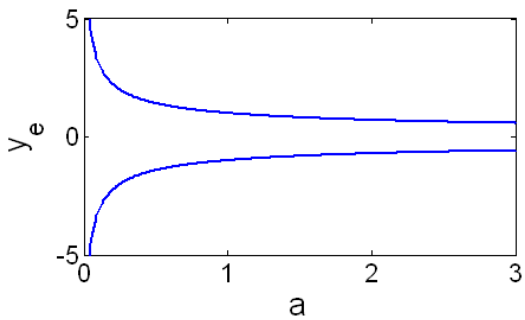
(c) $a = 4$

(d) $a = 6$

(e) $a = 8$

(f) Not enough information is given.

152. Which of the following differential equations is represented by the bifurcation diagram below?



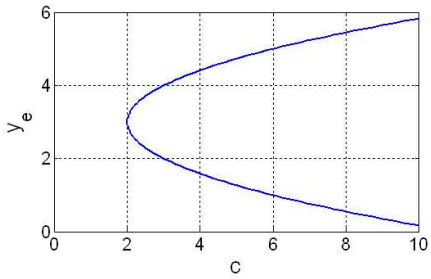
(a) $y' = y^2 + a$

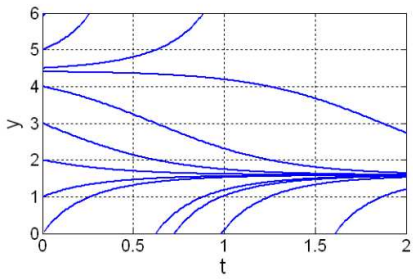
(b) $y' = ay^2 - 1$

(c) $y' = ay$

(d) $y' = y^2 + ay + 2$

153. The figure on the left is a bifurcation diagram, and the figure on the right plots several solution functions of this system for one specific value of the parameter. The figure on the right corresponds to what value of the parameter?





(a) $c = 0$

(b) $c = 2$

(c) $c = 4$

(d) $c = 6$

(e) $c = 8$

Linear Operators

154. Which differential operator is the appropriate one for the differential equation $ty'' + 2y' + ty = e^t$?

(a) $t\frac{d^2}{dt^2} + 2\frac{d}{dt} + t$

(b) $t\frac{d^2}{dt^2} + 2\frac{d}{dt} + t - e^t$

(c) $\frac{d^2}{dt^2}t + 2\frac{d}{dt} + t$

(d) $\frac{d^2}{dt^2} + 2\frac{d}{dt} + 1$

(e) None of the above

155. Let $L = t^2\frac{d^2}{dt^2} + 2t\frac{d}{dt} + 2$ be a differential operator. Evaluate $L[t^3]$.

(a) 0

(b) $14t^3$

(c) $2 + 12t^3$

(d) $6t^3 + 6t^2 + 2$

Second Order Differential Equations: Damping

156. Which of the following equations is not equivalent to $y'' + 3y' + 2y = 0$?

    (a) $2y'' + 6y' + 4y = 0$
    (b) $y'' = 3y' + 2y$
    (c) $-12y'' = 36y' + 24y$
    (d) $3y'' = -9y' - 6y$
    (e) All are equivalent.

157. Which of the following equations is equivalent to $y'' + \frac{2}{t}y' + \frac{3}{t^2}y = 0$?

    (a) $t^2y'' + 2ty' + 3y = 0$
    (b) $y'' + 2y' + 3y = 0$
    (c) $t^2y'' + \frac{t}{2}y' + \frac{1}{3}y = 0$
    (d) None are equivalent.

158. The motion of a child bouncing on a trampoline is modeled by the equation $p''(t) + 3p'(t) + 8p(t) = 0$ where p is in feet and t is in seconds. What will the child's acceleration be if he is 3 feet below equilibrium and moving up at 6 ft/s?

    (a) 6 ft/s^2 up
    (b) 6 ft/s^2 down
    (c) 42 ft/s^2 up
    (d) 42 ft/s^2 down
    (e) 39 ft/s^2 down
    (f) None of the above

159. The motion of a child on a trampoline is modeled by the equation $p''(t) + 2p'(t) + 3p(t) = 0$ where p is in feet and t is in seconds. Suppose we want the position function to be in inches instead of feet. How does this change the differential equation?

    (a) There is no change
    (b) $p''(t) + 24p'(t) + 36p(t) = 0$
    (c) $12p''(t) + 2p'(t) + 36p(t) = 0$
    (d) $144p''(t) + 24p'(t) + 3p(t) = 0$

(e) None of the above

160. A hydrogen atom is bound to a large molecule, and its distance from the molecule follows the equation $d'' + 4d' + 8d - 6 = 0$ where d is in picometers. At what distance from the molecule will the atom reach equilibrium?

(a) $d = 6$ pm.
(b) $d = \frac{3}{4}$ pm.
(c) $d = \frac{6}{13}$ pm
(d) No equilibrium exists.

161. When the space shuttle re-enters the Earth's atmosphere, the air resistance produces a force proportional to its velocity squared, and gravity produces an approximately constant force. Which of the following equations might model its position $p(t)$ if a and b are positive constants?

(a) $p'' + a(p')^2 + b = 0$
(b) $p'' - a(p')^2 + b = 0$
(c) $p'' + a(p')^2 + bp = 0$
(d) $p'' - a(p')^2 + bp = 0$
(e) None of the above

162. The differential equation $m\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + ky = 0$ with positive parameters m, γ, and k is often used to model the motion of a mass on a spring with a damping force. If γ was negative, what would this mean?

(a) This would be like a negative friction, making the oscillations speed up over time.
(b) This would be like a negative spring, that would push the object farther and farther from equilibrium.
(c) This would be like a negative mass, so that the object would accelerate in the opposite direction that the forces were pushing.
(d) None of the above

163. Test the following functions to see which is a solution to $y'' + 4y' + 3y = 0$.

(a) $y = e^{2t}$
(b) $y = e^{t}$
(c) $y = e^{-t}$

(d) $y = e^{-2t}$

(e) None of these are solutions.

(f) All are solutions.

164. Test the following functions to see which is a solution to $\frac{d^2g}{dx^2} + 2\frac{dg}{dx} + 2g = 0$.

(a) $g = e^x$

(b) $g = \sin x$

(c) $g = e^{-x} \sin x$

(d) None of these are solutions.

165. Suppose we want to solve the differential equation $y'' + ay' + by = 0$ and we conjecture that our solution is of the form $y = Ce^{rt}$. What equation do we get if we test this solution and simplify the result?

(a) $1 + ar + br^2 = 0$

(b) $C^2r^2 + Cr + c = 0$

(c) $Ce^{rt} + aCe^{rt} + bCe^{rt} = 0$

(d) $r^2 + ar + b = 0$

(e) None of the above

166. Suppose we want to solve the differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = 0$ and we conjecture that our solution is of the form $y = Ce^{rt}$. Solve the characteristic equation to determine what values of r satisfy the differential equation.

(a) $r = -2, -8$

(b) $r = -1, -4$

(c) $r = -3/2, +3/2$

(d) $r = 1, 4$

(e) None of the above

167. Find the general solution to $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0$.

(a) $y(t) = C_1e^{-t/2} + C_2e^{t/2}$

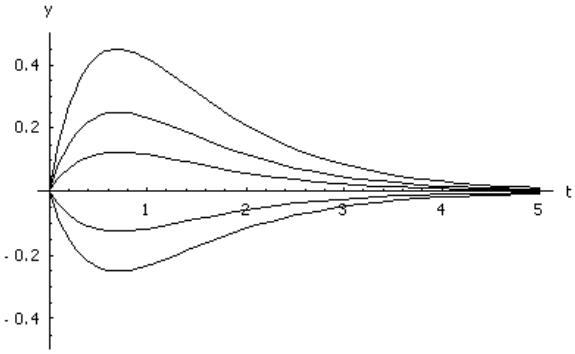
(b) $y(t) = C_1e^{-2t} + C_2e^{-t}$

(c) $y(t) = C_1e^{-2t} + C_2e^{t}$

(d) $y(t) = -2C_1e^{-2t} - C_2e^{-t}$

(e) None of the above

168. The graph below has five trajectories, call the top one y_1, the one below it y_2, down to the lowest one y_5. Which of these could be a solution of $y'' + 3y' + 2y = 0$ with $y(0) = 0$ and $y'(0) = 1$?



(a) y_1

(b) y_2

(c) y_3

(d) y_4

(e) y_5

169. What is the general solution to $f'' + 2f' + 2f = 0$?

(a) $f(x) = C_1e^{-x/2}\cos x + C_2e^{-x/2}\sin x$

(b) $f(x) = C_1e^{-x}\cos x + C_2e^{-x}\sin x$

(c) $f(x) = C_1e^{-x}\cos\frac{x}{2} + C_2e^{-x}\sin\frac{x}{2}$

(d) $f(x) = C_1 + C_2e^{-2x}$

(e) None of the above

170. The harmonic oscillator modeled by $mx'' + bx' + kx = 0$ with parameters $m = 1$, $k = 2$, and $b = 1$ is underdamped and thus oscillates. What is the period of the oscillations?

(a) $2\pi/\sqrt{7}$

(b) $4\pi/\sqrt{7}$

(c) $\sqrt{7}/2\pi$

(d) $\sqrt{7}/4\pi$

(e) None of the above.

171. A harmonic oscillator is modeled by $mx'' + bx' + kx = 0$. If we increase the parameter m slightly, what happens to the period of oscillation?

(a) The period gets larger.
(b) The period gets smaller.
(c) The period stays the same.

172. A harmonic oscillator is modeled by $mx'' + bx' + kx = 0$. If we increase the parameter k slightly, what happens to the period of oscillation?

(a) The period gets larger.
(b) The period gets smaller.
(c) The period stays the same.

173. A harmonic oscillator is modeled by $mx'' + bx' + kx = 0$. If we increase the parameter b slightly, what happens to the period of oscillation?

(a) The period gets larger.
(b) The period gets smaller.
(c) The period stays the same.

174. Classify the harmonic oscillator described below:

$$3\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + y = 0.$$

(a) underdamped
(b) overdamped
(c) critically damped
(d) no damping

175. Does the harmonic oscillator described below oscillate?

$$3\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + y = 0.$$

(a) Yes.
(b) No.

Linear Combinations and Independence of Functions

176. Which of the following expressions is a linear combination of the functions $f(t)$ and $g(t)$?

    (a) $2f(t) + 3g(t) + 4$
    (b) $f(t) - 2g(t) + t$
    (c) $2f(t)g(t) - 3f(t)$
    (d) $f(t) - g(t)$
    (e) All of the above
    (f) None of the above

177. **True or False** The function $h(t) = 4 + 3t$ is a linear combination of the functions $f(t) = (1 + t)^2$ and $g(t) = 2 - t - 2t^2$.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

178. **True or False** The function $h(t) = \sin(t + 2)$ is a linear combination of the functions $f(t) = \sin t$ and $g(t) = \cos t$.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

179. **True or False** $h(t) = t^2$ is a linear combination of $f(t) = (1 - t)^2$ and $g(t) = (1 + t)^2$.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

180. Let $y_1(t) = \sin(2t)$. For which of the following functions $y_2(t)$ will $\{y_1(t), y_2(t)\}$ be a linearly independent set?

(a) $y_2(t) = \sin(t)\cos(t)$
(b) $y_2(t) = 2\sin(2t)$
(c) $y_2(t) = \cos(2t - \pi/2)$
(d) $y_2(t) = \sin(-2t)$
(e) All of the above
(f) None of the above

181. Let $y_1(t) = e^{2t}$. For which of the following functions $y_2(t)$ will $\{y_1(t), y_2(t)\}$ be a linearly independent set?

(a) $y_2(t) = e^{-2t}$
(b) $y_2(t) = te^{2t}$
(c) $y_2(t) = 1$
(d) $y_2(t) = e^{3t}$
(e) All of the above
(f) None of the above

182. The functions $y_1(t)$ and $y_2(t)$ are linearly independent on the interval $a < t < b$ if

(a) for some constant k, $y_1(t) = ky_2(t)$ for $a < t < b$.
(b) there exists some $t_0 \in (a, b)$ and some constants c_1 and c_2 such that $c_1y_1(t_0) + c_2y_2(t_0) \neq 0$.
(c) the equation $c_1y_1(t) + c_2y_2(t) = 0$ holds for all $t \in (a, b)$ only if $c_1 = c_2 = 0$.
(d) the ratio $y_1(t)/y_2(t)$ is a constant function.
(e) All of the above
(f) None of the above

183. The functions $y_1(t)$ and $y_2(t)$ are linearly dependent on the interval $a < t < b$ if

(a) there exist two constants c_1 and c_2 such that $c_1y_1(t)+c_2y_2(t) = 0$ for all $a < t < b$.
(b) there exist two constants c_1 and c_2, not both 0, such that $c_1y_1(t)+c_2y_2(t) = 0$ for all $a < t < b$.
(c) for each t in (a, b), there exists constants c_1 and c_2 such that $c_1y_1(t)+c_2y_2(t) = 0$.

(d) for some $a < t_0 < b$, the equation $c_1 y_1(t_0) + c_2 y_2(t_0) = 0$ can only be true if $c_1 = c_2 = 0$.

(e) All of the above

(f) None of the above

184. The functions $y_1(t)$ and $y_2(t)$ are both solutions of a certain second-order linear homogeneous differential equation with continuous coefficients for $a < t < b$. Which of the following statements are true?

(i) The general solution to the ODE is $y(t) = c_1 y_1(t) + c_2 y_2(t)$, $a < t < b$.

(ii) $y_1(t)$ and $y_2(t)$ must be linearly independent, since they both are solutions.

(iii) $y_1(t)$ and $y_2(t)$ may be linearly dependent, in which case we do not know enough information to write the general solution.

(iv) The Wronskian of $y_1(t)$ and $y_2(t)$ must be nonzero for these functions.

(a) Only (i) and (ii) are true.

(b) Only (i) is true.

(c) Only (ii) and (iv) are true.

(d) Only (iii) is true.

(e) None are true.

185. Can the functions $y_1(t) = t$ and $y_2(t) = t^2$ be a linearly independent pair of solutions for an ODE of the form

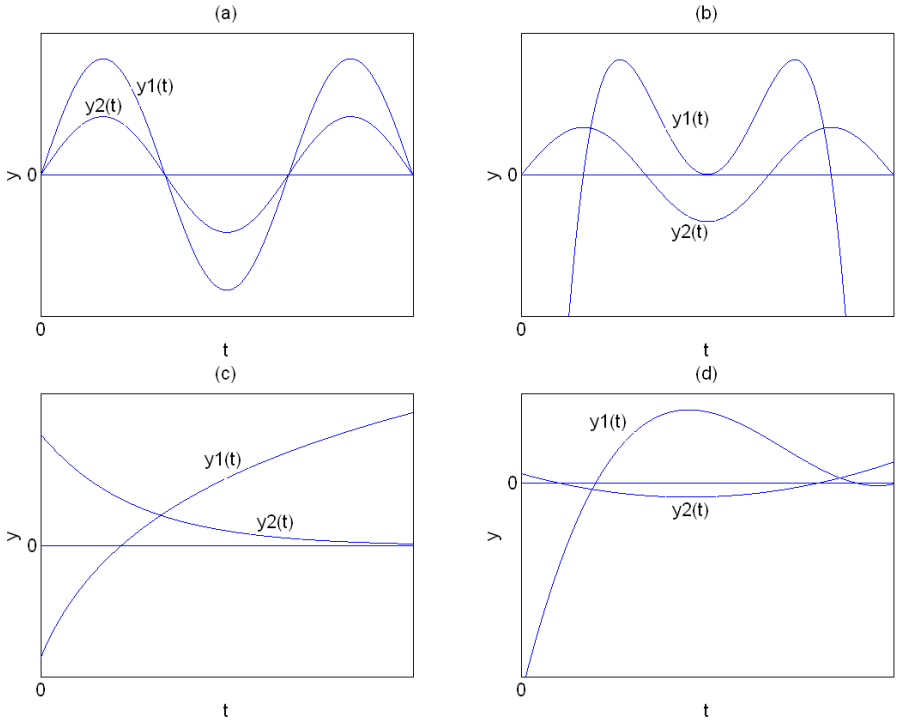
$$y'' + p(t)y' + q(t)y = 0 \qquad -1 \leq t \leq 1$$

where $p(t)$ and $q(t)$ are continuous functions?

(a) Yes

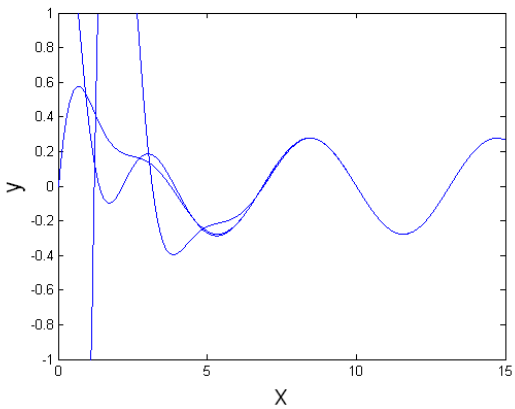
(b) No

186. Which pair of functions whose graphs are shown below could be linearly independent pairs of solutions to a second-order linear homogeneous differential equation?



Second Order Differential Equations: Forcing

187. The three functions plotted below are all solutions of $y'' + ay' + 4y = \sin x$. Is a positive or negative?



(a) a is positive.

(b) a is negative.

(c) $a = 0$.

(d) Not enough information is given.

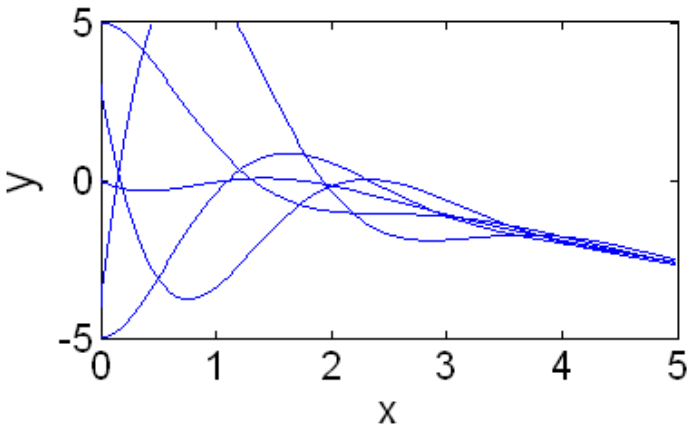
188. If we conjecture the function $y(x) = C_1 \sin 2x + C_2 \cos 2x + C_3$ as a solution to the differential equation $y''+4y = 8$, which of the constants is determined by the differential equation?

(a) C_1
(b) C_2
(c) C_3
(d) None of them are determined.

189. What will the solutions of $y'' + ay' + by = c$ look like if b is negative and a is positive.

(a) Solutions will oscillate at first and level out at a constant.
(b) Solutions will grow exponentially.
(c) Solutions will oscillate forever.
(d) Solutions will decay exponentially.

190. The functions plotted below are solutions to which of the following differential equations?



(a) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 3 - 3x$
(b) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 3e^{2x}$
(c) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = \sin \frac{2\pi}{9}x$
(d) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 2x^2 + 3x - 4$
(e) None of the above

191. The general solution to $f'' + 7f' + 12f = 0$ is $f(t) = C_1e^{-3t} + C_2e^{-4t}$. What should we conjecture as a particular solution to $f'' + 7f' + 12f = 5e^{-2t}$?

(a) $f(t) = Ce^{-4t}$
(b) $f(t) = Ce^{-3t}$
(c) $f(t) = Ce^{-2t}$
(d) $f(t) = C\cos 2t$
(e) None of the above

192. The general solution to $f'' + 7f' + 12f = 0$ is $f(t) = C_1e^{-3t} + C_2e^{-4t}$. What is a particular solution to $f'' + 7f' + 12f = 5e^{-6t}$?

(a) $f(t) = \frac{5}{6}e^{-6t}$
(b) $f(t) = \frac{5}{31}e^{-6t}$
(c) $f(t) = \frac{5}{20}e^{-6t}$
(d) $f(t) = e^{-3t}$
(e) None of the above

193. To find a particular solution to the differential equation

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \cos t,$$

we replace it with a new differential equation that has been "complexified." What is the new differential equation to which we will find a particular solution?

(a) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{2it}$
(b) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{3it}$
(c) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{it}$
(d) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-2it}$
(e) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-it}$
(f) None of the above.

194. To solve the differential equation $\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y = e^{it}$, we make a guess of $y_p(t) = Ce^{it}$. What equation results when we evaluate this in the differential equation?

(a) $-Ce^{it} + 3Cie^{it} + 2Ce^{it} = Ce^{it}$

(b) $-Ce^{it} + 3Cie^{it} + 2Ce^{it} = e^{it}$

(c) $Cie^{it} + 3Ce^{it} + 2Ce^{it} = e^{it}$

(d) $-Ce^{it} + 3Ce^{it} + 2Ce^{it} = Ce^{it}$

195. To solve the differential equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{it}$, we make a guess of $y_p(t) = Ce^{it}$. What value of C makes this particular solution work?

(a) $C = \frac{1+3i}{10}$

(b) $C = \frac{1-3i}{10}$

(c) $C = \frac{1+3i}{\sqrt{10}}$

(d) $C = \frac{1-3i}{\sqrt{10}}$

196. In order to find a particular solution to $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \cos t$, do we want the real part or the imaginary part of the particular solution $y_p(t) = Ce^{it}$ that solved the complexified equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{it}$?

(a) Real part

(b) Imaginary part

(c) Neither, we need the whole solution to the complexified equation.

197. What is a particular solution to $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \cos t$?

(a) $y_p(t) = \frac{3}{10}\cos t + \frac{1}{10}\sin t$

(b) $y_p(t) = \frac{-3}{10}\cos t + \frac{1}{10}\sin t$

(c) $y_p(t) = \frac{1}{10}\cos t + \frac{-3}{10}\sin t$

(d) $y_p(t) = \frac{1}{10}\cos t + \frac{3}{10}\sin t$

Beats and Resonance

198. Which of the following forced 2nd order equations has solutions exhibiting *resonance*?

    (a) $y'' + y = \cos(t)$
    (b) $y'' + y = 2\cos(t)$
    (c) $y'' + y = -2\cos(t)$
    (d) All of the above
    (e) None of the above

199. Which of the following forced 2nd order equations has solutions exhibiting *resonance*?

    (a) $2y'' + y = \cos(t)$
    (b) $2y'' + 4y = 2\cos(2t)$
    (c) $4y'' + y = -2\cos(t/2)$
    (d) All of the above
    (e) None of the above

200. Which of the following forced 2nd order equations has solutions exhibiting *resonance*?

    (a) $y'' + 2y = 10\cos(2t)$
    (b) $y'' + 4y = 8\cos(2t)$
    (c) $y'' + 2y = 6\cos(4t)$
    (d) All of the above
    (e) None of the above

201. Which of the following forced 2nd order equations has solutions clearly exhibiting *beats*?

    (a) $y'' + 3y = 10\cos(2t)$
    (b) $y'' + 1y = 2\cos(2t)$
    (c) $y'' + 9y = 1\cos(3t)$
    (d) All of the above
    (e) None of the above

202. The differential equation $y'' + 100y = 2\cos(\omega t)$ has solutions displaying *resonance* when

(a) $\omega = 10,000$
(b) $\omega = 10$
(c) $\omega = 9$
(d) All of the above
(e) None of the above

203. The differential equation $y'' + 100y = 2\cos(\omega t)$ has solutions displaying *beats* when

(a) $\omega = 10,000$
(b) $\omega = 10$
(c) $\omega = 9$
(d) All of the above
(e) None of the above

204. The differential equation $y'' + 4y = 2\cos(2t)$ has solutions clearly displaying

(a) beats
(b) damping
(c) resonance
(d) All of the above
(e) None of the above

205. The differential equation $4y'' + y = 2\cos(4t)$ has solutions clearly displaying

(a) beats
(b) damping
(c) resonance
(d) All of the above
(e) None of the above

206. The differential equation $4y'' + 4y = 2\cos(t)$ has solutions clearly displaying

(a) beats
(b) damping
(c) resonance
(d) All of the above
(e) None of the above

Second Order Differential Equations as Systems

207. A standard approach to converting second order equations such as $x'' = x' - 2x + 4$ is to introduce a new variable, y, such that:

    (a) $y' = x$
    (b) $y = x'$
    (c) $y = x$
    (d) $y' = x'$

208. A first-order system equivalent to the second order differential equation $x''+2x'+x = 2$ is:

    (a)
    $$\begin{aligned} x' &= y \\ y' &= x - 2x' + 2 \end{aligned}$$

    (b)
    $$\begin{aligned} x' &= y \\ y' &= -2x + y + 2 \end{aligned}$$

    (c)
    $$\begin{aligned} x' &= y \\ y' &= -x - 2y + 2 \end{aligned}$$

    (d)
    $$\begin{aligned} x' &= y \\ y' &= -x + 2y + 2 \end{aligned}$$

209. Which second-order differential equation is equivalent to the first-order system below?

$$\begin{aligned} x' &= y \\ y' &= 2x + 4y \end{aligned}$$

(a) $y' = 2x + 4x'$
(b) $x'' - 4x' - 2x = 0$
(c) $x'' + 4x' + 2x = 0$
(d) None of the above

210. Which second-order differential equation is equivalent to the first-order system below?

$$\begin{aligned} x' &= -3x + y \\ y' &= x - 2y \end{aligned}$$

(a) $x'' + 5x' + 5x = 0$
(b) $x' = -3x + x' + 3x$
(c) $y'' + 5y' + 5y = 0$
(d) $y' = -2y - (y' + 2y)$
(e) This system can not be converted to a second-order equation.

211. In the spring mass system described by $x'' = -2x' - 2x$, what does the variable x represent?

(a) The spring's displacement from equilibrium
(b) The mass's displacement from equilibrium
(c) The spring's velocity
(d) The mass's velocity
(e) None of the above

212. The spring mass system described by $x'' = -2x' - 2x$, can be converted to a first-order system by introducing the new variable $y = x'$. What does y represent?

(a) The mass's displacement from equilibrium
(b) The mass's velocity
(c) The mass's acceleration
(d) None of the above

213. A first-order system equivalent to the spring mass system $x'' = -2x' - 2x$ is:

(a)

$$\begin{aligned} x' &= y \\ y' &= -2x - 2x' \end{aligned}$$

(b)

$$\begin{aligned} x' &= y \\ y' &= -2y + 2x \end{aligned}$$

(c)

$$\begin{aligned} x' &= y \\ y' &= -2y - 2x \end{aligned}$$

(d)

$$\begin{aligned} x' &= y \\ y' &= 2y - 2x \end{aligned}$$

214. The solution to the spring mass system $x'' = -2x' - 2x$ is:

(a) $\begin{bmatrix} x \\ y \end{bmatrix} = c_1 e^{-2t} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_2 e^{t} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

(b) $\begin{bmatrix} y \\ x \end{bmatrix} = c_1 e^{-2t} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_2 e^{t} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

(c) $\begin{bmatrix} x \\ y \end{bmatrix} = e^{-t}\left(c_1 \cos t + c_2 \sin t\right) \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{-t}\left(-c_1 \sin t + c_2 \cos t\right) \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} x \\ y \end{bmatrix} = e^{-t}\left(c_1 \cos t + c_2 \sin t\right) \begin{bmatrix} 1 \\ -2 \end{bmatrix} + e^{-t}\left(-c_1 \sin t + c_2 \cos t\right) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

215. The position of the mass in the spring mass system $x'' = -2x' - 2x$ is given by:

(a) $y = -2c_1 e^{-t} \cos t - 2c_2 e^{-t} \sin t$

(b) $y = c_1 e^{-t}\left(\cos t - \sin t\right) + c_2 e^{-t}\left(\cos t + \sin t\right)$

(c) $x = c_1 e^{-t}\left(\cos t - \sin t\right) + c_2 e^{-t}\left(\cos t + \sin t\right)$

(d) $x = -2c_1 e^{-t} \cos t - 2c_2 e^{-t} \sin t$

Phase Portraits and Vector Fields of Systems

216. If we were graphing a vector field in the phase plane of the linear system $Y' = \begin{bmatrix} -4 & 2 \\ 2 & 4 \end{bmatrix} Y$, what slope would we graph when $y_1 = 1$ and $y_2 = 2$?

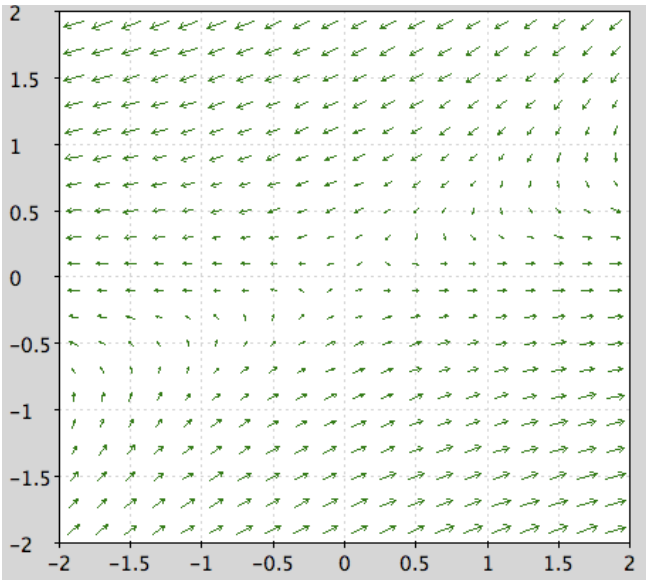
   (a) 0

   (b) ∞ (vertical)

   (c) 1

   (d) None of the above

217. Which linear system matches the direction field below?



   (a)

$$\begin{aligned} x' &= y \\ y' &= 2y - x \end{aligned}$$

   (b)

$$\begin{aligned} x' &= x - 2y \\ y' &= -y \end{aligned}$$

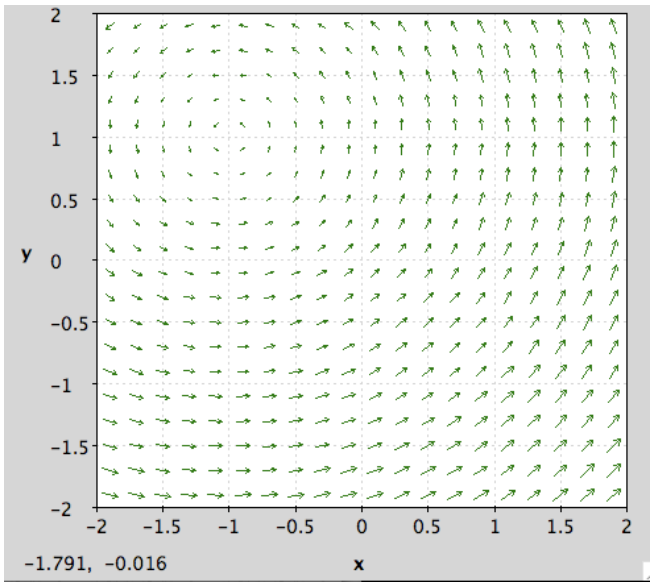
(c)

$$\begin{aligned} x' &= x^2 - 1 \\ y' &= -y \end{aligned}$$

(d)

$$\begin{aligned} x' &= x + 2y \\ y' &= -y \end{aligned}$$

218. Suppose you have the direction field below. At time $t = -2$, you know that $x = -2$ and $y = -.25$. What do you predict is the value of y when $t = 0$?



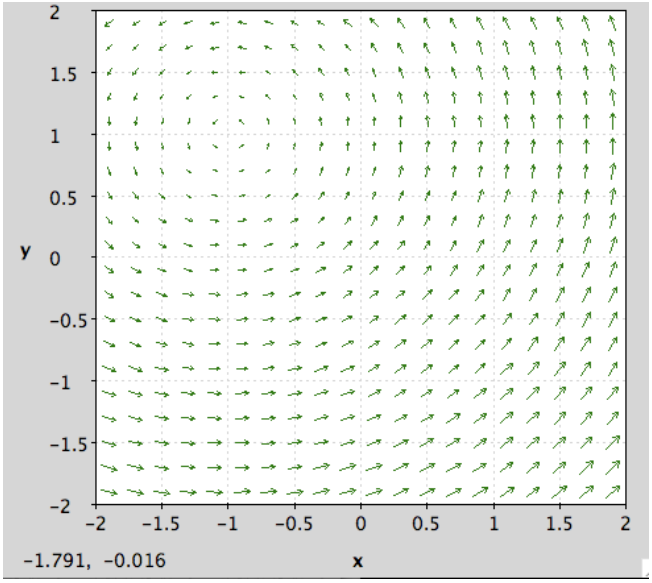
(a) $y \approx -1$

(b) $y \approx 1$

(c) $y \approx 0$

(d) We cannot tell from the information given.

219. Suppose you have the direction field below. We know that at time $t = 0$, we have $x = -2$ and $y = -.25$. The pair $(x(t), y(t))$ is a solution that satisfies the initial conditions. When $y(t) = 0$, about what should $x(t)$ be equal to?



(a) $x \approx -2$

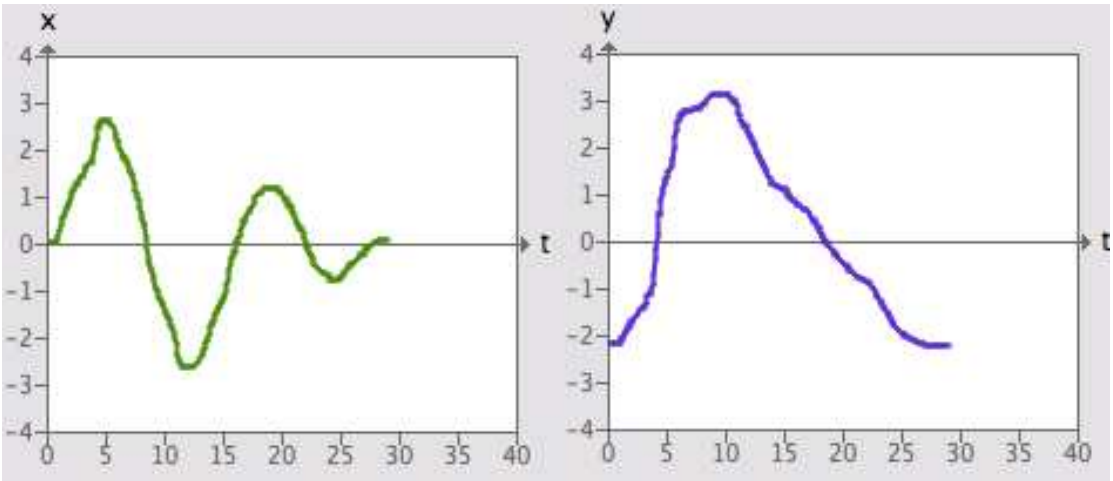
(b) $x \approx -1$

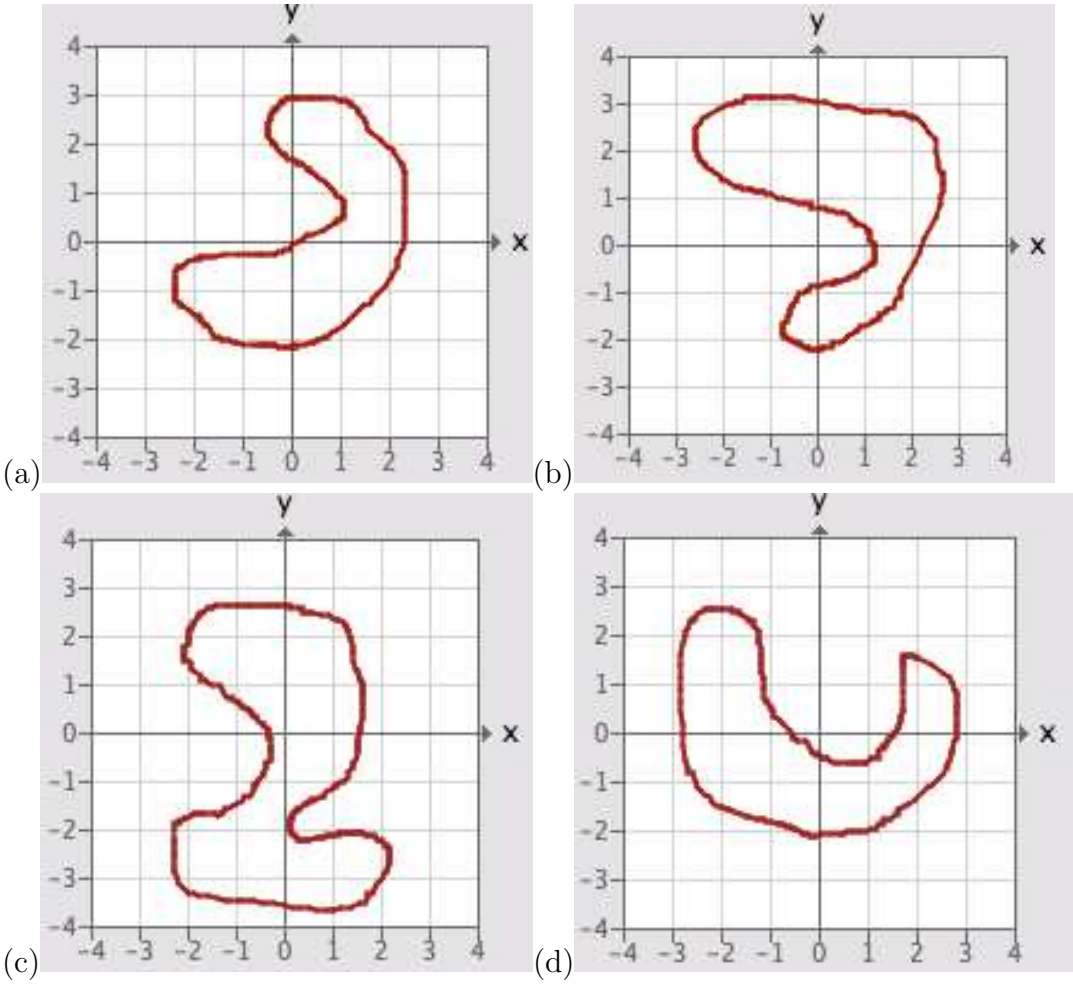
(c) $x \approx 0$

(d) $x \approx 1$

(e) We cannot tell from the information given.

220. Which of the following solution curves in the phase plane might correspond to the solution functions $x(t)$ and $y(t)$ graphed below.





Testing Solutions to Linear Systems

221. We want to test the solution $x_1 = -e^{-2t}$ and $x_2 = e^{-2t}$ in the system

$$\begin{aligned} x_1' &= x_1 + 3x_2 \\ x_2' &= 3x_1 + x_2 \end{aligned}$$

What equations result from substituting the solution into the equation?

(a)

$$\begin{aligned} -e^{-2t} &= -e^{-2t} + 3e^{-2t} \\ e^{-2t} &= -3e^{-2t} + e^{-2t} \end{aligned}$$

(b)

$$\begin{aligned} -e^{-2t} &= e^{-2t} - 3e^{-2t} \\ e^{-2t} &= 3e^{-2t} - e^{-2t} \end{aligned}$$

(c)

$$\begin{aligned} 2e^{-2t} &= -e^{-2t} + 3e^{-2t} \\ -2e^{-2t} &= -3e^{-2t} + e^{-2t} \end{aligned}$$

(d)

$$\begin{aligned} -2e^{-2t} &= e^{-2t} - 3e^{-2t} \\ 2e^{-2t} &= 3e^{-2t} - e^{-2t} \end{aligned}$$

(e) None of the above

222. Is $x_1 = x_2 = x_3 = e^t$ a solution to the system below?

$$\begin{aligned} x_1' &= 3x_1 - x_2 + x_3 \\ x_2' &= 2x_1 - x_3 \\ x_3' &= x_1 - x_2 + x_3 \end{aligned}$$

(a) Yes, it is a solution.
(b) No, it is not a solution because it does not satisfy $x_1' = 3x_1 - x_2 + x_3$.
(c) No, it is not a solution because it does not satisfy $x_2' = 2x_1 - x_3$.
(d) No, it is not a solution because it does not satisfy $x_3' = x_1 - x_2 + x_3$.
(e) No, it is not a solution because it doesn't work in any equation for all values of t.

223. Which of the following are solutions to the system below?

$$\begin{aligned} x_1' &= 4x_1 - x_2 \\ x_2' &= 2x_1 + x_2 \end{aligned}$$

(a)

$$\begin{aligned} x_1 &= e^{2t} \\ x_2 &= e^{2t} \end{aligned}$$

(b)

$$\begin{aligned} x_1 &= e^{2t} \\ x_2 &= 2e^{2t} \end{aligned}$$

(c)

$$\begin{aligned} x_1 &= e^{3t} \\ x_2 &= e^{-3t} \end{aligned}$$

(d) None of the above.

(e) All of the above.

224. Since we know that both $x_1 = x_2 = e^{3t}$ and $x_1 = e^{-t}, x_2 = -e^{-t}$ are solutions to the system

$$\begin{aligned} x_1' &= x_1 + 2x_2 \\ x_2' &= 2x_1 + x_2 \end{aligned}$$

Which of the following are also solutions?

(a)

$$\begin{aligned} x_1 &= 3e^{3t} - e^{-t} \\ x_2 &= 3e^{3t} + e^{-t} \end{aligned}$$

(b)

$$\begin{aligned} x_1 &= -e^{3t} - e^{-t} \\ x_2 &= -e^{3t} + e^{-t} \end{aligned}$$

(c)

$$\begin{aligned} x_1 &= 2e^{3t} + 4e^{-t} \\ x_2 &= -4e^{-t} + 2e^{3t} \end{aligned}$$

(d)

$$\begin{aligned} x_1 &= 0 \\ x_2 &= 0 \end{aligned}$$

(e) None of the above.

(f) All of the above.

225. Consider the system of differential equations,

$$y'(t) = \begin{bmatrix} 14 & 0 & -4 \\ 2 & 13 & -8 \\ -3 & 0 & 25 \end{bmatrix} y(t)$$

.

Which of the following functions solve this system?

(a) $y(t) = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} e^{-4t}$

(b) $y(t) = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} e^{6t}$

(c) $y(t) = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix} e^{13t}$

(d) None of the above.

(e) All of the above.

Euler's Method and Systems of Equations

226. We have the system of differential equations $x' = 3x - 2y$ and $y' = 4y^2 - 7x$. If we know that $x(0) = 2$ and $y(0) = 1$, estimate the values of x and y at $t = 0.1$.

(a) $x(0.1) = 4$, $y(0.1) = -10$
(b) $x(0.1) = 6$, $y(0.1) = -9$
(c) $x(0.1) = 2.4$, $y(0.1) = 0$
(d) $x(0.1) = 0.4$, $y(0.1) = -1$
(e) None of the above

227. We have the system of differential equations $x' = x(-x-2y+5)$ and $y' = y(-x-y+10)$. If we know that $x(4.5) = 3$ and $y(4.5) = 2$, estimate the values of x and y at $t = 4$.

(a) $x(4) = 0$, $y(4) = -3$
(b) $x(4) = 6$, $y(4) = 10$
(c) $x(4) = 6$, $y(4) = 7$
(d) None of the above

228. We have a system of differential equations for $\frac{dx}{dt}$ and $\frac{dy}{dt}$, along with the initial conditions that $x(0) = 5$ and $y(0) = 7$. We want to know the value of these functions when $t = 5$. Using Euler's method and $\Delta t = 1$ we get the result that $x(5) \approx 14.2$ and $y(5) \approx 23.8$. Next, we use Euler's method again with $\Delta t = 0.5$ and find that $x(5) \approx 14.6$ and $y(5) \approx 5.3$. Finally we use $\Delta t = 0.25$, finding that $x(5) \approx 14.8$ and $y(5) \approx -3.7$. What does this mean?

(a) Fewer steps means fewer opportunities for error, so $(x(5), y(5)) \approx (14.2, 23.8)$.
(b) Smaller stepsize means smaller errors, so $(x(5), y(5)) \approx (14.8, -3.7)$.
(c) We have no way of knowing whether any of these estimates is anywhere close to the true values of $(x(5), y(5))$.
(d) At these step sizes we can conclude that $x(5) \approx 15$, but we can only conclude that $y(5) < -3.7$.
(e) Results like this are impossible: We must have made an error in our calculations.

Modeling with Systems

229. In the predator - prey population model

$$\begin{aligned} \frac{dx}{dt} &= ax - \frac{ax^2}{N} - bxy \\ \frac{dy}{dt} &= cy + kxy \end{aligned}$$

with $a > 0$, $b > 0$, $c > 0$, $N > 0$, and $k > 0$,

which variable represents the predator population?

(a) x

(b) $\frac{dx}{dt}$

(c) y

(d) $\frac{dy}{dt}$

230. In which of the following predator - prey population models does the prey have the highest intrinsic reproduction rate?

(a)

$$\begin{aligned} P' &= 2P - 3Q * P \\ Q' &= -Q + 1/2Q * P \end{aligned}$$

(b)

$$\begin{aligned} P' &= P(1 - 4Q) \\ Q' &= Q(-2 + 3P) \end{aligned}$$

(c)

$$\begin{aligned} P' &= P(3 - 2Q) \\ Q' &= Q(-1 + P) \end{aligned}$$

(d)

$$\begin{aligned} P' &= 4P(1/2 - Q) \\ Q' &= Q(-1.5 + 2P) \end{aligned}$$

231. For which of the following predator - prey population models is the predator most successful at catching prey?

(a)

$$\begin{aligned} \frac{dx}{dt} &= 2x - 3x * y \\ \frac{dy}{dt} &= -y + 1/2x * y \end{aligned}$$

(b)

$$\begin{aligned}\frac{dx}{dt} &= x(1-4y)\\ \frac{dy}{dt} &= y(-2+3x)\end{aligned}$$

(c)

$$\begin{aligned}\frac{dx}{dt} &= x(3-2y)\\ \frac{dy}{dt} &= y(-1+x)\end{aligned}$$

(d)

$$\begin{aligned}\frac{dx}{dt} &= 4x(1/2-y)\\ \frac{dy}{dt} &= 2y(-1/2+x)\end{aligned}$$

232. In this predator - prey population model

$$\begin{aligned}\frac{dx}{dt} &= -ax+bxy\\ \frac{dy}{dt} &= cy-dxy\end{aligned}$$

with $a > 0$, $b > 0$, $c > 0$, and $d > 0$,

does the prey have limits to its population other than that imposed by the predator?

(a) Yes
(b) No
(c) Can not tell

233. In this predator - prey population model

$$\begin{aligned}\frac{dx}{dt} &= ax-\frac{ax^2}{N}-bxy\\ \frac{dy}{dt} &= cy+kxy\end{aligned}$$

with $a > 0$, $b > 0$, $c > 0$, and $k > 0$,

if the prey becomes extinct, will the predator survive?

(a) Yes

(b) No

(c) Can not tell

234. In this predator - prey population model

$$\begin{aligned} \frac{dx}{dt} &= ax - \frac{ax^2}{N} - bxy \\ \frac{dy}{dt} &= cy + kxy \end{aligned}$$

with $a > 0$, $b > 0$, $c > 0$, $N > 0$, and $k > 0$,

are there any limits on the prey's population other than the predator?

(a) Yes

(b) No

(c) Can not tell

235. On Komodo Island we have three species: Komodo dragons (K), deer (D), and a variety of plant (P). The dragons eat the deer and the deer eat the plant. Which of the following systems of differential equations could represent this scenario?

(a)

$$\begin{aligned} K' &= aK - bKD \\ D' &= cD + dKD - eDP \\ P' &= -fP + gDP \end{aligned}$$

(b)

$$\begin{aligned} K' &= -aK + bKD \\ D' &= -cD - dKD + eDP \\ P' &= fP - gDP \end{aligned}$$

(c)

$$\begin{aligned} K' &= aK - bKD + KP \\ D' &= cD + dKD - eDP \\ P' &= -fP + gDP - hKP \end{aligned}$$

(d)

$$\begin{aligned} K' &= -aK + bKD - KP \\ D' &= -cD - dKD + eDP \\ P' &= fP - gDP + hKP \end{aligned}$$

236. In the two species population model

$$\begin{aligned} R' &= 2R - bFR \\ F' &= -F + 2FR \end{aligned}$$

for what value of the parameter b will the system have a stable equilibrium?

(a) $b < 0$

(b) $b = 0$

(c) $b > 0$

(d) For no value of b

237. Two forces are fighting one another. x and y are the number of soldiers in each force. Let a and b be the offensive fighting capacities of x and y, respectively. Assume that forces are lost only to combat, and no reinforcements are brought in. What system represents this scenario?

(a)

$$\begin{aligned} \frac{dx}{dt} &= -ay \\ \frac{dy}{dt} &= -bx \end{aligned}$$

(b)

$$\begin{aligned} \frac{dx}{dt} &= -by \\ \frac{dy}{dt} &= -ax \end{aligned}$$

(c)

$$\begin{aligned} \frac{dx}{dt} &= y - a \\ \frac{dy}{dt} &= x - b \end{aligned}$$

(d)

$$\begin{aligned}\frac{dx}{dt} &= y - b \\ \frac{dy}{dt} &= x - a\end{aligned}$$

238. Two forces, x and y, are fighting one another. Let a and b be the fighting efficiences of x and y, respectively. Assume that forces are lost only to combat, and no reinforcements are brought in. How does the size of the y army change with respect to the size of the x army?

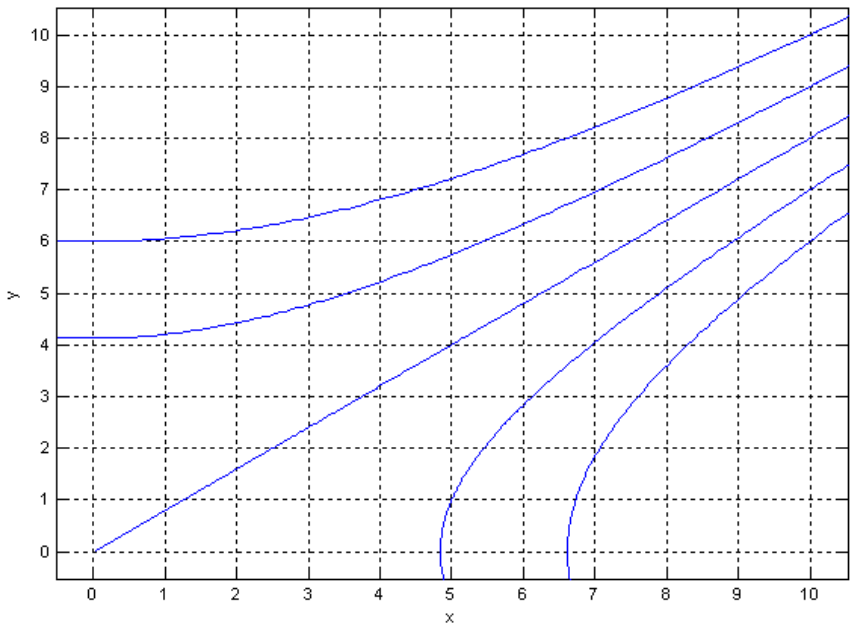
(a) $\frac{dy}{dx} = \frac{ax}{by}$

(b) $\frac{dy}{dx} = \frac{x}{y}$

(c) $\frac{dy}{dx} = \frac{y}{x}$

(d) $\frac{dy}{dx} = -by - ax$

239. Two forces, x and y, are fighting one another. Assume that forces are lost only to combat, and no reinforcements are brought in. Based on the phase plane below, if $x(0) = 10$ and $y(0) = 7$, who wins?



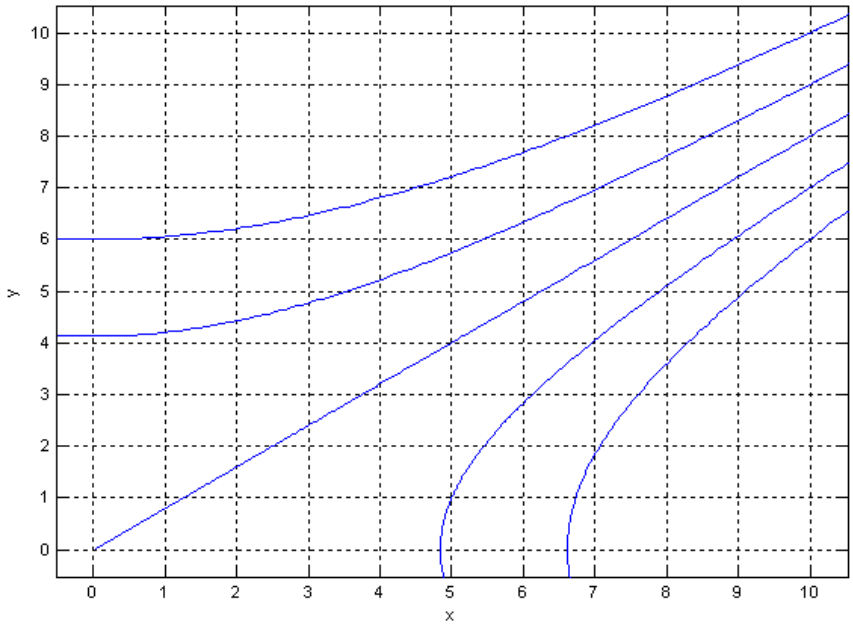
(a) x wins

(b) y wins

(c) They tie.

(d) Neither wins - both armies grow, and the battles escalate forever.

240. Two forces, x and y, are fighting one another. Assume that forces are lost only to combat, and no reinforcements are brought in. Based on the phase plane below, which force has a greater offensive fighting efficiency?



(a) x has the greater fighting efficiency.

(b) y has the greater fighting efficiency.

(c) They have the same fighting efficiencies.

241. Two forces, x and y, are fighting one another. Assume that forces are lost only to combat, and no reinforcements are brought in. You are the x-force, and you want to improve your chance of winning. Assuming that it would be possible, would you rather double your fighting efficiency or double your number of soldiers?

(a) Double the fighting efficiency

(b) Double the number of soldiers

(c) These would both have the same effect

Solutions to Linear Systems

242. Consider the linear system given by

$$\frac{d\vec{Y}}{dt} = \begin{bmatrix} 2 & 3 \\ 0 & -4 \end{bmatrix} \vec{Y}.$$

True or False: $\vec{Y_1}(t) = \begin{pmatrix} e^{2t} \\ 0 \end{pmatrix}$ is a solution.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

243. Consider the linear system $\frac{d\vec{Y}}{dt} = \begin{bmatrix} 2 & 3 \\ 0 & -4 \end{bmatrix} \vec{Y}$ with solution $\vec{Y_1}(t) = \begin{pmatrix} e^{2t} \\ 0 \end{pmatrix}$.

True or False: The function $k \cdot \vec{Y_1}(t)$ formed by multiplying $\vec{Y_1}(t)$ by a constant k is also a solution to the linear system.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

244. Consider the linear system $\frac{d\vec{Y}}{dt} = \begin{bmatrix} 2 & 3 \\ 0 & -4 \end{bmatrix} \vec{Y}$. The functions $\vec{Y_1}(t)$ and $\vec{Y_2}(t)$ are solutions to the linear system.

True or False: The function $\vec{Y_1}(t) + \vec{Y_2}(t)$ formed by adding the two solutions $\vec{Y_1}(t)$ and $\vec{Y_2}(t)$ is also a solution to the linear system.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

245. **True or False:** The functions $\vec{Y_1}(t) = \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix}$ and $\vec{Y_2}(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$ are linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

246. **True or False:** The functions $\vec{Y_1}(t) = \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix}$ and $\vec{Y_2}(t) = \begin{pmatrix} -2\sin(t) \\ -2\cos(t) \end{pmatrix}$ are linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

247. If we are told that the general solution to the linear homogeneous system $Y' = AY$ is $Y = c_1 e^{-4t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, then an equivalent form of the solution is

(a) $y_1 = -2c_1 e^{-4t} + c_2 e^{-4t}$ and $y_2 = 2c_1 e^{3t} + 3c_2 e^{3t}$
(b) $y_1 = -2c_1 e^{-4t} + 2c_2 e^{3t}$ and $y_2 = c_1 e^{-4t} + 3c_2 e^{3t}$
(c) $y_1 = -2c_1 e^{-4t} + c_1 e^{-4t}$ and $y_2 = 2c_2 e^{3t} + 3c_2 e^{3t}$
(d) $y_1 = -2c_1 e^{-4t} + 2c_1 e^{3t}$ and $y_2 = c_2 e^{-4t} + 3c_2 e^{3t}$
(e) All of the above
(f) None of the above

248. If $Y = e^{-2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + e^{-4t} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is a solution to the linear homogeneous system $Y' = AY$, which of the following is also a solution?

(a) $Y = 2e^{-2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
(b) $Y = 3e^{-2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} - 4e^{-4t} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$
(c) $Y = 1/4e^{-4t} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(d) All of the above

(e) None of the above

249. The eigenvalues and eigenvectors for the coefficient matrix A in the linear homogeneous system $Y' = AY$ are $\lambda_1 = 4$ with $v_1 =< 1, 2 >$ and $\lambda_2 = -3$ with $v_2 =< -2, 1 >$. What is a form of the solution?

(a) $Y = c_1 e^{-4t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

(b) $Y = c_1 e^{4t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{-3t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(c) $Y = c_1 e^{4t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{-3t} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

(d) $Y = c_1 e^{-4t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

250. You have a linear, homogeneous, system of differential equations with constant coefficients whose coefficient matrix A has the eigensystem: eigenvalues -5 and -2 and eigenvectors $< -1, 2 >$ and $< -4, 5 >$, respectively. Then a general solution to $\frac{d\vec{Y}}{dt} = A\vec{Y}$ is given by:

(a) $Y = \begin{bmatrix} -k_1 e^{-5t} + 2k_2 e^{-2t} \\ -4k_1 e^{-5t} + 5k_2 e^{-2t} \end{bmatrix}$

(b) $Y = \begin{bmatrix} -k_1 e^{-2t} - 4k_2 e^{-5t} \\ 2k_1 e^{-2t} + 5k_2 e^{-5t} \end{bmatrix}$

(c) $Y = \begin{bmatrix} -k_1 e^{-5t} - 4k_2 e^{-2t} \\ 2k_1 e^{-5t} + 5k_2 e^{-2t} \end{bmatrix}$

(d) $Y = \begin{bmatrix} -k_1 e^{-2t} + 2k_2 e^{-5t} \\ -4k_1 e^{-2t} + 5k_2 e^{-5t} \end{bmatrix}$

251. The eigenvalues and eigenvectors for the coefficient matrix A in the linear homogeneous system $Y' = AY$ are $\lambda_1 = 4$ with $v_1 =< 1, 2 >$ and $\lambda_2 = -3$ with $v_2 =< -2, 1 >$. In the long term, phase trajectories:

(a) become parallel to the vector $v_2 =< -2, 1 >$.

(b) tend towards positive infinity.

(c) become parallel to the vector $v_1 =< 1, 2 >$.

(d) tend towards 0.

(e) None of the above

252. If the eigenvalues and eigenvectors for the coefficient matrix A in the linear homogeneous system $Y' = AY$ are $\lambda_1 = -4$ with $v_1 =< 1, 2 >$ and $\lambda_2 = 3$ with $v_2 =< 2, 3 >$, is $y_a = e^{-4t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ a solution?

(a) Yes, it is a solution.

(b) No, it is not a solution because it does not contain λ_2.

(c) No, it is not a solution because it is a vector.

(d) No, it is not a solution because of a different reason.

253. The eigenvalues and eigenvectors for the coefficient matrix A in the linear homogeneous system $Y' = AY$ are $\lambda_1 = 4$ with $v_1 =< 1, 0 >$ and $\lambda_2 = 4$ with $v_2 =< 0, 1 >$. What is a form of the solution?

(a) $Y = c_1 e^{4t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{4t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(b) $Y = c_1 e^{-4t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{-4t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(c) $Y = c_1 e^{4t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 t e^{4t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(d) $Y = c_1 e^{-4t} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_2 e^{-4t} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

254. The system of differential equations $Y' = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} Y$ has eigenvalue $\lambda = 2$ with multiplicity 2, and all eigenvectors are multiples of $v =< 1, -1 >$. Testing $Y = e^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, we find that

(a) Y is a solution.

(b) Y is not a solution.

255. The system of differential equations $Y' = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} Y$ has eigenvalue $\lambda = 2$ with multiplicity 2, and all eigenvectors are multiples of $v =< 1, -1 >$. One solution to this equation is $Y = e^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Testing $Y = te^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, we find that

(a) $Y = te^{2t}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is also a solution.

(b) $Y = te^{2t}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is not a solution.

256. The eigenvalues and eigenvectors for the coefficient matrix A in the linear homogeneous system $Y' = AY$ are $\lambda = -4$ with multiplicity 2, and all eigenvectors are multiples of $v =< 1, -2 >$. What is the form of the general solution?

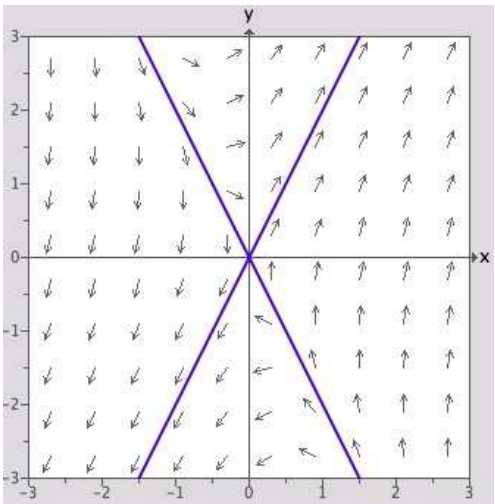
(a) $Y = c_1 e^{-4t}\begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 te^{-4t}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(b) $Y = c_1 e^{-4t}\begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 e^{-4t}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(c) $Y = c_1 e^{-4t}\begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2\left(te^{-4t}\begin{bmatrix} 1 \\ -2 \end{bmatrix} + e^{-4t}\begin{bmatrix} 0 \\ -1 \end{bmatrix}\right)$

Geometry of Systems

257. The differential equation $\dfrac{d\vec{Y}}{dt} = A\vec{Y}$ has two straight line solutions corresponding to eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ that are shown on the direction field below. We denote the associated eigenvalues by λ_1 and λ_2.



We can deduce that λ_1 is

(a) positive real

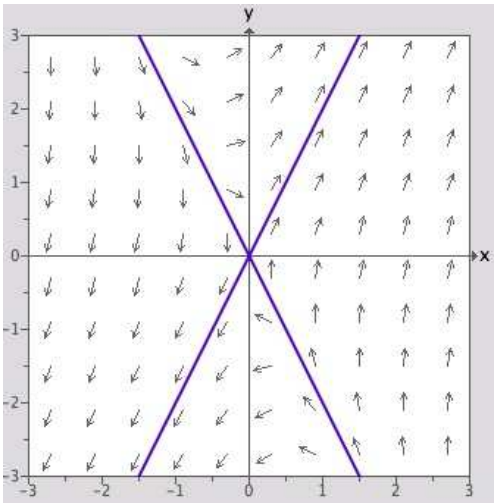
(b) negative real

(c) zero

(d) complex

(e) There is not enough information

258. The differential equation $\dfrac{d\vec{Y}}{dt} = A\vec{Y}$ has two straight line solutions corresponding to eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ that are shown on the direction field below. We denote the associated eigenvalues by λ_1 and λ_2.



We can deduce that λ_2 is

(a) positive real

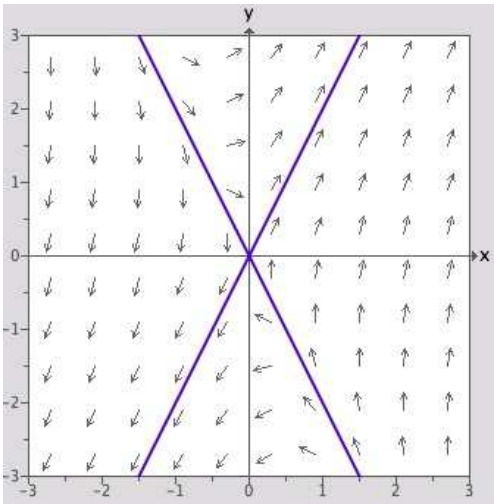
(b) negative real

(c) zero

(d) complex

(e) There is not enough information

259. The differential equation $\dfrac{d\vec{Y}}{dt} = A\vec{Y}$ has two straight line solutions corresponding to eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ that are shown on the direction field below.



Suppose we have a solution $\vec{Y}(t)$ to this system of differential equations which satisfies initial condition $\vec{Y}(t) = (x_0, y_0)$ where the point (x_0, y_0) is not on the line through the point $(1, -2)$. Which statement best describes the behavior of the solution as $t \to \infty$?

(a) The solution tends towards the origin.

(b) The solution moves away from the origin and asymptotically approaches the line through $< 1, 2 >$.

(c) The solution moves away from the origin and asymptotically approaches the line through $< 1, -2 >$.

(d) The solution spirals and returns to the point (x_0, y_0).

(e) There is not enough information.

260. Suppose you have a linear, homogeneous system of differential equations with constant coefficients whose coefficient matrix has the following eigensystem: eigenvalues -4 and -1 and eigenvectors $< 1, 1 >$ and $< -2, 1 >$ respectively. The function $\vec{Y}(t)$ is a solution to this system of differential equations which satisfies initial value $\vec{Y}(0) = (-15, 20)$. Which statement best describes the behavior of the solution as $t \to \infty$?

(a) The solution tends towards the origin.

(b) The solution moves away from the origin and asymptotically approaches the line through $< 1, 1 >$.

(c) The solution moves away from the origin and asymptotically approaches the line through $< -2, 1 >$.

(d) The solution spirals and returns to the point $(-15, 20)$.

(e) There is not enough information.

261. Suppose we have a linear, homogeneous system of differential equations with constant coefficients who coefficient matrix has the following eigensystem: eigenvalues -4 and -1 and eigenvectors $< 1, 1 >$ and $< -2, 1 >$ respectively. Suppose we have a solution $\vec{Y}(t)$ which satisfies $\vec{Y}(0) = (x_0, y_0)$ where the point (x_0, y_0) is not on the line through the point $(1, 1)$. How can we best describe the manner in which the solution $\vec{Y}(t)$ approaches the origin?

(a) The solution will approach the origin in the same manner as the line which goes through the point $(1, 1)$.

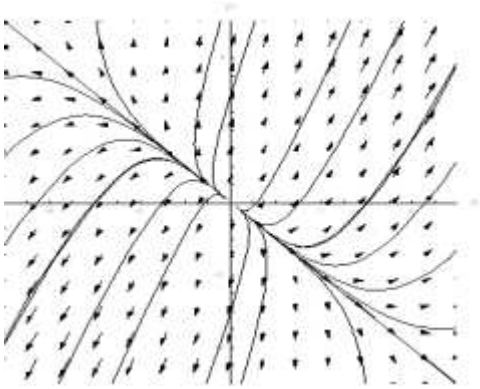
(b) The solution will approach the origin in the same manner as the line which goes through the point $(-2, 1)$.

(c) The solution will directly approach the origin in a straight line from the point (x_0, y_0).

(d) The answer can vary greatly depending on what the point (x_0, y_0) is.

(e) The solution doesn't approach the origin.

262. Using the phase portrait below for the system $Y' = AY$, we can deduce that the eigenvalues of the coefficient matrix A are:



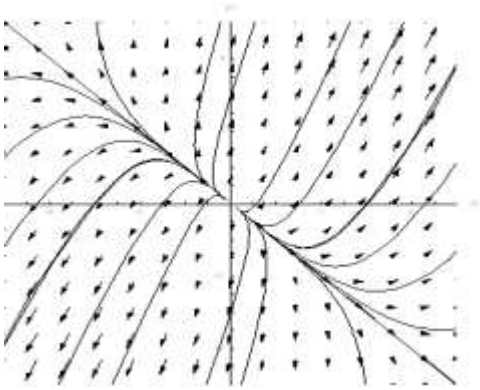
(a) both real

(b) both complex

(c) one real, one complex

(d) Not enough information is given

263. Using the phase portrait below for the system $Y' = AY$, we can deduce that the eigenvalues are:



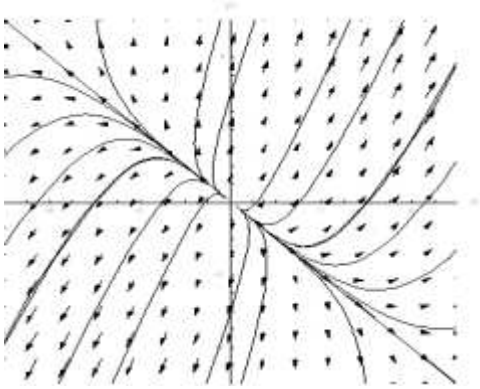
(a) of mixed sign

(b) both negative

(c) both positive

(d) Not enough information is given

264. Using the phase portrait below for the system $Y' = AY$, we can deduce that the dominant eigenvector is:



(a) $< -1, 1 >$

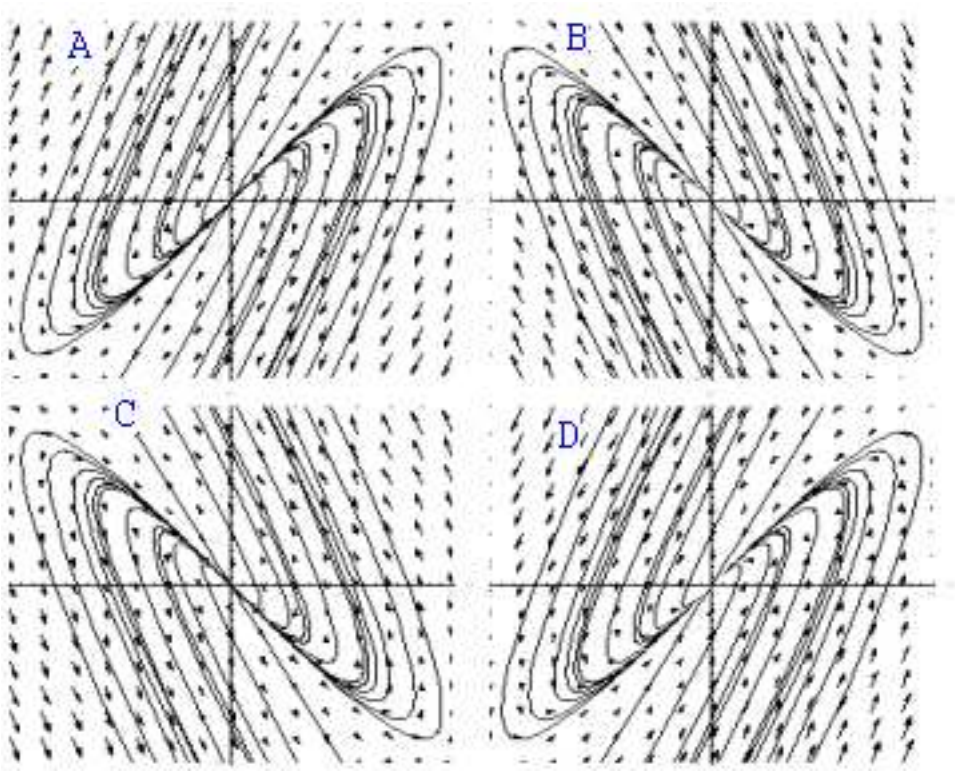
(b) $< 1, 3 >$

(c) $< 1, -2 >$

(d) There is no dominant eigenvector because there is no vector that is being approached by all of the solution curves.

(e) Not enough information is given

265. Which phase portrait below corresponds to the linear, homogeneous, system of differential equations with constant coefficients whose coefficient matrix has the following eigensystem: eigenvalues -5 and -2 and eigenvectors $< -1, 2 >$ and $< -4, 5 >$, respectively?



266. Classify the equilibrium point at the origin for the system

$$\frac{d\vec{Y}}{dt} = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \vec{Y}.$$

(a) Sink
(b) Source
(c) Saddle
(d) None of the above

Nonhomogeneous Linear Systems

267. Which of the following would be an appropriate guess for the particular solution to the forced ODE $y' = -3y + t^2$?

(a) $y_p = c_1 t^2$
(b) $y_p = c_1 + c_2 t + c_3 t^2$
(c) $y_p = c_1 e^{-3t} + c_2 t^2$
(d) $y_p = c_1 e^{-3t} + c_2 t^2 + c_3 t + c_4$

(e) $y_p = c_1 + c_2 t^2$

268. Which of the following would be an appropriate guess for the particular solution for the decoupled system $Y' = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} Y + \begin{bmatrix} 2e^{4t} \\ e^t \end{bmatrix}$?

(a) $\begin{bmatrix} c_1 e^{4t} + c_2 e^t \\ c_3 e^{4t} + c_4 e^t \end{bmatrix}$

(b) $\begin{bmatrix} c_1 e^t \\ c_2 e^{4t} \end{bmatrix}$

(c) $\begin{bmatrix} c_1 e^{4t} \\ c_2 e^t \end{bmatrix}$

(d) $\begin{bmatrix} 2c_1 e^{4t} \\ c_2 e^t \end{bmatrix}$

269. Which of the following would be an appropriate guess for the particular solution for the system $Y' = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix} Y + \begin{bmatrix} e^{4t} \\ e^t \end{bmatrix}$?

(a) $\begin{bmatrix} c_1 e^{4t} + c_2 e^t \\ c_3 e^{4t} + c_4 e^t \end{bmatrix}$

(b) $\begin{bmatrix} c_1 e^{4t} + c_2 e^t \\ c_3 e^t \end{bmatrix}$

(c) $\begin{bmatrix} c_1 e^{4t} \\ c_2 e^t \end{bmatrix}$

(d) $\begin{bmatrix} c_1 e^{4t} \\ c_2 e^{4t} + c_3 e^t \end{bmatrix}$

270. Which of the following is a particular solution for the system $Y' = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix} Y + \begin{bmatrix} 6e^{4t} \\ 2e^t \end{bmatrix}$?

(a) $Y_p = \begin{bmatrix} c_1 e^{4t} + c_2 e^t \\ c_3 e^{4t} + c_4 e^t \end{bmatrix}$

(b) $Y_p = \begin{bmatrix} 2.4e^{4t} + 0.2e^t \\ 1.2e^{4t} + 0.6e^t \end{bmatrix}$

(c) $Y_p = \begin{bmatrix} 1.05e^{4t} + 0.2e^t \\ 0.3e^{4t} + 0.6e^t \end{bmatrix}$

(d) More than one of the above

271. Which of the following is the general solution for the system $Y' = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix} Y + \begin{bmatrix} 6e^{4t} \\ 2e^{t} \end{bmatrix}$?

(a) $Y = k_1 e^{-4t} \begin{bmatrix} -1 \\ 2 \end{bmatrix} + k_2 e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1.05e^{4t} + 0.2e^{t} \\ 0.3e^{4t} + 0.6e^{t} \end{bmatrix}$

(b) $Y = k_1 e^{-4t} + k_2 e^{-t} + \begin{bmatrix} 1.05e^{4t} + 0.2e^{t} \\ 0.3e^{4t} + 0.6e^{t} \end{bmatrix}$

(c) $Y = k_1 e^{4t} \begin{bmatrix} -1 \\ 2 \end{bmatrix} + k_2 e^{t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1.05e^{4t} + 0.2e^{t} \\ 0.3e^{4t} + 0.6e^{t} \end{bmatrix}$

(d) $Y = k_1 e^{-4t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + k_2 e^{-t} \begin{bmatrix} -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1.05e^{4t} + 0.2e^{t} \\ 0.3e^{4t} + 0.6e^{t} \end{bmatrix}$

272. Which of the following would be an appropriate guess for the particular solution for the system $Y' = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix} Y + \begin{bmatrix} e^{-4t} \\ e^{-t} \end{bmatrix}$?

(a) $\begin{bmatrix} c_1 e^{-4t} + c_2 e^{-t} \\ c_3 e^{-4t} + c_4 e^{-t} \end{bmatrix}$

(b) $\begin{bmatrix} c_1 e^{-4t} + c_2 e^{-t} \\ c_3 e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} c_1 e^{-4t} \\ c_2 e^{-t} \end{bmatrix}$

(d) $\begin{bmatrix} c_1 e^{-4t} \\ c_2 e^{-4t} + c_3 e^{-t} \end{bmatrix}$

(e) None of the above

273. Which of the following would be an appropriate guess for the particular solution for the system $Y' = \begin{bmatrix} -2 & 0 \\ 2 & -3 \end{bmatrix} Y + \begin{bmatrix} e^{4t} \\ e^{t} \end{bmatrix}$?

(a) $\begin{bmatrix} c_1 e^{4t} + c_2 e^{t} \\ c_3 e^{4t} + c_4 e^{t} \end{bmatrix}$

(b) $\begin{bmatrix} c_1 e^{t} \\ c_2 e^{4t} + c_3 e^{t} \end{bmatrix}$

(c) $\begin{bmatrix} c_1 e^{4t} \\ c_2 e^{t} \end{bmatrix}$

(d) $\begin{bmatrix} c_1 e^{4t} \\ c_2 e^{4t} + c_3 e^{t} \end{bmatrix}$

274. Which of the following would be an appropriate guess for the particular solution for the system $Y' = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix} Y + \begin{bmatrix} \sin \pi t \\ 3t \end{bmatrix}$?

(a) $\begin{bmatrix} c_1 \sin \pi t + c_2 t \\ c_3 \sin \pi t + c_4 t \end{bmatrix}$

(b) $\begin{bmatrix} c_1 \sin \pi t + c_2 \cos \pi t + c_3 t + c_4 \\ c_5 \sin \pi t + c_6 \cos \pi t + c_7 t + c_8 \end{bmatrix}$

(c) $\begin{bmatrix} c_1 \sin \pi t + c_2 t + c_3 \\ c_4 \sin \pi t + c_5 t + c_6 \end{bmatrix}$

(d) $\begin{bmatrix} c_1 \sin \pi t \\ c_2 t + c_3 \end{bmatrix}$

Nonlinear Systems

275. The nonlinear system of differential equations given below has an equilibrium point at $(0, 0)$. Identify the system which represents a linear approximation of the nonlinear system around this point.

$$\frac{dx}{dt} = y + x^2$$
$$\frac{dy}{dt} = -2y + \sin x$$

(a)

$$\frac{dx}{dt} = y + 2x$$
$$\frac{dy}{dt} = -2y$$

(b)

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = -2y$$

(c)

$$\frac{dx}{dt} = y + 2x$$
$$\frac{dy}{dt} = -2y + x$$

(d)

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = -2y + x$$

276. For the nonlinear system given below, compute the Jacobian $J(x, y)$ that we associate to it.

$$\frac{dx}{dt} = x + 2xy$$
$$\frac{dy}{dt} = -2y + x^2$$

(a)

$$J(x, y) = \begin{pmatrix} 1 + 2x & 2y \\ 2x & -2 \end{pmatrix}$$

(b)

$$J(x, y) = \begin{pmatrix} 1 + 2y & 2x \\ 2x & -2 \end{pmatrix}$$

(c)

$$J(x, y) = \begin{pmatrix} 2x & 1 + 2y \\ -2 & 2x \end{pmatrix}$$

(d)

$$J(x, y) = \begin{pmatrix} 2x & -2 \\ 1 + 2y & 2x \end{pmatrix}$$

277. The nonlinear system given below has an equilibrium point at $(0, 0)$. Classify this point.

$$\frac{dx}{dt} = x + 2xy$$
$$\frac{dy}{dt} = -2y + x^2$$

(a) Sink
(b) Source
(c) Saddle
(d) Spiral Sink
(e) Spiral Source
(f) Center

Introduction to Partial Differential Equations

278. Which of the following functions satisfies the equation $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f$?

    (a) $f = e^{x+y}$
    (b) $f = 2x + 3y + 5$
    (c) $f = x^{0.2}y^{0.8}$
    (d) $f = x^2y^3$
    (e) More than one of the above

279. Which of the following functions satisfies the equation $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f$?

    (a) $f = 18\sqrt{xy}$
    (b) $f = 3x^{0.2}y^{0.8}$
    (c) $f = 5x^{0.4}y^{0.6}$
    (d) $f = -37x^{0.9}y^{0.1}$
    (e) All of the above

280. Does the function $f = 3x$ satisfy the partial differential equation $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f$?

    (a) Yes
    (b) No

281. Consider the heat diffusion equation: $\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial y^2}$ where $T(y,t)$ is a function which describes the temperature T (in °F) at position y (in meters) at time t (in hours). What are the units of the constant κ, usually called the thermal diffusivity?

    (a) meters squared per °F hour
    (b) meters squared per hour
    (c) °F per hour
    (d) °F per meters squared

282. At time $t = 0$ the temperature of a concrete slab is described by the function $T(x) = 10\cos\frac{\pi}{10}x + 3x + 40$ where $x = -10$ at one end of the slab and $x = +10$ at the other end. The way the temperature will change over time is controlled by the heat diffusion equation: $\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial x^2}$. At what rate is the temperature changing at $x = 0$ and $t = 0$ if $\kappa = 2$ meters squared per second?

(a) -2π °F per second

(b) $-\frac{\pi^2}{5}$ °F per second

(c) $\frac{\pi}{50}$ °F per second

(d) $-\frac{\pi^2}{10}$ °F per second

(e) None of the above

283. For what value of c does the function $T(y,t) = 3e^{ct}\sin 2\pi y$ satisfy the heat diffusion equation $\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial y^2}$ where $\kappa = 4$?

(a) $c = -16\pi^2$

(b) $c = 4$

(c) $c = 12\pi^2$

(d) $c = -6\pi$

(e) $c = -4\pi^2$

(f) None of the above

284. The function $T(y,t) = 2e^{-4t}\sin 5y$ is a solution of the heat diffusion equation $\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial y^2}$ where κ is in feet^2 per second. What are the units of the constants 2, 4, and 5?

(a) 2 feet^2 per second, 4 inverse seconds, 5 radians

(b) $2°F$ per second, 4 seconds, 5 inverse feet

(c) $2°F$, 4 inverse seconds, 5 radians per foot

(d) $2°F$, 4 seconds, 5 feet per radian

(e) None of the above

285. We have solved the heat diffusion equation to find a function that describes the temperature of a concrete slab that extends from $x = -10$ to $x = +10$, finding that $T = Ae^{kt}\cos(Bx) + Cx + D$, where $A = -25$, $B = \frac{\pi}{20}$, $C = 80$, and $D = -2$. Where is the slab warmest?

(a) In the center

(b) At $x = -10$

(c) At $x = +10$

(d) At another location

286. We want to find a function that describes the temperature evolution of a concrete slab that extends from $x = -15$ to $x = +15$, of the form $T = Ae^{kt}\cos(Bx) + C$. We know that initially the slab is hottest in the center, and that temperature decreases outward until it reaches the edges, which are both held at a fixed temperature. What value must B have?

(a) $B = \frac{\pi}{60}$
(b) $B = 15\pi^2$
(c) $B = \frac{\pi}{30}$
(d) $B = 30\pi$
(e) None of the above

287. We are solving the heat diffusion equation to find a function that describes how the temperature of a concrete slab changes over time, and find that the function is of the form $T = Ae^{kt}\sin(Bx) + C$. We know that the ends of the slab (at $x = 0$ and $x = 20$ meters) are both always held at a temperature of 90 degrees, and the center of the slab starts out at a temperature of 50 degrees before it starts to warm up. What are the values of A, B, and C?

(a) $A = 40$, $B = \pi/10$, $C = 90$
(b) $A = -40$, $B = \pi/20$, $C = 90$
(c) $A = -20$, $B = 20\pi$, $C = 70$
(d) $A = -40$, $B = \pi/10$, $C = 50$

288. The temperature of a concrete slab must satisfy the heat diffusion equation $\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial x^2}$ where $\kappa = 2$ meters squared per hour, with the boundary conditions that the temperature is always $60°F$ at the origin and at $x = 10$ meters. Which of the following could be $T(x,t)$?

(a) $T = 7e^{-t\pi^2 2/25}\sin\frac{\pi}{5}x + 60$
(b) $T = -18e^{-t\pi^2 8/25}\sin\frac{2\pi}{5}x + 60$
(c) $T = 60$
(d) $T = 3.34e^{-t\pi^2 18/25}\sin\frac{3\pi}{5}x + 60$
(e) All of the above

289. Suppose the function $T_0(x,t)$ solves the heat diffusion equation $\frac{\partial T}{\partial t} = \kappa\frac{\partial^2 T}{\partial x^2}$. Which of the following transformations of this function would NOT also be a solution?

(a) $T_1 = 3T_0$

(b) $T_2 = (T_0)^2$
(c) $T_3 = T_0 + 50$
(d) $T_4 = -T_0$
(e) All are solutions

290. Suppose we have two different functions $f(x,t)$ and $g(x,t)$ which each solve the heat diffusion equation $\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$. Which of the following would NOT also be a solution?

(a) $T = f + g$
(b) $T = 2f - 3g$
(c) $T = f + g + 10$
(d) $T = \frac{f}{g}$
(e) All are solutions

291. The wave equation $\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{C^2}\frac{\partial^2 \psi}{\partial t^2}$ allows us to understand the motion of water waves, sound waves, or light waves. If the function ψ gives us the height of the wave in inches as a function of position x in feet, and time t in seconds, what are the units of the constant C?

(a) feet squared per second squared
(b) seconds per foot
(c) feet per second
(d) inches per second
(e) None of the above

292. We conjecture that the function $\psi = A\sin(kx + \omega t)$ is a solution to the wave equation $\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{C^2}\frac{\partial^2 \psi}{\partial t^2}$. If $A = 10$, $k = 3$ and $C = 5$, what value could ω have?

(a) $\omega = 15$
(b) $\omega = 150$
(c) $\omega = 3/5$
(d) $\omega = 9/25$
(e) $\omega = 225$

293. The function $\psi = A\sin(kx + \omega t)$ is a solution to the wave equation $\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{C^2}\frac{\partial^2 \psi}{\partial t^2}$. If $A = 10$, $k = 3$ and $\omega = -5$, which direction is the wave traveling?

(a) In the $+x$ direction
(b) In the $-x$ direction
(c) Not enough information is given.

Integrating Factors

294. If y is a function of t, which of the following is $t\left(y' + \frac{1}{t}y\right)$ equivalent to?

   (a) $[ty]'$
   (b) $\left[\frac{1}{t}y\right]'$
   (c) ty'
   (d) $\frac{1}{t}y'$
   (e) None of the above

295. Which of the following is an integrating factor for $y' + 3ty = \sin t$?

   (a) $e^{\frac{3}{2}t^2}$
   (b) e^3
   (c) $e^{\sin t}$
   (d) $e^{-\cos t}$
   (e) e^{3y}
   (f) All of the above

296. Which of the following is an integrating factor for $y' + 2y = 3t$?

   (a) e^{2t}
   (b) e^{2t+5}
   (c) e^2e^{2t}
   (d) $7e^{2t}$
   (e) All of the above
   (f) None of the above

297. Which of the following is an integrating factor for $3y' + 6ty = 8t$?

   (a) e^{3t^2}
   (b) e^{t^2}
   (c) e^6
   (d) All of the above

(e) None of the above

(f) This problem cannot be solved with integrating factors.

298. Can integrating factors be used to solve $y' + 2ty = 1$?

(a) Yes. The solution is $y = e^{-t^2} \int e^{t^2} dt$.

(b) No - we cannot evaluate $\int e^{t^2} dt$.

299. The differential equation $y' + 2y = 3t$ is solved using integrating factors. The solution is $y = \frac{3}{2}t - \frac{3}{4} + Ce^{-2t}$. Which of the following statements describes the long-term behavior as $t \to \infty$?

(a) The solutions will approach zero, because $e^{-2t} \to 0$ as $t \to \infty$.

(b) The solutions will grow without bound because $\frac{3}{2}t \to \infty$ as $t \to \infty$.

(c) The long-term behavior depends on the inital condition; we need to know the value of C before we can answer this.

300. Which of the following is $e^{3t}(y' + 2y)$ equivalent to?

(a) $[e^{2t}y]'$

(b) $[e^{3t}y]'$

(c) $e^{3t}y'$

(d) $e^{2t}y'$

(e) None of the above

Power Series Solutions

301. What is the solution to $\frac{df}{dx} = f$?

(a) $f(x) = e^x$

(b) $f(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \cdots$

(c) Both of the above

302. Determine which of the following is a solution to the equation $y'' - xy = 0$ by taking derivatives and substituting them into the differential equation.

(a) $y = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \cdots$

(b) $y = x + \frac{1}{4!}x^4 + \frac{1}{7!}x^7 + \frac{1}{10!}x^{10} \cdots$
(c) $y = 1 + \frac{1}{2\cdot 3}x^3 + \frac{1}{2\cdot 3\cdot 5\cdot 6}x^6 + \frac{1}{2\cdot 3\cdot 5\cdot 6\cdot 8\cdot 9}x^9 + \cdots$
(d) $y = 1 + \frac{1}{2}x^2 + \frac{1}{2\cdot 4}x^4 + \frac{1}{2\cdot 4\cdot 6}x^6 + \cdots$
(e) None of the above

303. Write the series $\sum_{n=2}^{\infty}(n+1)a_n x^n$ as an equivalent series whose first term corresponds to $n = 0$ rather than $n = 2$.

(a) $\sum_{n=0}^{\infty}(n+1)a_n x^n$
(b) $\sum_{n=0}^{\infty}(n+3)a_{n+2}x^{n+2}$
(c) $\sum_{n=0}^{\infty}(n-1)a_{n-2}x^{n-2}$
(d) $\sum_{n=0}^{\infty}(n-1)a_{n+2}x^{n+2}$

304. What is the second derivative of the function defined by the power series $y(x) = \sum_{n=0}^{\infty} a_n x^n$?

(a) $y''(x) = \sum_{n=0}^{\infty} n(n-1)a_{n-2}x^{n-2}$
(b) $y''(x) = \sum_{n=0}^{\infty}(n+1)a_{n+1}x^n$
(c) $y''(x) = \sum_{n=0}^{\infty}(n+1)(n+2)a_{n+2}x^n$
(d) $y''(x) = \sum_{n=0}^{\infty}(n-2)(n-3)a_{n-2}x^{n-4}$

305. Find a power series in the form $y(x) = \sum_{n=0}^{\infty} a_n x^n$ that is a solution for the differential equation $y' = 3y$, by taking the derivative of this series, substituting both the series and its derivative into the differential equation, and simplifying the result.

(a) $a_{n+1} = \frac{3}{n+1}a_n$
(b) $a_{n+1} = \frac{-3}{n+1}a_n$
(c) $a_{n+1} = \frac{3}{n-1}a_n$
(d) $a_{n+1} = \frac{-3}{n-1}a_n$

306. Which of the following power series is a solution to the differential equation $y' = -2y$?

(a) $y(x) = 2 - 2x + 2x^2 - \frac{4}{3}x^3 + \frac{2}{3}x^4 + \cdots$
(b) $y(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \cdots$
(c) $y(x) = -2 - x - \frac{2}{3}x^2 - \frac{1}{2}x^3 + \cdots$
(d) $y(x) = 2 - 4x + 4x^2 - \frac{8}{3}x^3 + \frac{4}{3}x^4 + \cdots$

(e) $y(x) = -1 - 2x - 2x^2 - \frac{4}{3}x^3 - \frac{2}{3}x^4 + \cdots$

307. Find a power series in the form $y(x) = \sum_{n=0}^{\infty} a_n x^n$ that is a solution for the differential equation $y''+4y = 0$, by taking the second derivative of this series, substituting both the series and its second derivative into the differential equation, and finding a difference equation for a_n.

(a) $a_{n+2} = \frac{-4}{(n+1)(n+2)} a_n$

(b) $a_{n+2} = \frac{-4}{(n-1)(n-2)} a_n$

(c) $a_{n+2} = \frac{-4}{n(n+1)} a_n$

(d) $a_{n+2} = \frac{-4}{n(n-1)} a_n$

308. Which of the following is a solution to $(x-2)y'' - y = 0$?

(a) $x + \frac{1}{2}x^2 + \frac{1}{12}x^3 + \frac{1}{144}x^4 + \cdots$

(b) $(x-2) + \frac{1}{2}(x-2)^2 + \frac{1}{12}(x-2)^3 + \frac{1}{144}(x-2)^4 + \cdots$

(c) $(x+2) + \frac{1}{2}(x+2)^2 + \frac{1}{12}(x+2)^3 + \frac{1}{144}(x+2)^4 + \cdots$

(d) $-144 \cdot (x-2) - 72(x-2)^2 - 12(x-2)^3 - (x-2)^4 + \cdots$

(e) None of the above

(f) More than one of the above

309. Find a power series in the form $y(x) = \sum_{n=0}^{\infty} a_n (x-1)^n$ that is a solution for the differential equation $y'' - xy = 0$. Hint: After subsituting in the series for y and y'' it may be useful to write $x = 1 + (x-1)$.

(a) $a_{n+2} = \frac{a_n + a_{n-1}}{(n+1)(n+2)}$

(b) $a_{n+2} = \frac{a_n + a_{n-1}}{(n)(n+1)}$

(c) $a_{n+2} = \frac{a_n + a_{n+1}}{(n+1)(n+2)}$

(d) $a_{n+2} = \frac{a_n + a_{n+1}}{(n)(n+1)}$

Laplace Tranforms

310. **True or False** The Laplace transform method is the only way to solve some types of differential equations.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

311. Which of the following differential equations would be impossible to solve using the Laplace transform?

    (a) $\frac{dc}{dr} = 12c + 3\sin(2r) + 8\cos^2(3r+2)$
    (b) $\frac{d^2f}{dx^2} - 100\frac{df}{dx} = \frac{18}{ln(x)}$
    (c) $g'(b) = \frac{12}{g}$
    (d) $p''(q) = \frac{4}{q} + 96p' - 12p$

312. Suppose we know that zero is an equilibrium value for a certain homogeneous linear differential equation with constant coefficients. Further, suppose that if we begin at equilibrium and we add the nonhomogenous driving term $f(t)$ to the equation, then the solution will be the function $y(t)$. **True or False**: If instead we add the nonhomogeous driving term $2f(t)$, then the solution will be the function $2y(t)$.

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

313. Suppose we know that zero is an equilibrium value for a certain homogeneous linear differential equation with constant coefficients. Further, suppose that if we begin at equilibrium and we add a nonhomogenous driving term $f(t)$ to the equation that acts only for an instant, at $t = t_1$, then the solution will be the function $y(t)$. **True or False**: Even if we know nothing about the differential equation itself, using our knowledge of $f(t)$ and $y(t)$ we can infer how the system must behave for any other nonhomogeneous driving function $g(t)$ and for any other initial condition.

    (a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

314. The Laplace transform of a function $y(t)$ is defined to be a function $Y(s)$ so that $\mathcal{L}[y(t)] = Y(s) = \int_0^\infty y(t)e^{-st}dt$. If we have the function $y(t) = e^{5t}$, then what is its Laplace transform $\mathcal{L}[y(t)] = Y(s)$?

(a) $\mathcal{L}[y(t)] = \frac{1}{s-5}$ if $s > 5$.

(b) $\mathcal{L}[y(t)] = \frac{1}{s-5}$ if $s < 5$.

(c) $\mathcal{L}[y(t)] = \frac{1}{5-s}$ if $s > 5$.

(d) $\mathcal{L}[y(t)] = \frac{1}{5-s}$ if $s < 5$.

315. What is the Laplace transform of the function $y(t) = 1$?

(a) $\mathcal{L}[y(t)] = -\frac{1}{s}$ if $s > 0$.

(b) $\mathcal{L}[y(t)] = -\frac{1}{s}$ if $s < 0$.

(c) $\mathcal{L}[y(t)] = \frac{1}{s}$ if $s > 0$.

(d) $\mathcal{L}[y(t)] = \frac{1}{s}$ if $s < 0$.

(e) This function has no Laplace transform.

316. What is $\mathcal{L}[e^{3t}]$?

(a) $\mathcal{L}[e^{3t}] = \frac{1}{3-s}$ if $s < 3$.

(b) $\mathcal{L}[e^{3t}] = \frac{1}{s-3}$ if $s > 3$.

(c) $\mathcal{L}[e^{3t}] = \frac{1}{s+3}$ if $s > 3$.

(d) $\mathcal{L}[e^{3t}] = \frac{-1}{s+3}$ if $s < 3$.

317. Suppose we have the Laplace transform $Y(s) = \frac{1}{s-2}$ and we want the original function $y(t)$. What is $\mathcal{L}^{-1}\left[\frac{1}{s-2}\right]$?

(a) $\mathcal{L}^{-1}\left[\frac{1}{s-2}\right] = e^{-2t}$

(b) $\mathcal{L}^{-1}\left[\frac{1}{s-2}\right] = e^{2t}$

(c) This cannot be done

318. Suppose we know that the Laplace transform of a particular function $y(t)$ is the function $Y(s)$ so that $\mathcal{L}[y(t)] = Y(s)$. Now, suppose we multiply this function by 5 and take the Laplace transform. What can we say about $\mathcal{L}[5y(t)]$?

(a) $\mathcal{L}[5y(t)] = 5Y(s)$.
(b) $\mathcal{L}[5y(t)] = -5Y(s)$.
(c) $\mathcal{L}[5y(t)] = -\frac{1}{5}Y(s)$.
(d) $\mathcal{L}[5y(t)] = \frac{1}{5}e^{-5}Y(s)$.
(e) None of the above

319. Suppose we know that the Laplace transform of a particular function $f(t)$ is the function $W(s)$ and that the Laplace transform of another function $g(t)$ is the function $X(s)$ so that $\mathcal{L}[f(t)] = W(s)$ and $\mathcal{L}[g(t)] = X(s)$. Now, suppose we multiply these two functions together and take the Laplace transform. What can we say about $\mathcal{L}[f(t)g(t)]$?

(a) $\mathcal{L}[f(t)g(t)] = W(s)X(s)$.
(b) $\mathcal{L}[f(t)g(t)] = \frac{W(s)}{X(s)}$.
(c) $\mathcal{L}[f(t)g(t)] = W(s) + X(s)$.
(d) $\mathcal{L}[f(t)g(t)] = W(s) - X(s)$.
(e) None of the above

320. Suppose we know that the Laplace transform of a particular function $f(t)$ is the function $W(s)$ and that the Laplace transform of another function $g(t)$ is the function $X(s)$ so that $\mathcal{L}[f(t)] = W(s)$ and $\mathcal{L}[g(t)] = X(s)$. Now, suppose we add these two functions together and take the Laplace transform. What can we say about $\mathcal{L}[f(t) + g(t)]$?

(a) $\mathcal{L}[f(t) + g(t)] = W(s)X(s)$.
(b) $\mathcal{L}[f(t) + g(t)] = \frac{W(s)}{X(s)}$.
(c) $\mathcal{L}[f(t) + g(t)] = W(s) + X(s)$.
(d) $\mathcal{L}[f(t) + g(t)] = W(s) - X(s)$.
(e) None of the above

321. Suppose we have $Y(s) = \frac{1}{s-2} + \frac{5}{s+6}$ and we want the function $y(t)$. What is $\mathcal{L}^{-1}\left[\frac{1}{s-2} + \frac{5}{s+6}\right]$?

(a) $\mathcal{L}^{-1}\left[\frac{1}{s-2} + \frac{5}{s+6}\right] = e^{2t} + e^{6t/5}$
(b) $\mathcal{L}^{-1}\left[\frac{1}{s-2} + \frac{5}{s+6}\right] = e^{2t} + 5e^{-6t}$
(c) $\mathcal{L}^{-1}\left[\frac{1}{s-2} + \frac{5}{s+6}\right] = 5e^{2t}e^{-6t}$

(d) $\mathcal{L}^{-1}\left[\frac{1}{s-2}+\frac{5}{s+6}\right]=\frac{e^{2t}e^{6t}}{5}$

(e) This cannot be done

322. Suppose we know that $\mathcal{L}[y(t)]=Y(s)$ and we want to take the Laplace transform of the derivative of $y(t)$, $\mathcal{L}\left[\frac{dy}{dt}\right]$. To get started, recall the method of integration by parts, which tells us that $\int u\,dv = uv - \int v\,du$.

(a) $\mathcal{L}\left[\frac{dy}{dt}\right]=y(t)e^{-st}+sY(s)$.

(b) $\mathcal{L}\left[\frac{dy}{dt}\right]=y(t)e^{-st}-sY(s)$.

(c) $\mathcal{L}\left[\frac{dy}{dt}\right]=y(0)+sY(s)$.

(d) $\mathcal{L}\left[\frac{dy}{dt}\right]=y(0)-sY(s)$.

(e) $\mathcal{L}\left[\frac{dy}{dt}\right]=-y(0)+sY(s)$.

323. We know that $\mathcal{L}[y(t)]=Y(s)=\int_0^\infty y(t)e^{-st}dt$ and that $\mathcal{L}\left[\frac{dy}{dt}\right]=-y(0)+sY(s)$. What do we get when we take the Laplace transform of the differential equation $\frac{dy}{dt}=2y+3e^{-t}$ with $y(0)=2$?

(a) $2+sY(s)=2Y(s)+\frac{3}{s+1}$

(b) $2+sY(s)=\frac{1}{2}Y(s)+\frac{3}{s-1}$

(c) $-2+sY(s)=2Y(s)+\frac{3}{s+1}$

(d) $-2+sY(s)=2Y(s)-\frac{3}{s+1}$

(e) $2-sY(s)=\frac{1}{2}Y(s)+\frac{3}{1-s}$

324. We know that $\mathcal{L}[y(t)]=Y(s)$ and that $\mathcal{L}\left[\frac{dy}{dt}\right]=-y(0)+sY(s)$. Take the Laplace transform of the differential equation $\frac{dy}{dt}=5y+2e^{-3t}$ with $y(0)=4$ and solve for $Y(s)$.

(a) $Y(s)=\frac{2}{(-s-3)(s-5)}+\frac{4}{s-5}$

(b) $Y(s)=\frac{2}{(-s-3)(s-5)}+\frac{4}{s+5}$

(c) $Y(s)=\frac{2}{(s+3)(s-5)}+\frac{4}{s-5}$

(d) $Y(s)=\frac{2}{(s+3)(s+5)}+\frac{4}{s+5}$

325. Suppose that after taking the Laplace transform of a differential equation we solve for the function $Y(s)=\frac{3}{(s+2)(s-6)}$. This is equivalent to which of the following?

(a) $Y(s)=\frac{-3/8}{s+2}+\frac{3/8}{s-6}$

(b) $Y(s) = \frac{-8/3}{s+2} + \frac{8/3}{s-6}$

(c) $Y(s) = \frac{3/8}{s+2} + \frac{-3/8}{s-6}$

(d) $Y(s) = \frac{8/3}{s+2} + \frac{-8/3}{s-6}$

326. Find $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right]$.

(a) $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right] = 3e^{-2t}e^{6t}$

(b) $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right] = -\frac{3}{8}e^{-2t} + \frac{3}{8}e^{6t}$

(c) $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right] = \frac{3}{8}e^{2t} - \frac{3}{8}e^{-6t}$

(d) $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right] = e^{-3/8}e^{-2t} + e^{3/8}e^{6t}$

(e) $\mathcal{L}^{-1}\left[\frac{3}{(s+2)(s-6)}\right] = -\frac{3}{8}e^{2t} + \frac{3}{8}e^{-6t}$

327. Solve the differential equation $\frac{dy}{dt} = -4y + 3e^{2t}$ with $y(0) = 5$ using the method of Laplace transforms: First take the Laplace transform of the entire equation, then solve for $Y(s)$, use the method of partial fractions to simplify the result, and take the inverse transform to get the function $y(t)$.

(a) $y(t) = \frac{9}{2}e^{4t} + \frac{1}{2}e^{-2t}$

(b) $y(t) = \frac{7}{2}e^{4t} + \frac{3}{2}e^{-2t}$

(c) $y(t) = \frac{9}{2}e^{-4t} + \frac{1}{2}e^{2t}$

(d) $y(t) = \frac{7}{2}e^{-4t} + \frac{3}{2}e^{2t}$

328. Consider the Heaviside function, which is defined as

$$u_a(t) = \begin{cases} 0 & \text{if } t < a \\ 1 & \text{if } t \geq a \end{cases}$$

Suppose $a = 2$. What is the Laplace transform of this $\mathcal{L}[u_2(t)]$? Hint: It may be helpful to break this integral into two pieces, from 0 to 2 and from 2 to ∞.

(a) $\mathcal{L}[u_2(t)] = -\frac{1}{s}e^{2s} + \frac{1}{s}$

(b) $\mathcal{L}[u_2(t)] = \frac{1}{s}e^{-2s} - \frac{1}{s}$

(c) $\mathcal{L}[u_2(t)] = -\frac{1}{s}e^{-2s}$

(d) $\mathcal{L}[u_2(t)] = \frac{1}{s}e^{-2s}$

(e) $\mathcal{L}[u_2(t)] = \frac{1}{s}e^{2s}$

329. Find the Laplace transform of the function $u_5(t)e^{-(t-5)}$.

(a) $\mathcal{L}[u_5(t)e^{-(t-5)}] = \frac{e^5}{s+1}$
(b) $\mathcal{L}[u_5(t)e^{-(t-5)}] = \frac{e^{-5s}}{s+1}$
(c) $\mathcal{L}[u_5(t)e^{-(t-5)}] = \frac{e^5e^{-5s}}{s+1}$
(d) $\mathcal{L}[u_5(t)e^{-(t-5)}] = \frac{1}{s+1}$

330. Find the Laplace transform of the function $u_4(t)e^{3(t-4)}$.

(a) $\mathcal{L}[u_4(t)e^{3(t-4)}] = \frac{e^{-3s}}{s-4}$
(b) $\mathcal{L}[u_4(t)e^{3(t-4)}] = \frac{e^{3s}}{s+4}$
(c) $\mathcal{L}[u_4(t)e^{3(t-4)}] = \frac{e^{-4s}}{s+3}$
(d) $\mathcal{L}[u_4(t)e^{3(t-4)}] = \frac{e^{-4s}}{s-3}$
(e) $\mathcal{L}[u_4(t)e^{3(t-4)}] = \frac{e^{4s}}{s-3}$

331. Find $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right]$.

(a) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_2(t)e^{-2(t-3)}$
(b) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_2(t)e^{-3(t-2)}$
(c) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_2(t)e^{-3(t-3)}$
(d) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_3(t)e^{-2(t-3)}$
(e) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_3(t)e^{-2(t-2)}$
(f) $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{s+2}\right] = u_3(t)e^{-3(t-2)}$

332. Use Laplace transforms to solve the differential equation $\frac{dy}{dt} = -2y + 4u_2(t)$ with $y(0) = 1$.

(a) $y(t) = e^{-2t} + 2u_2(t)e^{-(t-2)} - 2u_2(t)e^{-2(t-2)}$
(b) $y(t) = e^{-2t} + 4u_2(t)e^{-(t-2)} - 4u_2(t)e^{-2(t-2)}$
(c) $y(t) = e^{-2t} + 2u_2(t) - 2u_2(t)e^{-2(t-2)}$
(d) $y(t) = e^{-2t} + 4u_2(t) - 4u_2(t)e^{-2(t-2)}$

333. What is the Laplace transform of the second derivative, $\mathcal{L}\left[\frac{d^2y}{dt^2}\right]$?

(a) $\mathcal{L}\left[\frac{d^2y}{dt^2}\right] = -y'(0) + s\mathcal{L}[y]$

(b) $\mathcal{L}\left[\frac{d^2y}{dt^2}\right] = -y'(0) - sy(0) + s^2\mathcal{L}[y]$

(c) $\mathcal{L}\left[\frac{d^2y}{dt^2}\right] = -y'(0) - sy(0) + s\mathcal{L}[y]$

(d) $\mathcal{L}\left[\frac{d^2y}{dt^2}\right] = -y'(0) + sy(0) - s^2\mathcal{L}[y]$

334. We know that the differential equation $y'' = -y$ with the initial condition $y(0) = 0$ and $y'(0) = 1$ is solved by the function $y = \sin t$. By taking the Laplace transform of this equation find an expression for $\mathcal{L}[\sin t]$.

(a) $\mathcal{L}[\sin t] = \frac{1}{s^2+1}$

(b) $\mathcal{L}[\sin t] = \frac{1}{s^2-1}$

(c) $\mathcal{L}[\sin t] = \frac{s}{s^2+1}$

(d) $\mathcal{L}[\sin t] = \frac{s}{s^2-1}$

335. Find an expression for $\mathcal{L}[\sin \omega t]$, by first using your knowledge of the second order differential equation and initial conditions $y(0)$ and $y'(0)$ that are solved by this function, and then taking the Laplace transform of this equation.

(a) $\mathcal{L}[\sin \omega t] = \frac{\omega}{s^2+1}$

(b) $\mathcal{L}[\sin \omega t] = \frac{1}{s^2\omega^2+\omega^2}$

(c) $\mathcal{L}[\sin \omega t] = \frac{1}{s^2+\omega^2}$

(d) $\mathcal{L}[\sin \omega t] = \frac{\omega}{s^2+\omega}$

(e) $\mathcal{L}[\sin \omega t] = \frac{\omega}{s^2+\omega^2}$

336. Find an expression for $\mathcal{L}[\cos \omega t]$, by first using your knowledge of the second order differential equation and initial conditions $y(0)$ and $y'(0)$ that are solved by this function, and then taking the Laplace transform of this equation.

(a) $\mathcal{L}[\cos \omega t] = \frac{s}{s^2+\omega^2}$

(b) $\mathcal{L}[\cos \omega t] = \frac{s\omega}{s^2+\omega^2}$

(c) $\mathcal{L}[\cos \omega t] = \frac{s\omega}{s^2-\omega^2}$

(d) $\mathcal{L}[\cos \omega t] = \frac{\omega}{s^2+\omega^2}$

(e) $\mathcal{L}[\cos \omega t] = \frac{\omega}{s^2-\omega^2}$

337. Find $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right]$.

(a) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 3\cos 2t$.

(b) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 3\cos 4t$.

(c) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 4\cos 3t$.

(d) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 3\sin 2t$.

(e) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 3\sin 4t$.

(f) $\mathcal{L}^{-1}\left[\frac{3s}{4+s^2}\right] = 4\sin 3t$.

338. Find $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right]$.

(a) $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right] = 5\sin\sqrt{3}t$.

(b) $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right] = 5\sin\sqrt{6}t$.

(c) $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right] = \frac{5}{2}\sin\sqrt{3}t$.

(d) $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right] = \frac{5}{2}\sin\sqrt{6}t$.

(e) $\mathcal{L}^{-1}\left[\frac{5}{2s^2+6}\right] = \frac{5}{2\sqrt{3}}\sin\sqrt{3}t$.

339. We know that the function $f(t)$ has a Laplace transform $F(s)$, so that $\mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}dt = F(s)$. What can we say about the Laplace transform of the product $\mathcal{L}[e^{at}f(t)]$?

(a) $\mathcal{L}[e^{at}f(t)] = e^{at}F(s)$.

(b) $\mathcal{L}[e^{at}f(t)] = e^{as}F(s)$.

(c) $\mathcal{L}[e^{at}f(t)] = F(s-a)$.

(d) $\mathcal{L}[e^{at}f(t)] = F(s+a)$.

(e) We cannot make a general statement about this Laplace transform without knowing $f(t)$.

340. Find $\mathcal{L}[3e^{4t}\sin 5t]$.

(a) $\mathcal{L}[3e^{4t}\sin 5t] = \frac{3}{(s-4)^2+25}$.

(b) $\mathcal{L}[3e^{4t}\sin 5t] = \frac{15}{(s-4)^2+25}$.

(c) $\mathcal{L}[3e^{4t}\sin 5t] = \frac{15}{(s+4)^2+25}$.

(d) $\mathcal{L}[3e^{4t}\sin 5t] = \frac{3\sqrt{5}}{(s-4)^2+5}$.

(e) $\mathcal{L}[3e^{4t}\sin 5t] = \frac{3\sqrt{5}}{(s+4)^2+5}$.

341. Find $\mathcal{L}^{-1}\left[\frac{8}{(s+3)^2+16}\right]$.

(a) $\mathcal{L}^{-1}\left[\frac{8}{(s+3)^2+16}\right] = 2e^{-3t}\sin 4t$

(b) $\mathcal{L}^{-1}\left[\frac{8}{(s+3)^2+16}\right] = 2e^{-4t}\sin 3t$

(c) $\mathcal{L}^{-1}\left[\frac{8}{(s+3)^2+16}\right] = 8e^{-3t}\sin 4t$

(d) $\mathcal{L}^{-1}\left[\frac{8}{(s+3)^2+16}\right] = 2e^{3t}\sin 4t$

342. $s^2 + 6s + 15$ is equivalent to which of the following?

(a) $s^2 + 6s + 15 = (s+2)^2 + 11$

(b) $s^2 + 6s + 15 = (s+3)^2 + 6$

(c) $s^2 + 6s + 15 = (s+5)^2 - 10$

(d) $s^2 + 6s + 15 = (s+6)^2 - 21$

343. Find $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right]$, by first completing the square in the denominator.

(a) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = 2e^{-3t}\sin\sqrt{6}t$

(b) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = 2e^{3t}\cos\sqrt{6}t$

(c) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = 2e^{-3t}\cos\sqrt{6}t$

(d) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = 2e^{3t}\sin\sqrt{6}t$

(e) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = e^{6t}\cos\sqrt{6}t$

(f) $\mathcal{L}^{-1}\left[\frac{2s+6}{s^2+6s+15}\right] = e^{-6t}\cos\sqrt{6}t$

344. Find $\mathcal{L}^{-1}\left[\frac{2s+8}{s^2+6s+15}\right]$, by first completing the square in the denominator.

(a) $\mathcal{L}^{-1}\left[\frac{2s+8}{s^2+6s+15}\right] = 2e^{-3t}\cos\sqrt{6}t$

(b) $\mathcal{L}^{-1}\left[\frac{2s+8}{s^2+6s+15}\right] = 2e^{-3t}\cos\sqrt{6}t + 2$

(c) $\mathcal{L}^{-1}\left[\frac{2s+8}{s^2+6s+15}\right] = 2e^{-3t}\cos\sqrt{6}t + 2e^{-3t}\sin\sqrt{6}t$

(d) $\mathcal{L}^{-1}\left[\frac{2s+8}{s^2+6s+15}\right] = 2e^{-3t}\cos\sqrt{6}t + \frac{2}{\sqrt{6}}e^{-3t}\sin\sqrt{6}t$

345. Suppose that after taking the Laplace transform of a differential equation we obtain the function $Y(s) = \frac{2s}{(s^2+16)(s^2+3)}$. This is equivalent to which of the following?

(a) $Y(s) = \frac{2s/19}{s^2+16} + \frac{2s/19}{s^2+3}$
(b) $Y(s) = \frac{-2s/19}{s^2+16} + \frac{2s/19}{s^2+3}$
(c) $Y(s) = \frac{2s/13}{s^2+16} + \frac{-2s/13}{s^2+3}$
(d) $Y(s) = \frac{-2s/13}{s^2+16} + \frac{2s/13}{s^2+3}$

346. Find $\mathcal{L}^{-1}\left[\frac{2s}{(s^2+16)(s^2+3)}\right]$.

(a) $\mathcal{L}^{-1}\left[\frac{2s}{(s^2+16)(s^2+3)}\right] = -\frac{2}{13}\cos 4t + \frac{2}{13}\cos\sqrt{3}t.$
(b) $\mathcal{L}^{-1}\left[\frac{2s}{(s^2+16)(s^2+3)}\right] = \frac{2}{13}\cos 4t - \frac{2}{13}\cos\sqrt{3}t.$
(c) $\mathcal{L}^{-1}\left[\frac{2s}{(s^2+16)(s^2+3)}\right] = -\frac{2}{52}\sin 4t + \frac{2}{13\sqrt{3}}\sin\sqrt{3}t.$
(d) $\mathcal{L}^{-1}\left[\frac{2s}{(s^2+16)(s^2+3)}\right] = \frac{2}{52}\sin 4t - \frac{2}{13\sqrt{3}}\sin\sqrt{3}t.$

347. Use Laplace transforms to solve the differential equation $y'' = -2y + 4\sin 3t$ if we know that $y(0) = y'(0) = 0$.

(a) $y = -\frac{6\sqrt{2}}{7}\sin\sqrt{2}t + \frac{4}{7}\sin 3t$
(b) $y = \frac{6\sqrt{2}}{7}\sin\sqrt{2}t - \frac{4}{7}\sin 3t$
(c) $y = \frac{12}{7}\sin\sqrt{2}t - \frac{12}{7}\sin 3t$
(d) $y = -\frac{12}{7}\cos\sqrt{2}t + \frac{12}{7}\cos 3t$

348. The Dirac delta function $\delta_a(t)$ is defined so that $\delta_a(t) = 0$ for all $t \neq a$, and if we integrate this function over any integral containing a, the result is 1. What would be $\int_5^{10} (\delta_3(t) + 2\delta_6(t) - 3\delta_8(t) + 5\delta_{11}(t))\, dt$?

(a) -1
(b) 0
(c) 4
(d) 5
(e) -30
(f) This integral cannot be determined.

349. Find $\int_5^{10} (\delta_6(t) \cdot \delta_7(t) \cdot \delta_8(t) \cdot \delta_9(t))\, dt$?

(a) -1
(b) 0
(c) 1
(d) 4
(e) 5
(f) This integral cannot be determined.

350. Find $\mathcal{L}[\delta_7(t)]$.

(a) 1
(b) 7
(c) e^{-7s}
(d) e^{-7t}
(e) This integral cannot be determined.

351. Find $\mathcal{L}[\delta_0(t)]$.

(a) 0
(b) 1
(c) e^{-s}
(d) e^{-t}
(e) This integral cannot be determined.

352. We have a system modeled as an undamped harmonic oscillator, that begins at equilibrium and at rest, so $y(0) = y'(0) = 0$, and that receives an impulse force at $t = 4$, so that it is modeled with the equation $y'' = -9y + \delta_4(t)$. Find $y(t)$.

(a) $y(t) = u_4(t) \sin 3t$
(b) $y(t) = \frac{1}{3} u_4(t) \sin 3t$
(c) $y(t) = u_4(t) \sin 3(t-4)$
(d) $y(t) = \frac{1}{3} u_4(t) \sin 3(t-4)$

353. We have a system modeled as an undamped harmonic oscillator, that begins at equilibrium and at rest, so $y(0) = y'(0) = 0$, and that receives an impulse forcing at $t = 5$, so that it is modeled with the equation $y'' = -4y + \delta_5(t)$. Find $y(t)$.

(a) $y(t) = u_5(t)\sin 2t$
(b) $y(t) = \frac{1}{2}u_5(t)\sin 2(t-5)$
(c) $y(t) = \frac{1}{2}u_2(t)\sin 2(t-5)$
(d) $y(t) = \frac{1}{5}u_2(t)\sin 5(t-2)$
(e) $y(t) = \frac{1}{5}u_5(t)\sin 2(t-2)$

354. For many differential equations, if we know how the equation responds to a nonhomogeneous forcing function that is a Dirac delta function, we can use this response function to predict how the differential equation will respond to any other forcing function and any initial conditions. For which of the following differential equations would this be impossible?

(a) $p''(q) = \frac{4}{q} + 96p' - 12p$
(b) $\frac{d^2m}{dn^2} - 100\frac{dm}{dn} + 14nm = \frac{18}{\ln(n)}$
(c) $\frac{d^2f}{dx^2} - 100\frac{df}{dx} = \frac{18}{\ln(x)}$
(d) $\frac{dc}{dr} = 12c + 3\sin(2r) + 8\cos^2(3r+2)$

355. The convolution of two functions $f * g$ is defined to be $\int_0^t f(t-u)g(u)du$. What is the convolution of the functions $f(t) = 2$ and $g(t) = e^{2t}$?

(a) $f * g = e^{2t} - 1$
(b) $f * g = 2e^{2t} - 2$
(c) $f * g = 4e^{2t} - 4$
(d) $f * g = \frac{1}{2}e^{2t} - \frac{1}{2}$
(e) This convolution cannot be computed.

356. What is the convolution of the functions $f(t) = u_2(t)$ and $g(t) = e^{-3t}$?

(a) $f * g = \frac{1}{3}e^{-3(t-2)} - \frac{1}{3}$
(b) $f * g = \frac{1}{3}e^{-3t} - \frac{1}{3}$
(c) $f * g = -\frac{1}{3}e^{-3(t-2)} + \frac{1}{3}$
(d) $f * g = -\frac{1}{3}e^{-3t} + \frac{1}{3}$
(e) This convolution cannot be computed.

357. Let $\zeta(t)$ be the solution to the initial-value problem $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = \delta_0(t)$, with $y(0) = y'(0) = 0^-$. Find an expression for $\mathcal{L}[\zeta]$.

(a) $\mathcal{L}[\zeta] = 0$

(b) $\mathcal{L}[\zeta] = s^2 + ps + q$

(c) $\mathcal{L}[\zeta] = \frac{1}{s^2+ps+q}$

(d) $\mathcal{L}[\zeta] = \frac{e^{-s}}{s^2+ps+q}$

358. Let $\zeta(t) = 2e^{-3t}$ be the solution to an initial-value problem $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = \delta_0(t)$, with $y(0) = y'(0) = 0^-$ for some specific values of p and q, so that $\mathcal{L}[\zeta] = \frac{1}{s^2+ps+q}$. What will be the solution of $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = 0$ if $y(0) = 0$ and $y'(0) = 5$?

(a) $y = 2e^{-3t}$

(b) $y = 10e^{-3t}$

(c) $y = \frac{2}{5}e^{-3t}$

(d) $y = 2e^{-15t}$

(e) It cannot be determined from the information given.

359. Let $\zeta(t)$ be the solution to the initial-value problem $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = \delta_0(t)$, with $y(0) = y'(0) = 0^-$. Now suppose we want to solve this problem for some other forcing function $f(t)$, so that we have $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = f(t)$. Which of the following is a correct expression for $\mathcal{L}[y]$?

(a) $\mathcal{L}[y] = 0$.

(b) $\mathcal{L}[y] = \mathcal{L}[f]\mathcal{L}[\zeta]$.

(c) $\mathcal{L}[y] = \frac{\mathcal{L}[f]}{\mathcal{L}[\zeta]}$.

(d) $\mathcal{L}[y] = \mathcal{L}[f\zeta]$.

(e) None of the above

360. Let $\zeta(t) = e^{-3t}$ be the solution to an initial-value problem $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = \delta_0(t)$, with $y(0) = y'(0) = 0^-$. Now suppose we want to solve this problem for the forcing function $f(t) = e^{-2t}$, so that we have $\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = f(t)$. Find $y(t)$.

(a) $y(t) = e^{-2t} - e^{-3t}$

(b) $y(t) = e^{-3t} - e^{-2t}$

(c) $y(t) = \frac{e^{-2t}-e^{-3t}}{5}$

(d) $y(t) = \frac{e^{2t}-e^{-3t}}{5}$