

# GoodQuestions for College Algebra

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While many of the following discussion/peer-learning style questions were created solely by the author, several questions have been taken directly or with minor modifications from the GoodQuestions for Calculus project which the author participated in while a graduate student in the mathematics department at Cornell University. A full listing of the questions created during that project is available on line at [www.math.cornell.edu/GoodQuestions/](http://www.math.cornell.edu/GoodQuestions/) and [www.math.cornell.edu/~GoodQuestions/](http://www.math.cornell.edu/~GoodQuestions/).

1. If the line  $x = 2$  hits the graph of a curve in two places, then the graph doesn't represent a function.

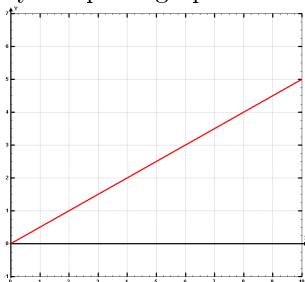
A) This is true, because the definition of a function says that no vertical line touches the graph more than one time.

B) This is false, because every vertical line must hit the graph twice to keep the graph from representing a function.

C) This is true, because the  $y$ -values at the two points where the line touches the curve both correspond to  $x = 2$ .

D) This is false, because there might be another vertical line which only hits the curve at one point.

2. Suppose that you were riding your bicycle up the graph:



Would the graph be a good graph of your distance covered with respect to time (in hours)? (Choose the best answer)

A) This is a good graph, since as long as I don't stop, the distance should be increasing.

B) This is not a good graph for the whole trip, since my speed won't always be the same.

C) This is a good graph, since my average speed should be the same even if my actual speed isn't constant.

D) This isn't a good graph, since the world is round.

3. Imagine that there is a rope around the equator of the earth. Add a 60 feet segment of rope to it. The new rope is held in a circular shape centered about the earth. Then the tallest of the following that can walk beneath the rope without touching it is:

- A) An amoeba
- B) An Ant
- C) A Dog
- D) Me (the student)

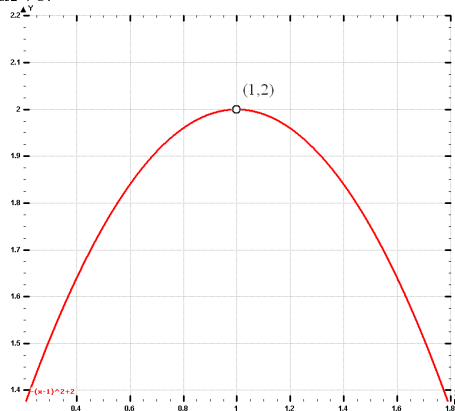
4. Given two infinite decimals  $a = .3939393939\dots$  and  $b = .67766777666\dots$ , their sum  $a + b$

- A) is not defined because the sum of a rational and an irrational number is not defined.
- B) is not a number because not all infinite decimals are real numbers.
- C) can be defined precisely by using successively better approximations.
- D) is not a real number because the pattern may not be predictable indefinitely.

5. Let  $f$  be the function defined by  $f(x) = x + x^2$  and let  $g$  be the function defined by  $g(u) = u + u^2$ , for all real numbers  $x$  and  $u$ . Then,

- A)  $f$  and  $g$  are exactly the same functions
- B) if  $x$  and  $u$  are different numbers,  $f$  and  $g$  are different functions
- C) not enough information is given to determine if  $f$  and  $g$  are the same.

6. Consider the following graph of the function  $f$ , which indicates that the point  $(1,2)$  is not included in the graph of the curve:



- A) The maximum is the largest real number which is  $< 2$ .
- B) There is no maximum, because  $f$  gets as close as you could ask to 2, but is never equal to 2.
- C) The maximum is at  $(1, 2)$  since the function switches from increasing to decreasing there.
- D) There is no maximum because there is no open interval around  $x = 1$  such that  $f(x) < 2$  on the entire interval.

7. A system of two linear equations can have a solution set of no points, one point, or infinitely many points.

A) This is true, because these are the only possibilities for any system of equations, linear or not.

B) This is false, there could be other possibilities for the number of solutions

C) This is true because any two lines in a plane must intersect in one point, no points or all their points.

D) Not enough information is given to tell if this is true or not - you need to know what the two equations are.

8. If a function  $f$  is increasing then its inverse must be decreasing.

A) Always True

B) Sometimes True

C) Never True

9. Suppose you start with two rabbits in generation one, and the number of rabbits doubles in each subsequent generation. Which kind of function would you need to answer the question, "How many generations will it take to experience an 18 fold increase in the number of rabbits?"

A) A linear function would help to answer this question.

B) A logarithmic function would help to answer this question.

C) An exponential function would help to answer this question.

D) A quadratic function would help to answer this question.

10. What is the significance of the phrase: the line tangent to the graph of  $e^x$  at each point of the graph has slope which is equal to the height of the exponential function at the tangent point.

A) Since functions with constant slope are best for modeling, this means that we shouldn't use exponential functions to model data.

B) This statement is not significant since it is true for all exponential functions with any base number.

C) Since the slope of the line is like the rate of change, this means that the rate of change of the  $e^x$  depends on the size of  $e^x$ .

D) Actually, slope is like the average rate of change, so the real rate of change of  $e^x$  might not be changing, just the average rate of change.

11. Suppose you are modeling some data. From one point to the next, the average rate of change of the data is positive for a while, and then the average rate of change switches to being negative.

Which of these kinds of function might best be used to model this data?

A) exponential   B) logarithmic   C) quadratic   D) linear

12. Starting from  $f(x)$ , to graph of  $7f(2x + 3) - 4$  we should transform in this order:

- A) Down -4, vert. stretch by 7, left by 3 and horiz. shrink by 2.
- B) Left by three, horiz. shrink by 2, vert. stretch by 7, down 4.
- C) Horiz. shrink by 2, left by three, vert. stretch by 7, down by 4.
- D) It doesn't matter what order, just do all four transformations.

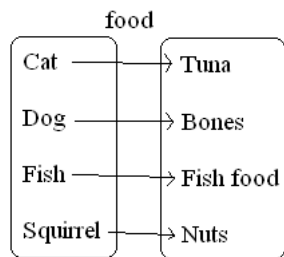
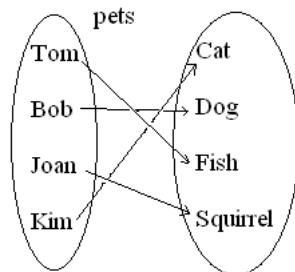
13. If a function  $f$  is increasing then it must be one-to-one.

- A) Always True
- B) Sometimes True
- C) Never True

14. If  $f(x) = \frac{x^2-4}{x-2}$  and  $g(x) = x + 2$ , then we can say the functions  $f$  and  $g$  are equal.

- A) True.
- B) False.

15.  
For these two functions,



A) It makes sense to compose  $\text{pets} \circ \text{food}$  since the domain of  $\text{food}$  matches up with the range of  $\text{pets}$ .

B) It does not make sense to compose  $\text{food} \circ \text{pets}$  since range of  $\text{pets}$  is the same as the domain of  $\text{food}$ .

C) It does not make sense to compose  $\text{pets} \circ \text{food}$  since the range of  $\text{food}$  is not the same as the domain of  $\text{pets}$ .

D) It never makes sense to compose functions unless they are real functions.

16. Suppose that on an exam, you are asked to find the exact solutions to a 5th degree polynomial. You should

- A) skip it, the Abel-Ruffini Theorem says there are no solutions.

B) do it, the Fundamental Theorem of Algebra guarantees that there are solutions.

C) do it, just use your graphing calculator to find the solutions.

D) skip it, unless crossing your fingers and guessing the solutions works pretty fast.

17. If you know that polynomial  $P(x)$  has degree 5 and  $d(x)$  has degree 2 then

A)  $Q(x)$  has degree 3 and  $R(x)$  has degree zero

B)  $Q(x)$  has degree 3 and  $R(x)$  has degree 1

C)  $Q(x)$  has degree 5 and  $R(x)$  has degree 1

D) Not enough information to tell the degree of  $R$ .

18. You come across an advertisement in a magazine for an investment opportunity that guarantees that your investments with their company will "Grow Exponentially Fast!". You invest \$5,000 bucks, since everybody knows exponential growth is really fast. Five years later, your investment is worth \$5,250. You should

A) definitely sue, since no exponential function could grow that slow.

B) still expect to get rich, but plan to live a really long time.

C) do nothing, since your investment will be worth way more within the next few years.

19. You were once exactly three feet tall.

A) True

B) False

20. If you use completing the square to help draw the graph of a quadratic by just transforming the graph of  $x^2$ , which of these will you never need to use? (Hint: try to remember our example,  $2x^2 - 10x + 14$ .)

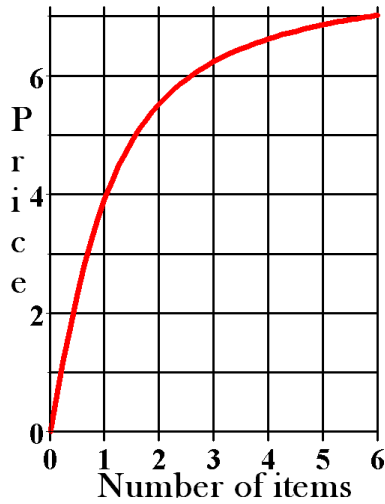
A) vertical shift

B) vertical stretch or shrink

C) horizontal shift

D) horizontal stretch or shrink

21.



Suppose this is a graph of the cost for buying some number of items in a store. If there is a buy one get one (BOGO) free sale plus 3 dollars off of your total order, then we should

A) move the graph down 3, and then horizontally stretch by a factor of 2.

B) move the graph down by 3 and vertically stretch by 2.

C) move the graph left by 3 and horizontally stretch by a factor of 2.

D) Horizontally stretch the graph by 2 and then move it down 3.

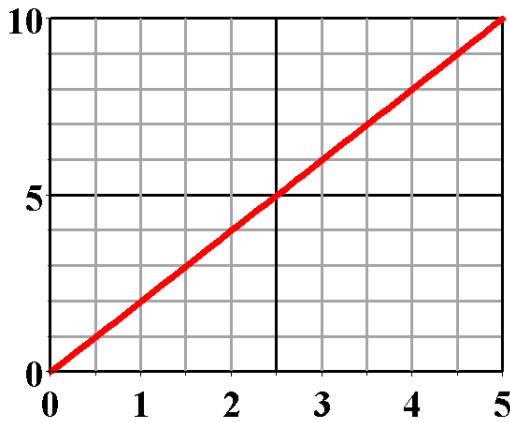
22. A drunk man is discovered searching for his keys beneath a street lamp. When asked where he lost his keys, he quickly pointed up the street and said, “about 50 yards that way”. When asked why he was searching here, under the lamp, instead of where the keys were lost, the drunk answered, “the light is better here.”

A) The drunk’s problem is that his model is too complicated - he won’t be able to compute a result.

B) The drunk’s problem is that his model is too simple - he might be able to compute a result, but the possible error is so large that his solution will be meaningless.

23. The following is a graph of the speed of an object traveling down a straight road.

How far does the object travel between time 2 and 4?



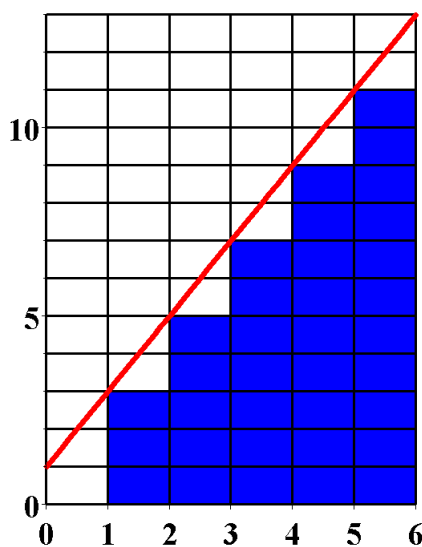
- A) more than 5 units.
- B) less than 7 units
- C) Exactly 6 units
- D) All of these

24. We have seen that we can compare Arithmetic sequences to linear equations. Let's try to push this idea a little bit further. You may want to pick an example, like  $2, 5, 8, \dots$  and draw a picture of the sequence with the linear equation it goes with to help you see the truth. The number  $S_n$  of an Arithmetic Series is the most like

- A) The slope of the line which corresponds to the arithmetic sequence.
- B) The  $y$ -intercept of this line.
- C) The height of the line when  $x = n$ .
- D) The area between the line and the  $x$ -axis from  $x = 0$  to  $x = n$ .
- E)  $S_n$  is not really like any of these.

25. Linear equations and arithmetic sequences are related since

- A) for either, the change in height only depends on the change in  $x$  or in  $n$ , not on which  $x$  or  $n$  we started with.
- B) for either, the total area under the line or the sum of the terms of the sequence starting from zero grows quadratically.
- C) the amount of area under the graph from  $x$  to  $x + 2$  grows linearly as  $x$  grows, and the sum of  $a_n + a_{n+1}$  grows linearly as  $n$  grows.
- D) all of the above.



26. Suppose you are putting tile on a square floor and you need to decide how wide the floor should be. The flooring worker tells you that the cost for tile for a 10 foot wide floor would be \$235. Next you find out that it would cost \$122 more to build a 12 foot wide floor, or \$347. If you wanted to build a 14 foot wide floor, it would cost

- A) \$459, another 122\$ more.
- B) less than \$459 but you couldn't say how much less.
- C) less than \$459, and you could figure out how much less.
- D) more than \$459, but you couldn't say how much more.
- E) more than \$459, and you could figure out how much more.

27. You toss a pebble into a pond. You notice that the radius of the circular wave that your pebble created is growing at 10 centimeters per second, so that after one second, the area of the circle is  $20\pi$ . About how long will it take for

the area of the circle to double?

- A) About 1 second.
- B) Less than 1 second, but you couldn't say how much less.
- C) Less than 1 second, and you could figure out how much less.
- D) Longer than 1 second, but you couldn't say how much longer.
- E) Longer than 1 second, and you could figure out how much longer.

28. At some time since you were born, your weight in pounds was exactly equal to your height in inches.

- A) True
- B) False

29. You already know that when you divide a number by 7 the remainder has to be smaller than 7.

It seems like when you divide a polynomial  $P(x)$  by another polynomial  $d(x)$  the remainder  $r(x)$  should be smaller than  $d(x)$ . This means that

- A) the lead coefficient of  $r(x)$  is smaller than the lead coefficient of  $d(x)$ .
- B) the degree of  $r(x)$  is smaller than the degree of  $d(x)$ .
- C) the lead term of  $r(x)$  is smaller than the lead term of  $d(x)$ .
- D) the number of terms in  $r(x)$  is smaller than the number of terms in  $d(x)$ .
- E) actually, the remainder doesn't have to be smaller when you do polynomial division.

30. Getting something finite after adding up infinitely many numbers

- A) doesn't make sense - you should get something infinitely large.
- B) doesn't make sense - you can never finish adding all the numbers.
- C) does make sense - you could get as close as anybody could ask, even if you can't finish.
- D) does make sense as long as the numbers get smaller and smaller and you add them faster and faster.

31. If a function is always increasing then it must have an inverse.

- A) True
- B) False

32. If a function  $f$  is increasing then its inverse must be decreasing.

- A) True
- B) False

33. The slope of the line tangent to the graph of  $\ln(x)$  at  $x=1$  is

- A)  $\ln(1)$
- B) 1

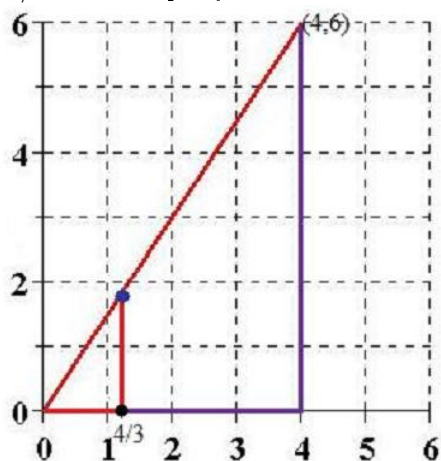


- C)  $e$
- D) You can't tell without using calculus.

34. The inequality  $|x-7|>3$  must have a solution which is the union of two intervals because

- A) the inequality is true for all numbers which are close enough to seven on either side.
- B) all inequalities involving absolute values have a solution which is the union of two intervals.
- C) on the number line there are two directions in which  $x$  can move more than three units away from seven.
- D) Actually, there is only one interval in the solution to this inequality.

35. Suppose you want to find the coordinates of the point which is one-third of the way from  $(0,0)$  to  $(4,6)$ . This graph with the short red line drawn at  $x = 4/3$  would help if you combine it with



- A) the Pythagorean Theorem.
- B) the sum of the angles of a triangle is 180 degrees.
- C) the principle of similar triangles.
- D) the principle of parallel lines.

36. Since  $(1 + \frac{.05}{360})^{360}$  is really close to  $e^{.05}$ , daily compounding and continuous compounding aren't really very different.

- A) True
  - B) False
  - C) It depends
37. The formula

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

- A) works for every value of  $n$  and all values of  $r$  except for  $r = 1$ .
- B) works for  $n = \infty$ .
- C) can be used to find the sum of the infinite series if  $|r| < 1$ .
- D) both A and C are true.
- E) All of the above.