

MathQuest: Linear Algebra

Systems of Equations

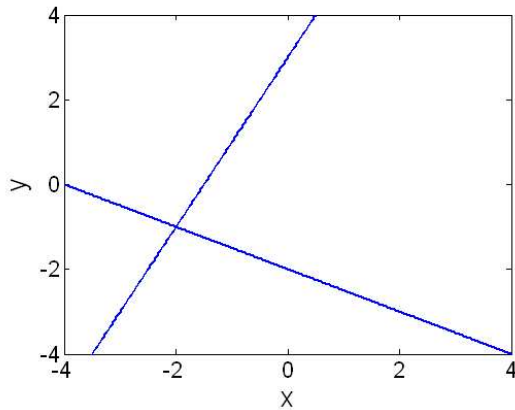
1. What is the solution to the following system of equations?

$$2x + y = 3$$

$$3x - y = 7$$

- (a) $x = 4$ and $y = -5$
- (b) $x = 4$ and $y = 5$
- (c) $x = 2$ and $y = -1$
- (d) $x = 2$ and $y = 1/2$
- (e) There are an infinite number of solutions to this system.
- (f) There are no solutions to this system.

2. Which of the following systems of equations could be represented in the graph below?



- (a) $3x + 3y = -6$, $x + 2y = 3$
- (b) $x - y = -5$, $2x + y = 4$
- (c) $-8x + 4y = 12$, $2x + 4y = -8$
- (d) $-x + 3y = 9$, $2x - y = 4$

3. What is the solution to the following system of equations?

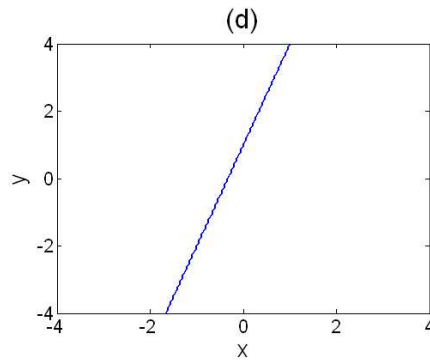
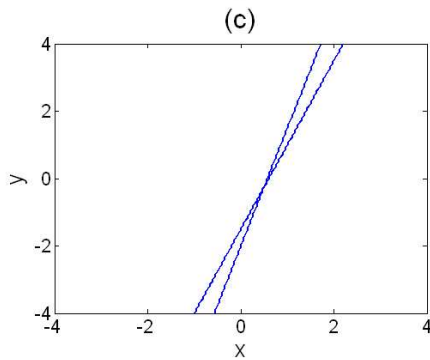
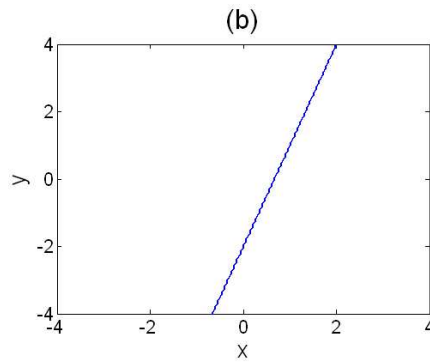
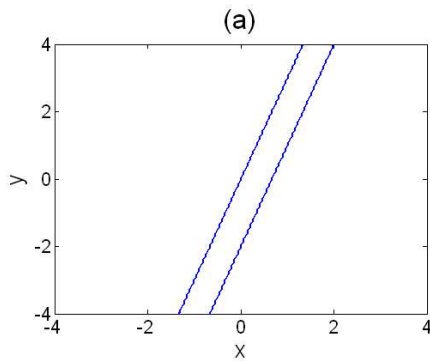
$$2x + y = 3$$

$$4x + 2y = 6$$

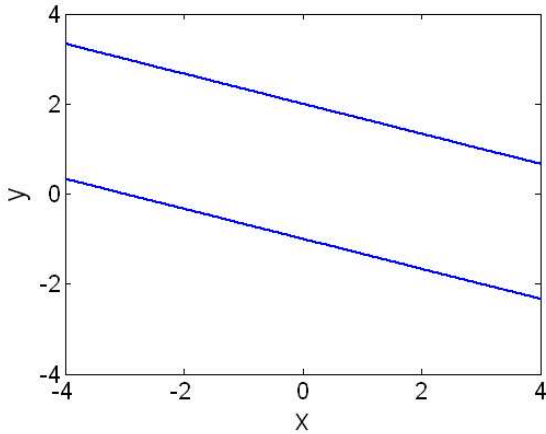
- (a) $x = 0$ and $y = 0$
- (b) $x = 2$ and $y = -1$
- (c) $x = 0$ and $y = 1$
- (d) $x = 0$ and $y = 3$
- (e) There are an infinite number of solutions to this system.
- (f) There are no solutions to this system.

4. Which of the graphs below could represent the following linear system?

$$\begin{aligned} 3x - y &= 2 \\ -9x + 3y &= -6 \end{aligned}$$



5. Which of the following systems of equations could be represented in the graph below?



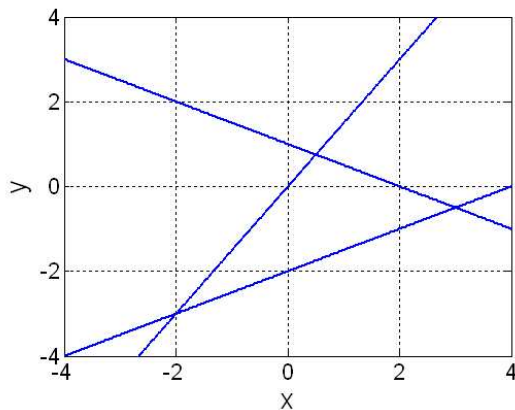
- (a) $-x + 3y = 6, 2x + 6y = -6$
- (b) $-x + 3y = 6, 2x + 6y = 12$
- (c) $x + 3y = 6, 2x + 6y = 12$
- (d) $x + 3y = 6, x + 3y = -3$

6. What is the solution to the following system of equations?

$$\begin{aligned} -3x + 2y &= 4 \\ 12x - 8y &= 10 \end{aligned}$$

- (a) $x = -4/3$ and $y = 0$
- (b) $x = 1/2$ and $y = -1/2$
- (c) $x = 0$ and $y = 2$
- (d) $x = 1/3$ and $y = 5/2$
- (e) There are an infinite number of solutions to this system.
- (f) There are no solutions to this system.

7. We have a system of three linear equations with two unknowns, as plotted in the graph below. How many solutions does this system have?



- (a) 0
- (b) 1
- (c) 2
- (d) 3
- (e) Infinite

8. A system of linear equations could *not* have exactly _____ solutions.

- (a) 0
- (b) 1
- (c) 2
- (d) infinite
- (e) All of these are possible numbers of solutions to a system of linear equations.

9. The system

$$\begin{aligned}x + y &= 2 \\2x + 2y &= 4\end{aligned}$$

has an infinite number of solutions. Which of the following describes the set of solutions to this system?

- (a) $x = 1$ and $y = 1$
- (b) $x = 2 - t$ and $y = t$
- (c) x and y could each be anything.
- (d) None of the above

10. Which of the following options describes the set of solutions to the system below?

$$\begin{aligned}x + y &= 1 \\x - y &= 0 \\2x + y &= 3\end{aligned}$$

- (a) $x = 1 - t$ and $y = t$
- (b) $x = 1$ and $y = 1$
- (c) no solution exists
- (d) None of the above

11. Which of the following options describes the set of solutions to the system below?

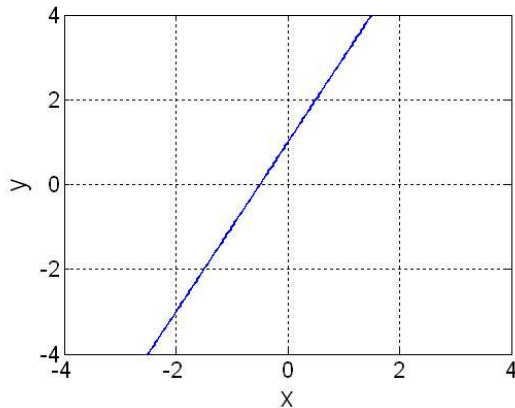
$$\begin{aligned}x + y &= 2 \\2x - y &= -2 \\x - 2y &= -4\end{aligned}$$

- (a) $x = t$ and $y = 2 - t$
- (b) $x = 0$ and $y = 2$
- (c) no solution exists
- (d) None of the above

12. $x = 3 - 2t$ and $y = t$ represent the set of solutions to a system of equations. What line in \mathbb{R}^2 does this set of solutions represent?

- (a) $x + 2y = 3$
- (b) $x - 2y = 3$
- (c) $x + y = 3 - t$
- (d) It is impossible to answer this question with the information given.

13. The set of solutions to a system of linear equations is plotted below. Which of the following parameterizations represents this solution set?



- (a) $x = 2t$ and $y = 4t + 1$
- (b) $x = \frac{1}{2}t - \frac{1}{2}$ and $y = t$
- (c) $x = t - 1$ and $y = 2t - 1$
- (d) $x = t$ and $y = 2t + 1$
- (e) All of the above

14. A certain mini-golf course does not list their prices. I paid \$26.25 for 3 children and 4 adults. The group in front of me had paid \$25.50 for 6 children and 2 adults. Which system of equations would allow us to determine the prices for children and adults?

(a)

$$\begin{aligned}3x + 6y &= 26.25 \\4x + 2y &= 25.50\end{aligned}$$

(b)

$$\begin{aligned}3x + 4y &= 26.25 \\6x + 2y &= 25.50\end{aligned}$$

(c)

$$\begin{aligned}26.25x + 25.50y &= 51.75 \\9x + 6y &= 15\end{aligned}$$

(d)

$$\begin{aligned}(26.25/3)x + (26.25/4)y &= 0 \\(25.50/6)x + (25.50/6)y &= 0\end{aligned}$$

15. A system of 3 linear equations with 3 variables could not have exactly _____ solutions.

(a) 0

(b) 1

(c) 2

(d) 3

(e) More than one of (a)-(d) are impossible.

(f) All of (a)-(d) are possible numbers of solutions.

16. A linear equation with two variables can be geometrically represented as a line in \mathfrak{R}^2 . How can we best represent a linear equation with three variables?

(a) As a line in \mathfrak{R}^2

(b) As a line in \mathfrak{R}^3

(c) As a plane in \mathfrak{R}^3

(d) As a volume in \mathfrak{R}^3

17. We find that a system of three linear equations in three variables has an infinite number of solutions. How could this happen?
- (a) We have three equations for the same plane.
 - (b) At least two of the equations must represent the same plane.
 - (c) The three planes intersect along a line.
 - (d) The planes represented are parallel.
 - (e) More than one of the above are possible.
18. We consider a system of three linear equations in three variables, and visualize the graph of each equation as a plane in \mathfrak{R}^3 . Suppose no solutions exist to this system. This means that
- (a) all three planes must be parallel.
 - (b) at least two of the planes must be parallel.
 - (c) at least two of the equations represent the same plane.
 - (d) none of these planes ever intersects with another.
 - (e) None of the above
19. We have a system of four linear equations in four variables. We can think about the graph of each equation as a 3-dimensional volume in \mathfrak{R}^4 . Which of the following could geometrically represent the solutions to this system?
- (a) A point in \mathfrak{R}^4
 - (b) A line in \mathfrak{R}^4
 - (c) A plane in \mathfrak{R}^4
 - (d) A three dimensional volume in \mathfrak{R}^4
 - (e) All of the above
 - (f) None of the above
20. How can we geometrically represent the parametric equations $x = 2t$, $y = -t + 1$, and $z = t$?
- (a) A line in \mathfrak{R}^2
 - (b) A line in \mathfrak{R}^3
 - (c) A plane in \mathfrak{R}^3
 - (d) A volume in \mathfrak{R}^3

21. A system of 5 linear equations and 7 variables could not have exactly _____ solutions.

- (a) 0
- (b) 1
- (c) infinite
- (d) More than one of these is impossible.
- (e) All of these are possible numbers of solutions.

22. A system of 8 linear equations and 6 variables could not have exactly _____ solutions.

- (a) 0
- (b) 1
- (c) infinite
- (d) More than one of these is impossible.
- (e) All of these are possible numbers of solutions.

23. Howard's store sells three blends of flour: standard, extra wheat, and extra soy. Each is a blend of whole wheat flour and soy flour, and the table below shows how many pounds of each type of flour is needed to make one pound of each blend.

whole wheat flour	soy flour	
0.5	0.5	Standard Blend
0.8	0.2	Extra Wheat
0.3	0.7	Extra Soy

A customer comes in who wants one pound of a blend that is 60% wheat and 40% soy. Which system of equations below would allow us to solve for the amount of each blend needed to fulfill this special request?

(a)

$$0.5x_1 + 0.5x_2 = 1$$

$$0.8x_1 + 0.2x_2 = 1$$

$$0.3x_1 + 0.7x_2 = 1$$

(b)

$$0.5x_1 + 0.5x_2 = 0.6$$

$$0.8x_1 + 0.2x_2 = 0.4$$

$$0.3x_1 + 0.7x_2 = 0$$

(c)

$$\begin{aligned}0.5x_1 + 0.8x_2 + 0.3x_3 &= 1 \\0.5x_1 + 0.2x_2 + 0.7x_3 &= 1\end{aligned}$$

(d)

$$\begin{aligned}0.5x_1 + 0.8x_2 + 0.3x_3 &= 0.6 \\0.5x_1 + 0.2x_2 + 0.7x_3 &= 0.4\end{aligned}$$

24. In the previous question you set up a system of equations so that you could find the amount of each blend needed to make a new mixture. How many solutions must this system have? (You do not need to solve the system.)

- (a) 0
- (b) 1
- (c) 2
- (d) 3
- (e) Infinite

25. The previous two questions dealt with the system

$$\begin{aligned}0.5x_1 + 0.8x_2 + 0.3x_3 &= 0.6 \\0.5x_1 + 0.2x_2 + 0.7x_3 &= 0.4\end{aligned}$$

In the context given, what quantity or unit does 0.6 represent?

- (a) pounds
- (b) %
- (c) pounds²
- (d) pounds per %
- (e) 0.6 does not have units