MathQuest: Linear Algebra

Gaussian Elimination

1. Which of the following operations on an augmented matrix could change the solution set of a system?

   (a) Interchanging two rows
   (b) Multiplying one row by any constant
   (c) Adding one row to an other
   (d) Adding a multiple of one row to an other
   (e) None of the above
   (f) More than one of the above (which ones?)

2. Which of the following matrices is NOT row equivalent to the one below? In other words, which matrix could you NOT get from the matrix below through elementary row operations?

$$\begin{bmatrix} 2 & 1 & 3 & 1 \\ 0 & 1 & 3 & 4 \\ 1 & 2 & 0 & 4 \end{bmatrix}$$

   (a)
$$\begin{bmatrix} 2 & 4 & 0 & 8 \\ 0 & 1 & 3 & 4 \\ 2 & 1 & 3 & 1 \end{bmatrix}$$

   (b)
$$\begin{bmatrix} 2 & 1 & 3 & 1 \\ 0 & 1 & 3 & 4 \\ 1 & 3 & 3 & 8 \end{bmatrix}$$

   (c)
$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 0 & 3 & 4 \\ 2 & 1 & 0 & 4 \end{bmatrix}$$

   (d) More than one of the above
   (e) All are possible through elementary row operations.

3. Which of the following matrices is row equivalent to the one below? In other words, which matrix could you get from the matrix below through elementary row operations?

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a)

$$\begin{bmatrix} 2 & 5 & 7 \\ 0 & 1 & 3 \\ 4 & 8 & 1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 3 & 1 & 5 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} -3 & 1 & 3 \\ -2 & 1 & 0 \\ 3 & 9 & 2 \end{bmatrix}$$

(d) More than one of the above

(e) All are possible through elementary row operations.

4. Which of the following matrices is NOT row equivalent to the one below? In other words, which matrix could you NOT get from the matrix below through elementary row operations?

$$\begin{bmatrix} 6 & 0 & 4 & 7 \\ 2 & 0 & 1 & 9 \\ 5 & 0 & 3 & 5 \end{bmatrix}$$

(a)

$$\begin{bmatrix} 12 & 0 & 8 & 14 \\ 2 & 0 & 1 & 9 \\ 1 & 0 & 1 & 2 \end{bmatrix}$$

(b)

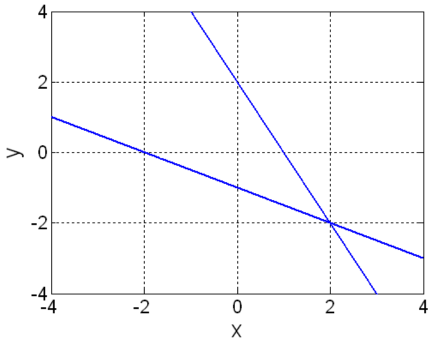
$$\begin{bmatrix} 12 & 0 & 8 & 14 \\ 0 & 0 & 1 & -20 \\ 2 & 1 & 3 & 0 \end{bmatrix}$$

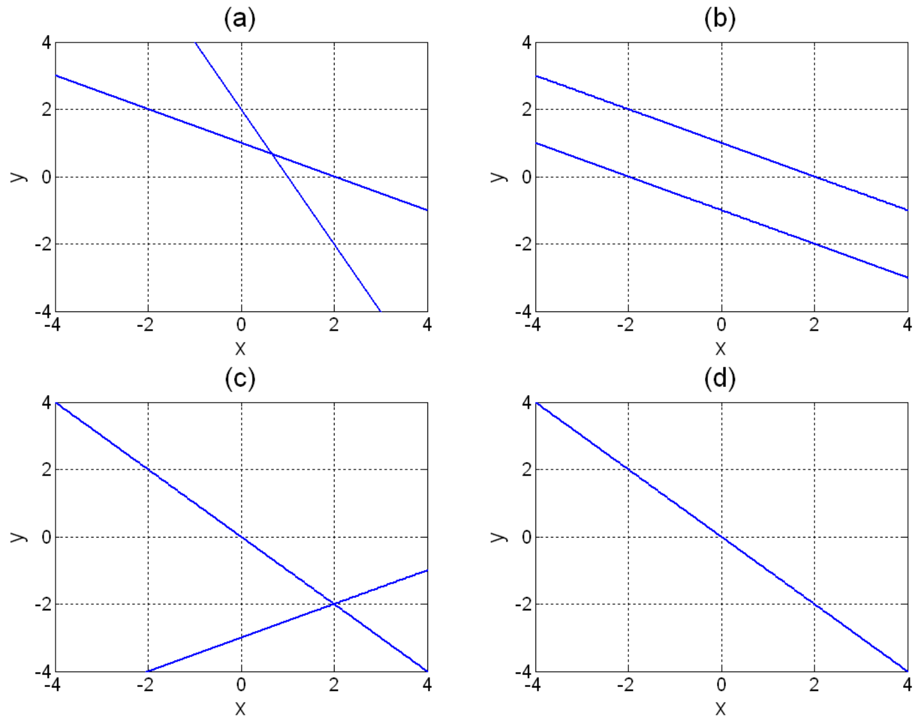
(c)

$$\begin{bmatrix} 6 & 0 & 4 & 7 \\ 2 & 0 & 1 & 9 \\ 7 & 0 & 4 & 14 \end{bmatrix}$$

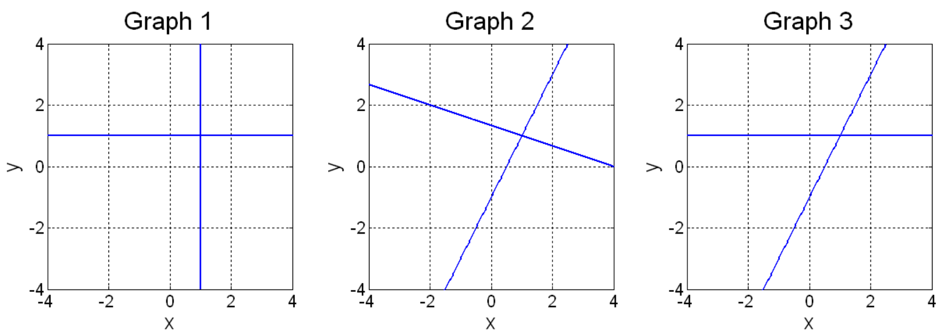
(d) All are possible through elementary row operations.

5. A linear system of equations is plotted below. We create an augmented matrix to represent this linear system, then perform a series of elementary row operations. Which of the following graphs could represent the result of these row operations?





6. We have a system of two linear equations and two unknowns which we solve by performing Gaussian elimination on an augmented matrix. Along the way we create the graphs below, showing geometrical representations of the initial system, the system at an intermediate step in the row reduction process, and the system after it has been put into reduced row echelon form. Put these graphs in order, starting with the initial system and ending with the system in reduced row echelon form.



(a) Graph 2, Graph 3, Graph 1

(b) Graph 1, Graph 3, Graph 2

(c) Graph 1, Graph 2, Graph 3

(d) Graph 2, Graph 1, Graph 3

(e) Graph 3, Graph 2, Graph 1

7. What is the value of a so that the linear system represented by the following matrix would have infinitely many solutions?

$$\begin{bmatrix} 2 & 6 & 8 \\ 1 & a & 4 \end{bmatrix}$$

(a) $a = 0$

(b) $a = 2$

(c) $a = 3$

(d) $a = 4$

(e) This is not possible.

(f) More than one of the above

8. We start with a system of two linear equations in two variables and we translate this system into an augmented matrix M. After performing Gaussian elimination, putting this matrix into reduced row echelon form, we get the matrix R which tells us that this system has no solution. How could we geometrically represent the linear equations contained in the rows of the augmented matrix R?

(a) We can represent the equations of R as two parallel lines.

(b) We can represent the equations of R as two lines that may not be parallel.

(c) We can represent the equations of R as a single line.

(d) The equations of R cannot be represented geometrically.

9. We start with a system of three linear equations in three variables and we translate this system into an augmented matrix M. After performing Gaussian elimination, putting this matrix into reduced row echelon form, we get the matrix R which tells us that this system has no solution. How could we best geometrically represent the linear equations contained in the rows of the augmented matrix M?

   (a) We can represent the equations of M as three parallel lines.
   (b) We can represent the equations of M as three parallel planes.
   (c) We can represent the equations of M as three planes, where at least two must be parallel.
   (d) We can represent the equations of M as three planes, where none of the planes ever intersects with another.
   (e) We can represent the equations of M as three planes, which do not share any points in common.
   (f) The equations of M cannot be represented geometrically.

10. Howard's store sells three blends of flour: standard, extra wheat, and extra soy. Each is a blend of whole wheat flour and soy flour, and the table below shows how many pounds of each type of flour is needed to make one pound of each blend.

whole wheat flour	soy flour	
0.5	0.5	Standard Blend
0.8	0.2	Extra Wheat
0.3	0.7	Extra Soy

A customer comes in who wants one pound of a blend that is 60% wheat and 40% soy. We can solve the following system of equations to determine the amount of Standard Blend (x_1), Extra Wheat Blend (x_2), and Extra Soy Blend (x_3) needed to create this special mixture.

$$\begin{aligned} 0.5x_1 + 0.8x_2 + 0.3x_3 &= 0.6 \\ 0.5x_1 + 0.2x_2 + 0.7x_3 &= 0.4 \end{aligned}$$

If we form an augmented matrix for this system, the reduced row echelon form is $R = \begin{bmatrix} 1 & 0 & 5/3 & 2/3 \\ 0 & 1 & -2/3 & 1/3 \end{bmatrix}$.

If the store is out of Extra Soy Blend, how much of each of the other blends is needed?

   (a) 2/3 pound of Standard Blend and 1/3 pound of Extra Wheat Blend
   (b) 5/3 pound of Standard Blend and 2/3 pound of Extra Wheat Blend
   (c) There are an infinite number of options for the amounts of Standard and Extra Wheat Blend.

(d) It is not possible to create this mixture without Extra Soy Blend.

11. Referring to the previous question, if the store is out of Extra Wheat Blend (x_2), how much of each of the other blends is needed to make the special mixture?

    (a) 2/3 pound of Standard Blend and 1/3 pound of Extra Soy Blend
    (b) 1/6 pound of Standard Blend and 1/2 pound of Extra Soy Blend
    (c) There are an infinite number of options for the amounts of Standard Blend and Extra Wheat Blend.
    (d) It is not possible to create this mixture without Extra Wheat Blend.

12. Referring to the previous two questions, what values are realistic for x_3 in this context?

    (a) x_3 can be any value.
    (b) $x_3 \geq 0$
    (c) $\frac{2}{3} \leq x_3 \leq \frac{5}{3}$
    (d) $-\frac{1}{2} \leq x_3 \leq \frac{2}{5}$
    (e) $0 \leq x_3 \leq \frac{2}{5}$

13. Let R be the reduced row echelon form of a $n \times n$ matrix A. Then

    (a) R is the identity.
    (b) R has at least one row of zeros.
    (c) None of the above.
    (d) All of the above are possible but there exist also other possibilities.
    (e) The two possibilities above are the only ones.
    (f) We can't tell without having the matrix A.