Solution Sets of Linear Systems

1. Which of the following are solutions to the system of equations?

$$\begin{aligned} 2x + y + 2z &= 0 \\ -x + 2y - 6z &= 0 \end{aligned}$$

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ 6 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \\ -2 \end{bmatrix}$

(e) None of the above.

(f) More than one of the above.

2. What is the solution to the following system of equations?

$$\begin{aligned} x + 2y + z &= 0 \\ x + 3y - 2z &= 0 \end{aligned}$$

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \\ 1 \end{bmatrix} s$

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 3 \\ 0 \end{bmatrix}$

(e) None of the above.

(f) More than one of the above.

3. What is the solution to the following system of equations?

$$\begin{aligned} x + 2y + z &= 3 \\ x + 3y - 2z &= 4 \end{aligned}$$

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} s$

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

(d) None of the above.

(e) More than one of the above.

4. What is the solution to the following system of equations?

$$\begin{aligned} x + 2y + z &= -2 \\ x + 3y - 2z &= 1 \end{aligned}$$

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

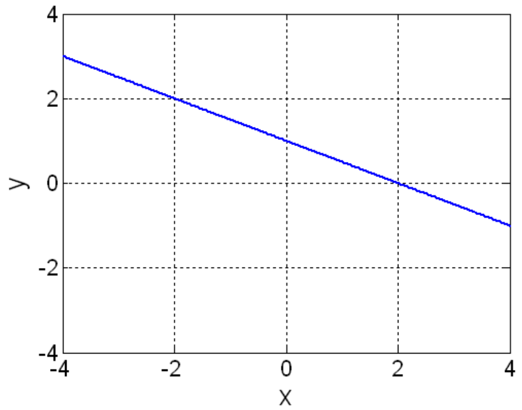
(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -3 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 7 \\ -3 \\ -1 \end{bmatrix} s$

(d) None of the above.

(e) More than one of the above.

5. The set of solutions to a system of linear equations is plotted below. Which of the following expressions represents this solution set?



(a) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1/2 \end{bmatrix} s$

(b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -1/2 \\ 2 \end{bmatrix} s$

(c) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1/2 \\ 2 \end{bmatrix} s$

(d) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} s$

(e) None of the above.

(f) More than one of the above.

6. The set of solutions to a linear system are represented by the expression below. How can we geometrically represent this solution set?

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} t$$

(a) As a line in $\Re^2$

(b) As a line in $\Re^3$

(c) As a plane in $\Re^3$

(d) As a volume in $\Re^3$

(e) None of the above

7. Let $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$. If R is the reduced row echelon form of the augmented matrix for the system $Ax = b$, what are the solutions to that system?

(a) $x_1 = 1, x_2 = 1,$ and $x_3 = 2$
(b) $x_1 = 1, x_2 = 1, x_3 = 2,$ and $x_4 = 0$
(c) $x_1 = -t, x_2 = -t, x_3 = -2t,$ and $x_4 = t$
(d) There are no solutions to this system.

8. Let $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$. If R is the reduced row echelon form of the coefficient matrix for the system $Ax = 0$, what are the solutions to that system?

(a) $x_1 = 1, x_2 = 1,$ and $x_3 = 2$
(b) $x_1 = 1, x_2 = 1, x_3 = 2,$ and $x_4 = 0$
(c) $x_1 = -t, x_2 = -t, x_3 = -2t,$ and $x_4 = t$
(d) There are no solutions to this system.

9. Let matrix R be the reduced row echelon form of matrix A. **True or False** The solutions to $Rx = 0$ are the same as the solutions to $Ax = 0$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

10. Let matrix R be the reduced row echelon form of matrix A. **True or False** The solutions to $Rx = b$ are the same as the solutions to $Ax = b$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

11. Consider a homogeneous linear system with n unknowns. Suppose the reduced row echelon form of its augmented matrix has $r \leq n$ nonzero rows. We can conclude that:

(a) $x_1 = 0, x_2 = 0, \ldots, x_n = 0$ is a solution to the system.
(b) The system has $n - r$ free variables (parameters).
(c) The system has infinitely many solutions.
(d) None of the above.
(e) More than one of the above.