Solution Sets of Linear Systems

1. Which of the following are solutions to the system of equations?

\[
\begin{align*}
2x + y + 2z &= 0 \\
-x + 2y - 6z &= 0
\end{align*}
\]

(a) \[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
-2 \\
2 \\
1
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
2 \\
-2 \\
-1
\end{bmatrix}
\]

(c) \[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
-6 \\
6 \\
3
\end{bmatrix}
\]

(d) \[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
4 \\
-4 \\
-2
\end{bmatrix}
\]

(e) None of the above.

(f) More than one of the above.

2. What is the solution to the following system of equations?

\[
\begin{align*}
x + 2y + z &= 0 \\
x + 3y - 2z &= 0
\end{align*}
\]

(a) \[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
-7 \\
3 \\
1
\end{bmatrix} s
\]

(b) \[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
7 \\
-3 \\
1
\end{bmatrix} s
\]

(c) \[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
7 \\
-3 \\
0
\end{bmatrix}
\]

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3. What is the solution to the following system of equations?

\[ \begin{align*}
  x + 2y + z &= 3 \\
  x + 3y - 2z &= 4
\end{align*} \]

(a) \[ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} s \]

(b) \[ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s \]

(c) \[ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s \]

(d) None of the above.

(e) More than one of the above.

4. What is the solution to the following system of equations?

\[ \begin{align*}
  x + 2y + z &= -2 \\
  x + 3y - 2z &= 1
\end{align*} \]

(a) \[ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ -1 \end{bmatrix} s \]

(b) \[ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -3 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s \]

(c) \[ \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 7 \\ -3 \\ -1 \end{bmatrix} s \]

(d) None of the above.

(e) More than one of the above.
5. The set of solutions to a system of linear equations is plotted below. Which of the following expressions represents this solution set?

![Graph showing linear equation]

(a) \[
\begin{pmatrix} x \\ y \\ \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ \end{pmatrix} + \begin{pmatrix} 1 \\ -1/2 \\ \end{pmatrix} s
\]
(b) \[
\begin{pmatrix} x \\ y \\ \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ \end{pmatrix} + \begin{pmatrix} -1/2 \\ 2 \\ \end{pmatrix} s
\]
(c) \[
\begin{pmatrix} x \\ y \\ \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \end{pmatrix} + \begin{pmatrix} -1/2 \\ 2 \\ \end{pmatrix} s
\]
(d) \[
\begin{pmatrix} x \\ y \\ \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ \end{pmatrix} s
\]
(e) None of the above.
(f) More than one of the above.

6. The set of solutions to a linear system are represented by the expression below. How can we geometrically represent this solution set?

\[
\begin{pmatrix} x \\ y \\ z \\ \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \\ 0 \\ \end{pmatrix} + \begin{pmatrix} -7 \\ 1 \\ 0 \\ \end{pmatrix} s + \begin{pmatrix} 2 \\ 0 \\ 1 \\ \end{pmatrix} t
\]

(a) As a line in \( \mathbb{R}^2 \)
(b) As a line in \( \mathbb{R}^3 \)
(c) As a plane in \( \mathbb{R}^3 \)
(d) As a volume in \( \mathbb{R}^3 \)
(e) None of the above

7. Let \( R = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \). If \( R \) is the reduced row echelon form of the augmented matrix for the system \( Ax = b \), what are the solutions to that system?
(a) $x_1 = 1, x_2 = 1$, and $x_3 = 2$
(b) $x_1 = 1, x_2 = 1, x_3 = 2$, and $x_4 = 0$
(c) $x_1 = -t, x_2 = -t, x_3 = -2t$, and $x_4 = t$
(d) There are no solutions to this system.

8. Let $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$. If $R$ is the reduced row echelon form of the coefficient matrix for the system $Ax = 0$, what are the solutions to that system?

(a) $x_1 = 1, x_2 = 1$, and $x_3 = 2$
(b) $x_1 = 1, x_2 = 1, x_3 = 2$, and $x_4 = 0$
(c) $x_1 = -t, x_2 = -t, x_3 = -2t$, and $x_4 = t$
(d) There are no solutions to this system.

9. Let matrix $R$ be the reduced row echelon form of matrix $A$. True or False The solutions to $Rx = 0$ are the same as the solutions to $Ax = 0$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

10. Let matrix $R$ be the reduced row echelon form of matrix $A$. True or False The solutions to $Rx = b$ are the same as the solutions to $Ax = b$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

11. Consider a homogeneous linear system with $n$ unknowns. Suppose the reduced row echelon form of its augmented matrix has $r \leq n$ nonzero rows. We can conclude that:

(a) $x_1 = 0, x_2 = 0, \ldots, x_n = 0$ is a solution to the system.
(b) The system has $n - r$ free variables (parameters).
(c) The system has infinitely many solutions.
(d) None of the above.
(e) More than one of the above.