Linear Combinations

1. If $u = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ and $v = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$, what is $2u - 3v$?

   (a) $\begin{bmatrix} -4 \\ 4 \\ 23 \end{bmatrix}$

   (b) $\begin{bmatrix} 8 \\ 4 \\ -7 \end{bmatrix}$

   (c) $\begin{bmatrix} 8 \\ 4 \\ 23 \end{bmatrix}$

   (d) $\begin{bmatrix} 7 \\ 6 \\ 2 \end{bmatrix}$

2. Write $z = \begin{bmatrix} -5 \\ 3 \\ 16 \end{bmatrix}$ as a linear combination of $x = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$ and $y = \begin{bmatrix} -3 \\ 2 \\ 6 \end{bmatrix}$.
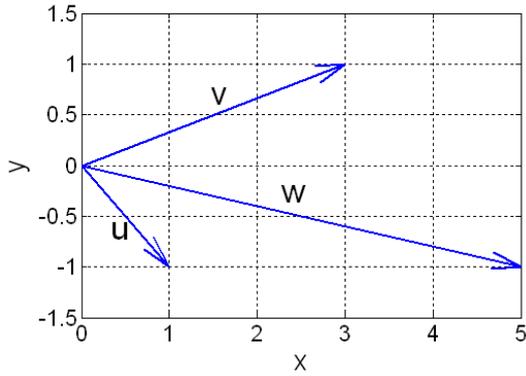
   (a) $z = -5x$

   (b) $z = -2x + y$

   (c) $z = x + 2y$

   (d) $z = 2x + y$

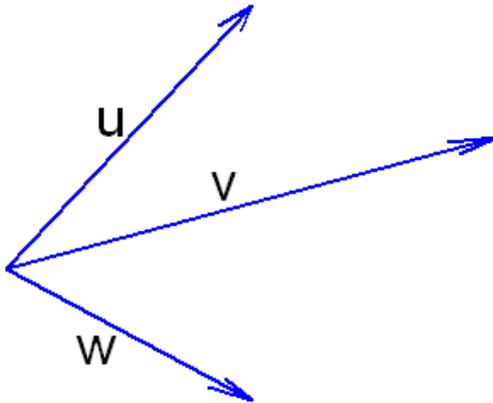   (e) z cannot be written as a linear combination of x and y.

   (f) None of the above

3. Write the vector w as a linear combination of u and v.

(a) $w = 2u + v$

(b) $w = u + v$

(c) $w = -u + v$

(d) $w = u - v$

(e) w cannot be written as a linear combination of u and v.

4. Write the vector w as a linear combination of u and v.

(a) $w = 2u + v$

(b) $w = u + v$

(c) $w = -u + v$

(d) $w = u - v$

(e) w cannot be written as a linear combination of u and v.

5. Write $z = \begin{bmatrix} 2 \\ -2 \\ 6 \end{bmatrix}$ as a linear combination of $x = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$ and $y = \begin{bmatrix} 4 \\ -5 \\ 7 \end{bmatrix}$.

(a) $z = x + y$

(b) $z = -x + y$

(c) $z = 3x + 2y$

(d) $z = -3x + y$

(e) z cannot be written as a linear combination of x and y.

(f) None of the above

6. Suppose we have the vectors $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$. Which of the following is *not* a linear combination of these?

(a) $\begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix}$

(b) $\begin{bmatrix} 8 \\ 0 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 8 \\ 1 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$

(e) $\begin{bmatrix} 40 \\ 5 \\ 15 \end{bmatrix}$

(f) More than one of the above is not a linear combination of the given vectors.

7. Suppose we have the vectors $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$. Which of the following is true?

(a) Every vector in $\Re^3$ can be written as a linear combination of these vectors.

(b) Some, but not all, vectors in $\Re^3$ can be written as a linear combination of these vectors.

(c) Every vector in $\Re^2$ can be written as a linear combination of these vectors.

(d) More than one of the above is true.

(e) None of the above are true.

8. Which of the following vectors can be written as a linear combination of the vectors (1, 0) and (0, 1)?

   (a) $(2, 0)$
   (b) $(-3, 1)$
   (c) $(0.4, 3.7)$
   (d) All of the above

9. How do you describe the set of all linear combinations of the vectors (1, 0) and (0, 1)?

   (a) A point
   (b) A line segment
   (c) A line
   (d) $\Re^2$
   (e) $\Re^3$

10. Which of the following vectors can be written as a linear combination of the vectors (1, 0, 0), (0, 1, 0), and (0, 0, 1)?

   (a) $(0, 2, 0)$
   (b) $(-3, 0, 1)$
   (c) $(0.4, 3.7, -1.5)$
   (d) All of the above

11. How do you describe the set of all linear combinations of the vectors (1, 0, 0), (0, 1, 0), (0, 0, 1)?

   (a) A point
   (b) A line segment
   (c) A line
   (d) $\Re^2$
   (e) $\Re^3$

12. How do you describe the set of all linear combinations of the vectors (1, 2, 0) and (-1, 1, 0)?

   (a) A point

(b) A line

(c) A plane

(d) $\Re^2$

(e) $\Re^3$

13. Let z be any vector from $\Re^3$. If we have a set V of unknown vectors from $\Re^3$, how many vectors must be in V to guarantee that z can be written as a linear combination of the vectors in V?

(a) 2

(b) 3

(c) 4

(d) It is not possible to make such a guarantee.

14. Suppose y and z are both solutions to $Ax = b$. **True or False** All linear combinations of y and z also solve $Ax = b$. (You should be prepared to support your answer with either a proof or a counterexample.)

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

15. Suppose y and z are both solutions to $Ax = 0$. **True or False** All linear combinations of y and z also solve $Ax = 0$. (You should be prepared to support your answer with either a proof or a counterexample.)

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

16. Lucinda owns two ice cream parlors. The first ice cream shop sells 5 gallons of vanilla ice cream and 8 gallons of chocolate ice cream each day. The daily sales at the second store are 6 gallons of vanilla ice cream and 10 gallons of chocolate ice cream. The daily sales at stores one and two can be represented by the vectors $s_1 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ and $s_2 = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$, respectively. In this context, what interpretation can be given to the vector $15s_1$?

(a) $15s_1$ shows the number of people that can be served with 15 gallons of vanilla ice cream.

(b) $15s_1$ shows the gallons of vanilla and chocolate ice cream sold by store 1 in 15 days.

(c) $15s_1$ gives the total revenue from selling 15 gallons of ice cream at store 1.

(d) $15s_1$ represents the number of days it will take to sell 15 gallons of ice cream at store 1.

17. Lucinda owns two ice cream parlors. The first ice cream shop sells 5 gallons of vanilla ice cream and 8 gallons of chocolate ice cream each day. The daily sales at the second store are 6 gallons of vanilla ice cream and 10 gallons of chocolate ice cream. The daily sales at stores one and two can be represented by the vectors $s_1 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ and $s_2 = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$, respectively. The stores are run by different managers, and they are not always able to be open the same number of days in a month. If store 1 is open for c_1 days in March, and store 2 is open for c_2 days in March, which of the following represents the total sales of each flavor of ice cream between the two stores?

(a) $c_1 s_1 + c_2 s_2$

(b) $\begin{bmatrix} 5 & 6 \\ 8 & 10 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

(c) $\begin{bmatrix} 5c_1 \\ 8c_1 \end{bmatrix} + \begin{bmatrix} 6c_2 \\ 10c_2 \end{bmatrix}$

(d) All of the above

(e) None of the above

18. Lucinda owns two ice cream parlors. The first ice cream shop sells 5 gallons of vanilla ice cream and 8 gallons of chocolate ice cream each day. The daily sales at the second store are 6 gallons of vanilla ice cream and 10 gallons of chocolate ice cream. The daily sales at stores one and two can be represented by the vectors $s_1 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ and $s_2 = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$, respectively. Lucinda is getting ready to close her ice cream parlors for the winter. She has a total of 39 gallons of vanilla ice cream in her warehouse, and 64 gallons of chocolate ice cream. She would like to distribute the ice cream to the two stores so that it is used up before the stores close for the winter. How much ice cream should she take to each store? The stores may stay open for different number of days, but no store may run out of ice cream before the end of the day on which it closes.

(a) Lucinda should take 3 gallons of each kind of ice cream to store 1 and 4 gallons of each kind to store 2.

(b) Lucinda should take 3 gallons of vanilla to each store and 4 gallons of chocolate to each store.

(c) Lucinda should take 15 gallons of vanilla and 24 gallons of chocolate to store 1, and she should take 24 gallons of vanilla and 40 gallons of chocolate to store 2.

(d) Lucinda should take 15 gallons of vanilla and 32 gallons of chocolate to store 1, and she should take 18 gallons of vanilla and 40 gallons of chocolate to store 2.

(e) This cannot be done unless ice cream is thrown out or a store runs out of ice cream before the end of the day.