Linear Independence

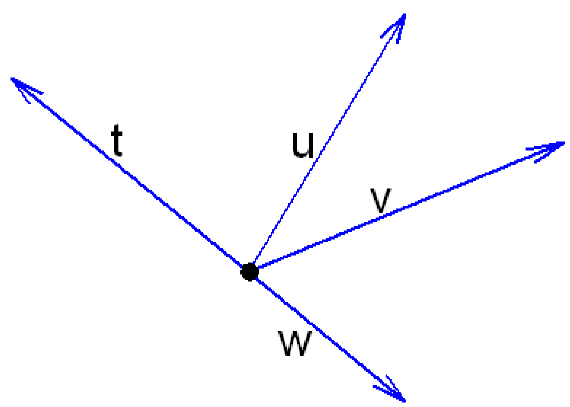
1. **True or False** The following vectors are linearly independent: (1,0,0), (0,0,2), (3,0,4)

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

2. Which set of vectors is linearly independent?

   (a) $(2,3),(8,12)$
   (b) $(1,2,3),(4,5,6),(7,8,9)$
   (c) $(-3,1,0),(4,5,2),(1,6,2)$
   (d) None of these sets are linearly independent.
   (e) Exactly two of these sets are linearly independent.
   (f) All of these sets are linearly independent.

3. Which subsets of the set of the vectors shown below are linearly dependent?



   (a) u, w
   (b) t, w
   (c) t, v
   (d) t, u, v

(e) None of these sets are linearly dependent.

(f) More than one of these sets is linearly dependent.

4. Suppose you wish to determine whether a set of vectors is linearly independent. You form a matrix with those vectors as the columns, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. What do you decide?

    (a) These vectors are linearly independent.

    (b) These vectors are not linearly independent.

5. Suppose you wish to determine whether a set of vectors $\{v_1, v_2, v_3, v_4\}$ is linearly independent. You form the matrix $A = [v_1 v_2 v_3 v_4]$, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. You now decide to write v_4 as a linear combination of v_1, v_2, and v_3. Which is a correct linear combination?

    (a) $v_4 = v_1 + v_2$

    (b) $v_4 = -v_1 - 2v_3$

    (c) v_4 cannot be written as a linear combination of v_1, v_2, and v_3.

    (d) We cannot determine the linear combination from this information.

6. Suppose you wish to determine whether a set of vectors $\{v_1, v_2, v_3, v_4\}$ is linearly independent. You form the matrix $A = [v_1 v_2 v_3 v_4]$, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. You now decide to write v_3 as a linear combination of v_1, v_2, and v_4. Which is a correct linear combination?

    (a) $v_3 = (1/2)v_1 - (1/2)v_4$

    (b) $v_3 = (1/2)v_1 + (1/3)v_2$

    (c) $v_3 = 2v_1 + 3v_2$

    (d) $v_3 = -2v_1 - 3v_2$

    (e) v_3 cannot be written as a linear combination of v_1, v_2, and v_4.

    (f) We cannot determine the linear combination from this information.

7. Suppose you wish to determine whether a set of vectors $\{v_1, v_2, v_3, v_4\}$ is linearly independent. You form the matrix $A = [v_1 v_2 v_3 v_4]$, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. You now decide to write v_2 as a linear combination of v_1, v_3, and v_4. Which is a correct linear combination?

   (a) $v_2 = 3v_3 + v_4$
   (b) $v_2 = -3v_3 - v_4$
   (c) $v_2 = v_4 - 3v_3$
   (d) $v_2 = -v_1 + v_4$
   (e) v_2 cannot be written as a linear combination of v_1, v_3, and v_4.
   (f) We cannot determine the linear combination from this information.

8. Are the vectors $\left\{ \begin{bmatrix} 1 \\ 4 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -14 \\ 13 \\ 7 \\ -19 \end{bmatrix} \right\}$ linearly independent?

   (a) Yes, they are linearly independent.
   (b) No, they are not linearly independent.

9. To determine whether a set of n vectors from $\Re^n$ is independent, we can form a matrix A whose columns are the vectors in the set and then put that matrix in reduced row echelon form. If the vectors are linearly independent, what will we see in the reduced row echelon form?

   (a) A row of all zeros.
   (b) A row that has all zeros except in the last position.
   (c) A column of all zeros.
   (d) An identity matrix.

10. To determine whether a set of fewer than n vectors from $\Re^n$ is independent, we can form a matrix A whose columns are the vectors in the set and then put that matrix in reduced row echelon form. If the vectors are linearly independent, what will we see in the reduced row echelon form?

   (a) An identity submatrix with zeros below it.
   (b) A row that has all zeros except in the last position.

(c) A column that is not an identity matrix column.
(d) A column of all zeros.

11. If the columns of A are not linearly independent, how many solutions are there to the system $Ax = 0$?
    (a) 0
    (b) 1
    (c) infinite
    (d) Not enough information is given.

12. **True or False** A set of 4 vectors from $\Re^3$ could be linearly independent.
    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

13. **True or False** A set of 2 vectors from $\Re^3$ must be linearly independent.
    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

14. **True or False** A set of 3 vectors from $\Re^3$ could be linearly independent.
    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

15. **True or False** A set of 5 vectors from $\Re^4$ could be linearly independent.
    (a) True, and I am very confident
    (b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

16. Which statement is equivalent to saying that v_1, v_2, and v_3 are linearly independent vectors?

    (a) The only solution to $c_1v_1 + c_2v_2 + c_3v_3 = 0$ is $c_1 = c_2 = c_3 = 0$.

    (b) v_3 cannot be written as a linear combination of v_1 and v_2.

    (c) No vector is a multiple of any other.

    (d) Exactly two of (a), (b), and (c) are true.

    (e) All three statements are true.