

## Dimension and Rank

1. Let  $A = \begin{bmatrix} 5 & 4 & -8 & 1 \\ 1 & 3 & 4 & 8 \\ 0 & 2 & 1 & 3 \\ -1 & -2 & 4 & 1 \end{bmatrix}$ . The reduced row echelon form of  $A$  is  $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ .

What is the rank of  $A$ ?

- (a) 0
  - (b) 1
  - (c) 2
  - (d) 3
  - (e) 4
2. Suppose a  $4 \times 4$  matrix  $A$  has rank 3. Are the columns of  $A$  linearly independent?
- (a) Yes, they are linearly independent.
  - (b) No, they are not linearly independent.
  - (c) We do not have enough information to decide.
3. Suppose a  $4 \times 4$  matrix  $A$  has rank 4. How many solutions does the system  $Ax = b$  have?
- (a) 0
  - (b) 1
  - (c) Infinite
  - (d) Not enough information is given.
4. Suppose a  $4 \times 4$  matrix  $A$  has rank 3. How many solutions does the system  $Ax = b$  have?
- (a) 0
  - (b) 1
  - (c) Infinite
  - (d) Not enough information is given.

5. Suppose a  $4 \times 4$  matrix  $A$  has rank 3. If it is known that  $(4, 5, 0, 1)$  is a solution to the system  $Ax = b$ , then how many solutions does  $Ax = b$  have?
- (a) 1
  - (b) Infinite
  - (c) Not enough information is given.
6. Suppose a  $5 \times 5$  matrix  $A$  has rank 3. If it is known that  $(-1, 4, 2, 0, 3)$  is a solution to the system  $Ax = b$ , then how many parameters does the solution set have?
- (a) 0
  - (b) 1
  - (c) 2
  - (d) 3
  - (e) 4
  - (f) Not enough information is given.
7. **True or False** If  $AX = BX$  for all matrices  $X$  where the products are defined, then  $A$  and  $B$  have to be the same matrix.
8. **True or False** If  $Ax = Bx$  for all vectors  $x$  where the products are defined, then  $A$  and  $B$  have to be the same matrix.