

Dimension and Rank

1. Let $A = \begin{bmatrix} 5 & 4 & -8 & 1 \\ 1 & 3 & 4 & 8 \\ 0 & 2 & 1 & 3 \\ -1 & -2 & 4 & 1 \end{bmatrix}$. The reduced row echelon form of A is $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

What is the rank of A ?

- (a) 0
 - (b) 1
 - (c) 2
 - (d) 3
 - (e) 4
2. Suppose a 4×4 matrix A has rank 3. Are the columns of A linearly independent?
- (a) Yes, they are linearly independent.
 - (b) No, they are not linearly independent.
 - (c) We do not have enough information to decide.
3. Suppose a 4×4 matrix A has rank 4. How many solutions does the system $Ax = b$ have?
- (a) 0
 - (b) 1
 - (c) Infinite
 - (d) Not enough information is given.
4. Suppose a 4×4 matrix A has rank 3. How many solutions does the system $Ax = b$ have?
- (a) 0
 - (b) 1
 - (c) Infinite
 - (d) Not enough information is given.

5. Suppose a 4×4 matrix A has rank 3. If it is known that $(4, 5, 0, 1)$ is a solution to the system $Ax = b$, then how many solutions does $Ax = b$ have?
- (a) 1
 - (b) Infinite
 - (c) Not enough information is given.
6. Suppose a 5×5 matrix A has rank 3. If it is known that $(-1, 4, 2, 0, 3)$ is a solution to the system $Ax = b$, then how many parameters does the solution set have?
- (a) 0
 - (b) 1
 - (c) 2
 - (d) 3
 - (e) 4
 - (f) Not enough information is given.
7. **True or False** If $AX = BX$ for all matrices X where the products are defined, then A and B have to be the same matrix.
- (a) True, and I am very confident
 - (b) True, but I am not very confident
 - (c) False, but I am not very confident
 - (d) False, and I am very confident
8. **True or False** If $Ax = Bx$ for all vectors x where the products are defined, then A and B have to be the same matrix.
- (a) True, and I am very confident
 - (b) True, but I am not very confident
 - (c) False, but I am not very confident
 - (d) False, and I am very confident