Matrix Inverses

1. Which of the following matrices does not have an inverse?
   (a) \[
   \begin{bmatrix}
   1 & 2 \\
   3 & 4 
   \end{bmatrix}
   \]
   (b) \[
   \begin{bmatrix}
   2 & 2 \\
   4 & 4 
   \end{bmatrix}
   \]
   (c) \[
   \begin{bmatrix}
   -1 & 0 \\
   0 & 3 
   \end{bmatrix}
   \]
   (d) \[
   \begin{bmatrix}
   0 & 4 \\
   2 & 0 
   \end{bmatrix}
   \]
   (e) More than one of the above do not have inverses.
   (f) All have inverses.

2. When we put a matrix \( A \) into reduced row echelon form, we get the matrix \[
   \begin{bmatrix}
   1 & 2 \\
   0 & 0 
   \end{bmatrix}
   \]
   This means that
   (a) Matrix \( A \) has no inverse.
   (b) The matrix we have found is the inverse of matrix \( A \).
   (c) Matrix \( A \) has an inverse, but this isn’t it.
   (d) This tells us nothing about whether \( A \) has an inverse.

3. Let \( A = \begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix} \). What is \( A^{-1} \)?
   (a) \[
   \begin{bmatrix}
   0 & 4 \\
   2 & 0 
   \end{bmatrix}
   \]
   (b) \[
   \begin{bmatrix}
   4 & 0 \\
   0 & 2 
   \end{bmatrix}
   \]
   (c) \[
   \begin{bmatrix}
   0 & 1/4 \\
   1/2 & 0 
   \end{bmatrix}
   \]
   (d) \[
   \begin{bmatrix}
   0 & 1/2 \\
   1/4 & 0 
   \end{bmatrix}
   \]
4. We find that for a square coefficient matrix \( A \), the homogeneous matrix equation
\[
AX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},
\]
has only the trivial solution \( X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \). This means that

(a) Matrix \( A \) has no inverse.
(b) Matrix \( A \) has an inverse.
(c) This tells us nothing about whether \( A \) has an inverse.

5. **True or False** If \( A \), \( B \), and \( C \) are square matrices and we know that \( AB = AC \), this means that matrix \( B \) is equal to matrix \( C \).

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

6. **True or False** Suppose that \( A \), \( B \), and \( C \) are square matrices, and \( CA = B \), and \( A \) is invertible. This means that \( C = A^{-1}B \).

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

7. We know that \((5A)^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\). What is matrix \( A \)?

(a) \( \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \)
(b) \( \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \)
(c) \( \begin{bmatrix} 1/5 & 0 \\ 0 & 1/5 \end{bmatrix} \)
(d) \( \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix} \)
(e) There is no matrix \( A \) which solves this equation.
8. A and B are invertible matrices. If $AB = C$, then what is the inverse of $C$?

(a) $C^{-1} = A^{-1}B^{-1}$
(b) $C^{-1} = B^{-1}A^{-1}$
(c) $C^{-1} = AB^{-1}$
(d) $C^{-1} = BA^{-1}$
(e) More than one of the above is true.
(f) Just because $A$ and $B$ have inverses, this doesn’t mean that $C$ has an inverse.

9. Let $A$ be a $2 \times 2$ matrix. The inverse of $3A$ is

(a) $\frac{1}{3}A^{-1}$
(b) $\frac{1}{3}A^{-1}$
(c) $A^{-1}$
(d) $3A^{-1}$
(e) Not enough information is given.

10. If $A$ is an invertible matrix, what else must be true?

(a) If $AB = C$ then $B = A^{-1}C$.
(b) $A^2$ is invertible.
(c) $A^T$ is invertible.
(d) $5A$ is invertible.
(e) The reduced row echelon form of $A$ is $I$.
(f) All of the above must be true.