MathQuest: Linear Algebra

Matrix Inverses

1. Which of the following matrices does not have an inverse?

   (a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

   (b) $\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}$

   (c) $\begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$

   (d) $\begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$

   (e) More than one of the above do not have inverses.

   (f) All have inverses.

2. When we put a matrix A into reduced row echelon form, we get the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$. This means that

   (a) Matrix A has no inverse.

   (b) The matrix we have found is the inverse of matrix A.

   (c) Matrix A has an inverse, but this isn't it.

   (d) This tells us nothing about whether A has an inverse.

3. Let $A = \begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$. What is A^{-1}?

   (a) $\begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$.

   (b) $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$.

   (c) $\begin{bmatrix} 0 & 1/4 \\ 1/2 & 0 \end{bmatrix}$.

   (d) $\begin{bmatrix} 0 & 1/2 \\ 1/4 & 0 \end{bmatrix}$.

4. We find that for a square coefficient matrix A, the homogeneous matrix equation $AX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, has only the trivial solution $X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. This means that

   (a) Matrix A has no inverse.

   (b) Matrix A has an inverse.

   (c) This tells us nothing about whether A has an inverse.

5. **True or False** If A, B, and C are square matrices and we know that $AB = AC$, this means that matrix B is equal to matrix C.

   (a) True, and I am very confident

   (b) True, but I am not very confident

   (c) False, but I am not very confident

   (d) False, and I am very confident

6. **True or False** Suppose that A, B, and C are square matrices, and $CA = B$, and A is invertible. This means that $C = A^{-1}B$.

   (a) True, and I am very confident

   (b) True, but I am not very confident

   (c) False, but I am not very confident

   (d) False, and I am very confident

7. We know that $(5A)^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. What is matrix A?

   (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

   (c) $\begin{bmatrix} 1/5 & 0 \\ 0 & 1/5 \end{bmatrix}$

   (d) $\begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$

   (e) There is no matrix A which solves this equation.

8. A and B are invertible matrices. If $AB = C$, then what is the inverse of C?

   (a) $C^{-1} = A^{-1}B^{-1}$
   (b) $C^{-1} = B^{-1}A^{-1}$
   (c) $C^{-1} = AB^{-1}$
   (d) $C^{-1} = BA^{-1}$
   (e) More than one of the above is true.
   (f) Just because A and B have inverses, this doesn't mean that C has an inverse.

9. Let A be a 2×2 matrix. The inverse of $3A$ is

   (a) $\frac{1}{9}A^{-1}$
   (b) $\frac{1}{3}A^{-1}$
   (c) A^{-1}
   (d) $3A^{-1}$
   (e) Not enough information is given.

10. If A is an invertible matrix, what else must be true?

    (a) If $AB = C$ then $B = A^{-1}C$.
    (b) A^2 is invertible.
    (c) A^T is invertible.
    (d) $5A$ is invertible.
    (e) The reduced row echelon form of A is I.
    (f) All of the above must be true.