Eigenvalues and Eigenvectors

1. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   (a) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 3 \\ 3 \end{bmatrix}$

   (c) $\begin{bmatrix} 3 & 3 \end{bmatrix}$

   (d) $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$

   (e) None of the above

2. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^2 \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   (a) $\begin{bmatrix} 3 \\ 3 \end{bmatrix}$

   (b) $\begin{bmatrix} 6 \\ 6 \end{bmatrix}$

   (c) $\begin{bmatrix} 9 \\ 9 \end{bmatrix}$

   (d) $\begin{bmatrix} 12 \\ 12 \end{bmatrix}$

   (e) None of the above

   (f) This matrix multiplication is impossible.

3. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^4 \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   (a) $\begin{bmatrix} 27 \\ 27 \end{bmatrix}$

   (b) $\begin{bmatrix} 81 \\ 81 \end{bmatrix}$

   (c) $\begin{bmatrix} 243 \\ 243 \end{bmatrix}$

   (d) $\begin{bmatrix} 729 \\ 729 \end{bmatrix}$

(e) None of the above

4. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   (a) $\begin{bmatrix} 3n \\ 3n \end{bmatrix}$

   (b) $3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   (c) $n^3 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   (d) $3^n \begin{bmatrix} n \\ n \end{bmatrix}$

   (e) $\begin{bmatrix} 3 \\ 3 \end{bmatrix}^n$

   (f) More than one of the above

5. Suppose A is an $n \times n$ matrix, c is a scalar, and x is an $n \times 1$ vector. If $Ax = cx$, what is A^2x?

   (a) $2cx$

   (b) c^2x

   (c) cx

   (d) None of the above

6. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

   (a) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 3 \\ -3 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

   (d) $\begin{bmatrix} -3 \\ 3 \end{bmatrix}$

   (e) None of the above

7. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

   (a) $(-1)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(b) $(-1)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(c) $(-3)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) $\begin{bmatrix} (-1)^n \\ (-1)^{n+1} \end{bmatrix}$

(e) None of the above

(f) More than one of the above

8. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

(a) $3^n \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

(b) $2^n \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

(c) $6^n \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(d) $3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

9. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} -5 \\ 5 \end{bmatrix}$

(a) $3^n \begin{bmatrix} -5 \\ 5 \end{bmatrix}$

(b) $(-1)^n \begin{bmatrix} -5 \\ 5 \end{bmatrix}$

(c) $(-5)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) $5 \begin{bmatrix} (-1)^n \\ (-1)^n \end{bmatrix}$

(e) None of the above

(f) More than one of the above

10. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(a) $\begin{bmatrix} 3 \\ 15 \end{bmatrix}$

(b) $\begin{bmatrix} -1 \\ -5 \end{bmatrix}$

(c) $\begin{bmatrix} 11 \\ 7 \end{bmatrix}$

(d) $\begin{bmatrix} 7 \\ 11 \end{bmatrix}$

(e) None of the above

11. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(a) $11^n \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(b) $7^n \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(c) $\begin{bmatrix} 11^n \\ 7^n \end{bmatrix}$

(d) $\begin{bmatrix} 25 \\ 29 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

12. Write the vector $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 5 \end{bmatrix} = \frac{1}{2}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$

(c) $\begin{bmatrix} 1 \\ 5 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 1 \end{bmatrix} - 2\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) None of the above

(e) More than one of the above

13. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(a) $-1 \times 3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 3 \times (-2)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(b) $3 \times (-1)^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + (-2) \times 3^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) $3 \times 3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + (-2) \times (-1)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) $3 \times 3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + (-1) \times (-2)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

14. Which of the following is an eigenvector of the matrix $\begin{bmatrix} 2 & -1 \\ -4 & -1 \end{bmatrix}$?

(a) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ -4 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

15. Which of the following is an eigenvector of the matrix $\begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$?

(a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} -3 \\ -3 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ -2/3 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

16. Suppose the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has an eigenvalue 1 with associated eigenvector $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. What is $A^{50}x$?

(a) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(b) $\begin{bmatrix} a^{50} & b^{50} \\ c^{50} & d^{50} \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} 2^{50} \\ 3^{50} \end{bmatrix}$

(e) Way too hard to compute.

17. Vector x is an eigenvector of matrix A. If $x = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $Ax = \begin{bmatrix} 4 \\ 12 \end{bmatrix}$, then what is the associated eigenvalue?

(a) 1

(b) 3

(c) 4

(d) Not enough information is given.

18. Which of the following is an eigenvector of $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$? (You should be able to answer this by checking the vectors given, rather than by finding the eigenvectors of A from scratch.)

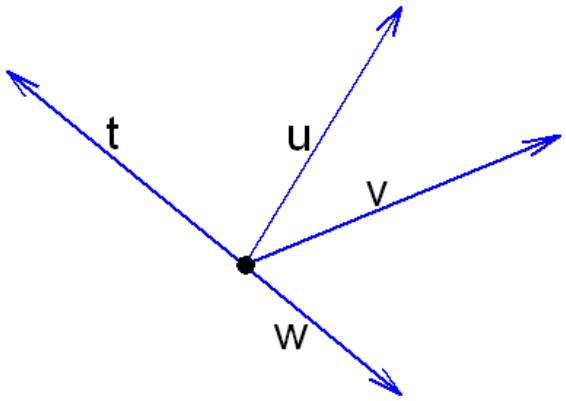
(a) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) None of the above

19. The vector t is an eigenvector of the matrix A. What could be the result of the product At?



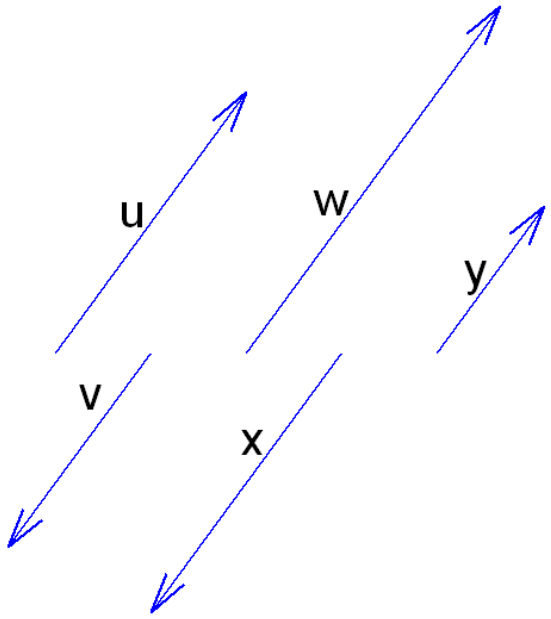
(a) $At = u$

(b) $At = v$

(c) $At = w$

(d) None of the above

20. The vector u is an eigenvector of the matrix A and $Au = v$, where the vectors u and v are shown below. What could be the result of the product Av?



(a) $Av = u$

(b) $Av = v$

(c) $Av = w$

(d) $Av = x$

(e) $Av = y$

21. $\begin{bmatrix} 4/3 \\ 1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$. What is the associated eigenvalue? (Think! Don't solve for all the eigenvalues and eigenvectors.)

(a) 4/3

(b) 5

(c) -2

22. The matrix $A = \begin{bmatrix} -1 & 4 \\ 3 & 0 \end{bmatrix}$ has an eigenvalue 3 with associated eigenvector $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Let $y = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. Which of the following statements is true?

(a) $Ax = 3x$

(b) $Ay = 3y$

(c) For any scalars c and d, $A(cx + dy) = 3(cx + dy)$

(d) All of the above are true.

(e) Only (a) and (b) are true.

23. The matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ has an eigenvalue 2 with associated eigenvectors $x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. Which of the following statements is true?

(a) $Ax = 2x$

(b) $Ay = 2y$

(c) For any scalars c and d, $A(cx + dy) = 2(cx + dy)$.

(d) For any nonzero scalars c and d, $cx + dy$ is an eigenvector of A corresponding to the eigenvalue 2.

(e) All of the above are true.

(f) Only (a) and (b) are true.

24. **True or False** Any nonzero linear combination of two eigenvectors of a matrix A is an eigenvector of A.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

25. If w is an eigenvector of A, how does the vector Aw compare geometrically to the vector w?

(a) Aw is a rotation of w.

(b) Aw is a reflection of w in the x-axis.

(c) Aw is a reflection of w in the y-axis.

(d) Aw is parallel to w but may have a different length.

26. What does it mean if 0 is an eigenvalue of a matrix A?

(a) The determinant of A is zero.

(b) The columns of A are linearly dependent.

(c) There are an infinite number of solutions to the system $Ax = 0$.

(d) All of the above

(e) None of the above

27. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 6 \\ 0 & 4 & 2 \end{bmatrix}$ and note that all of the rows sum to six. Which of the following is true?

(a) $w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector of A.

(b) 6 is an eigenvalue of A.

(c) Both statements are true.

(d) Neither statement is true.