Vector Spaces and Subspaces

1. Which property of vector spaces is not true for the following set?

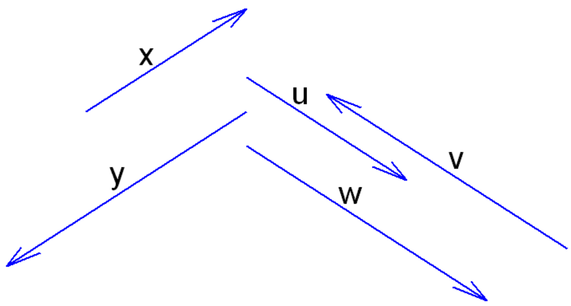
$$\left\{ \begin{bmatrix} -2 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$$

   (a) Closure under vector addition
   (b) Existence of an additive identity
   (c) Existence of an additive inverse for each vector
   (d) None of the above

2. A vector subspace does *not* have to satisfy which of the following properties?
   (a) Associativity under vector addition
   (b) Existence of an additive identity
   (c) Commutativity under vector addition
   (d) A vector subspace must satisfy all of the above properties.
   (e) A vector subspace need not satisfy any of the above properties.

3. A vector space does *not* have to satisfy which of the following properties?
   (a) Closure under vector addition
   (b) Closure under scalar multiplication
   (c) Closure under vector multiplication
   (d) A vector subspace must satisfy all of the above properities.
   (e) A vector subspace need not satisfy any of the above properties.

4. Which of the following sets of vectors are contained within a proper subspace of $\Re^2$?



i. x, y ii. u, v, w

iii. x, v iv. y, u, w

(a) i, ii, iii, and iv
(b) ii, iii, and iv only
(c) i and ii only
(d) ii and iv only
(e) iii and iv only
(f) ii only

5. The set of all 2×2 matrices with determinant equal to zero is not a vector subspace. Why?

(a) 2×2 matrices are not vectors.
(b) With matrices, AB need not equal BA.
(c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$ is not in the set.
(d) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is not in the set.
(e) None of the above

6. Which of the following sets of vectors is a basis for $\Re^3$?

(a) $\{(1,0,0),(0,1,0),(0,0,1)\}$
(b) $\{(1,0,1),(1,1,0),(1,1,1)\}$
(c) $\{(2,0,0),(0,5,0),(0,0,8)\}$
(d) All are bases for $\Re^3$.

7. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$. Which of the following sets has the same span as the set of all three vectors $\{v_1, v_2, v_3\}$?

(a) $\{v_1, v_2\}$
(b) $\{v_2, v_3\}$
(c) $\{v_1, v_3\}$
(d) None of the above

(e) More than one of the above

8. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$. Which of the following vectors is *not* in the subspace of $\Re^3$ spanned by $\{v_1, v_2, v_3\}$?

   (a) $(1, 0, 0)$
   (b) $(4, 1, 1)$
   (c) $(3, 3, 6)$
   (d) All of these are in the subspace of $\Re^3$ spanned by $\{v_1, v_2, v_3\}$.

9. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$. Geometrically, what is the subspace spanned by the set $\{v_1, v_2, v_3\}$?

   (a) a point
   (b) a line
   (c) a plane
   (d) a volume
   (e) all of $\mathcal{R}^3$

10. Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, w = \begin{bmatrix} k \\ 2 \\ -3 \end{bmatrix}$. For how many values of k will the vector w be in the subspace spanned by $\{v_1, v_2, v_3\}$?

    (a) No values of k - vector w will never be in this subspace
    (b) Exactly one value of k will work.
    (c) Any value of k will work.

11. Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, w = \begin{bmatrix} k \\ 8 \\ 11 \end{bmatrix}$. For how many values of k will the vector w be in the subspace spanned by $\{v_1, v_2, v_3\}$?

    (a) No values of k - vector w will never be in this subspace

(b) Exactly one value of k will work.

(c) Any value of k will work.

12. Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, w = \begin{bmatrix} 1 \\ k \\ k \end{bmatrix}$. For how many values of k will the vector w be in the subspace spanned by $\{v_1, v_2, v_3\}$?

    (a) No values of k - vector w will never be in this subspace

    (b) Exactly one value of k will work.

    (c) Any value of k will work.