Fundamental Vector Subspaces

1. How many linearly independent columns are there in the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

   (a) 2
   (b) 1
   (c) 0

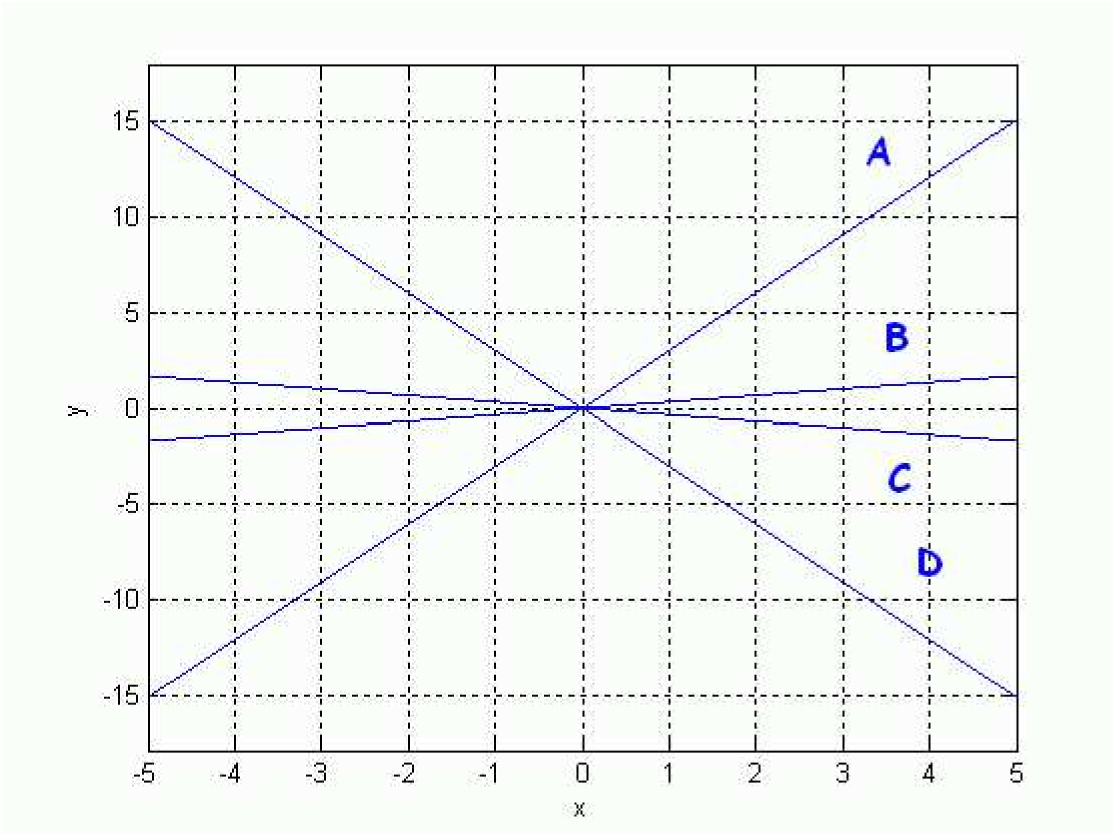
2. The *column space* of a matrix A is the set of vectors that can be created by taking all linear combinations of the columns of A. Is the vector $b = \begin{bmatrix} -4 \\ 12 \end{bmatrix}$ in the column space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

   (a) Yes, since we can find a vector x so that $Ax = b$.
   (b) Yes, since $-2\begin{bmatrix} 1 \\ 3 \end{bmatrix} - 1\begin{bmatrix} 2 \\ 6 \end{bmatrix} = \begin{bmatrix} -4 \\ 12 \end{bmatrix}$.
   (c) No, because there is no vector x so that $Ax = b$.
   (d) No, because there we can't find c_1 and c_2 such that $c_1\begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2\begin{bmatrix} 2 \\ 6 \end{bmatrix} = \begin{bmatrix} -4 \\ 12 \end{bmatrix}$.
   (e) More than one of the above
   (f) None of the above

3. The column space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ is

   (a) the set of all linear combinations of the columns of A.
   (b) a line in $\Re^2$.
   (c) the set of all multiples of the vector $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.
   (d) All of the above
   (e) None of the above

4. Which line in the graph below represents the column space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?



(a) line A
(b) line B
(c) line C
(d) line D
(e) None of the above

5. How many solutions x are there to $Ax = 0$ where $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(a) 0 solutions
(b) 1 solution
(c) 2 solutions
(d) Infinite number of solutions

6. The *null space* of a matrix A is the set of all vectors x that are solutions of $Ax = 0$. Which of the following vectors is in the null space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(a) $x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

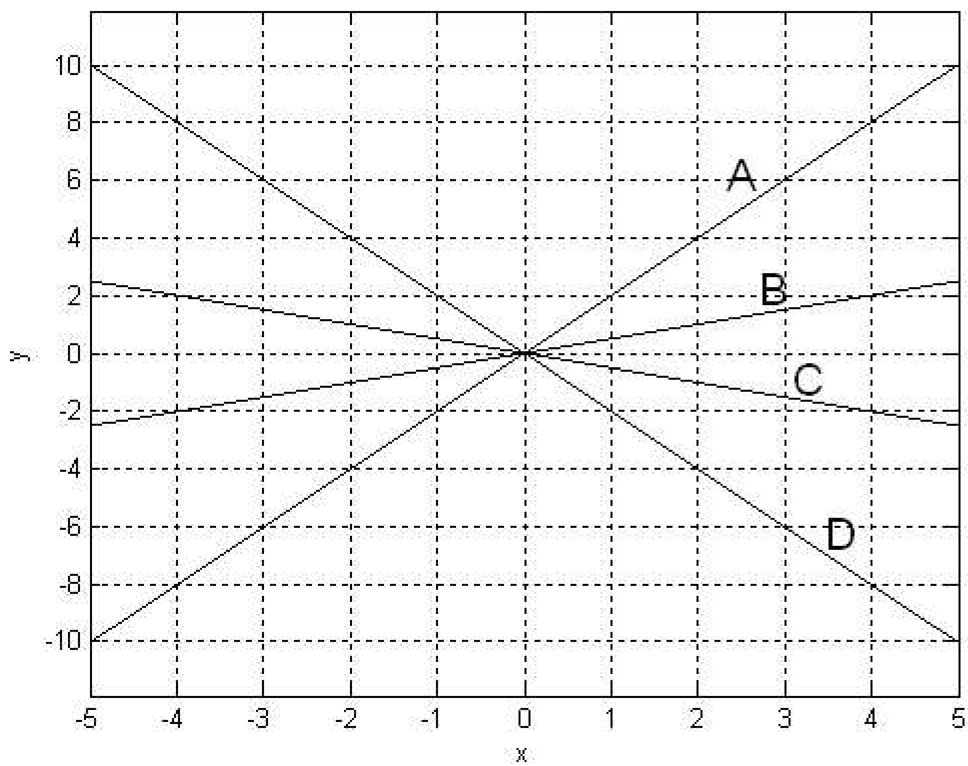
(b) $x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(c) $x = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

(d) All of the above

(e) None of the above

7. Which line in the graph below represents the null space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?



(a) line A

(b) line B

(c) line C

(d) line D

(e) None of the above

8. The *row space* of a matrix A is the set of vectors that can be created by taking all linear combinations of the rows of A. Which of the following vectors is in the row space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

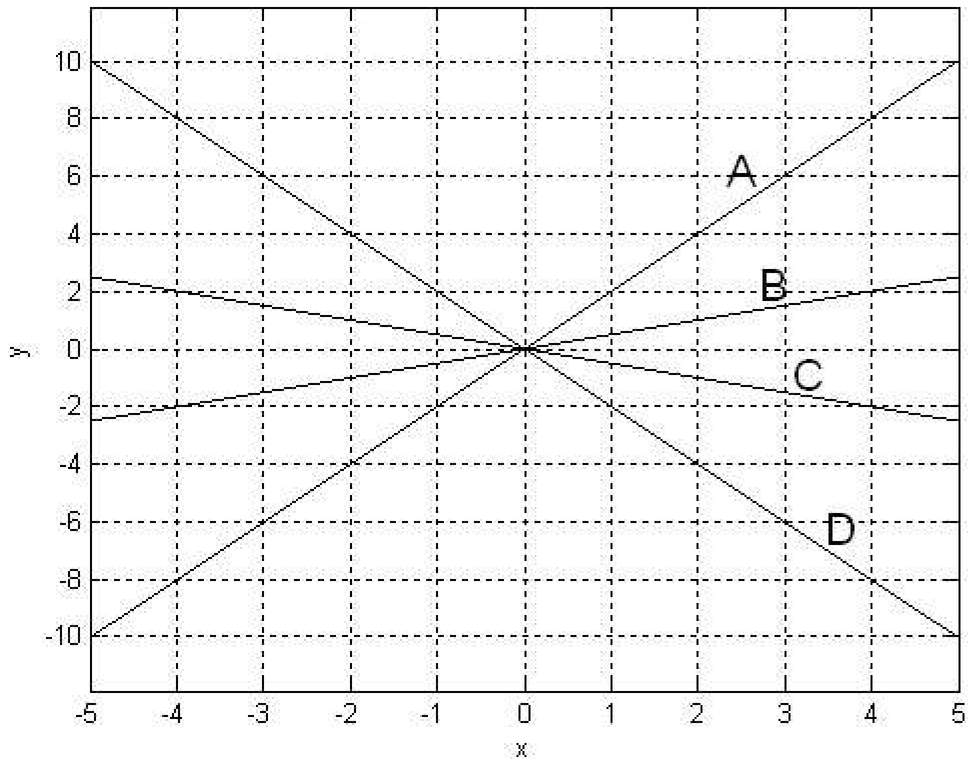
   (a) $x = \begin{bmatrix} -2 & 4 \end{bmatrix}$
   (b) $x = \begin{bmatrix} 4 & 8 \end{bmatrix}$
   (c) $x = \begin{bmatrix} 0 & 0 \end{bmatrix}$
   (d) $x = \begin{bmatrix} 8 & 4 \end{bmatrix}$
   (e) More than one of the above
   (f) None of the above

9. **True or False:** The row space of a matrix A is the same as the column space of A^T.

10. The row space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ consists of

   (a) All linear combinations of the columns of A^T.
   (b) All multiples of the vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.
   (c) All linear combinations of the rows of A.
   (d) All of the above
   (e) None of the above

11. Which line in the graph below represents the row space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?



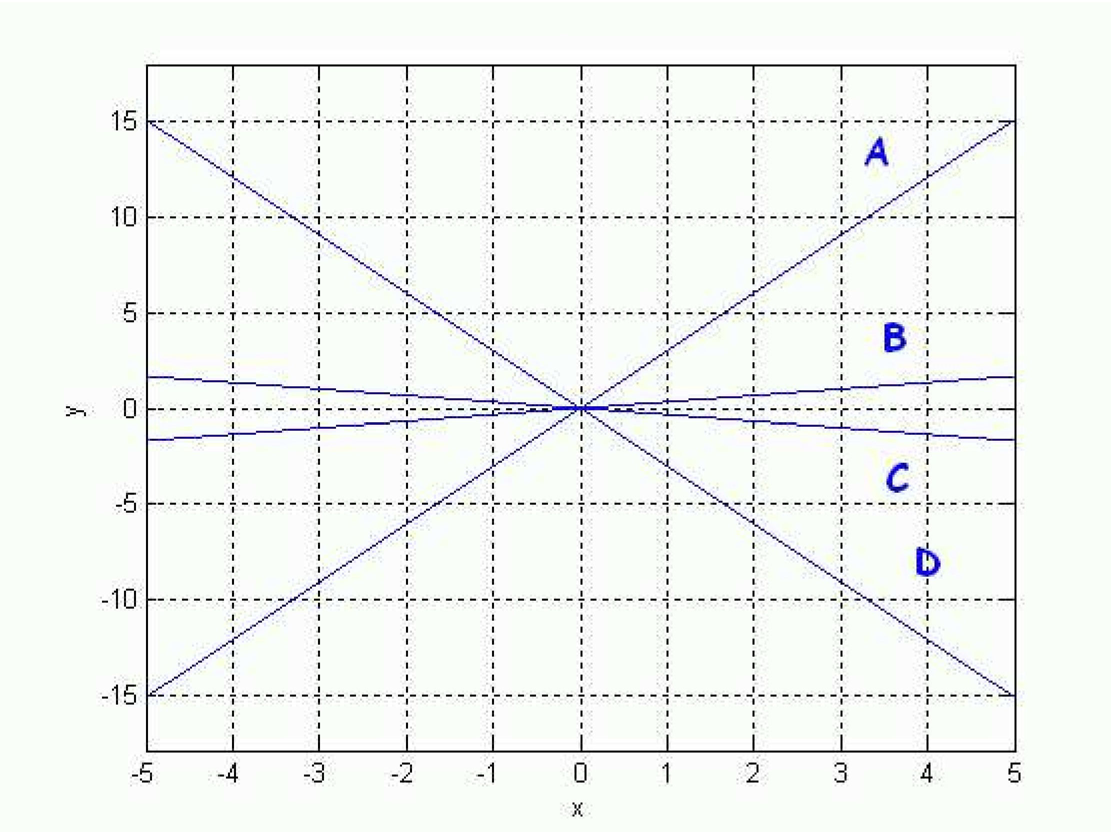
(a) line A
(b) line B
(c) line C
(d) line D
(e) None of the above

12. The *left null space* of a matrix A is the set of vectors x that solve $xA = 0$. Which of the following vectors is in the left null space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(a) $x = \begin{bmatrix} -2 & 1 \end{bmatrix}$
(b) $x = \begin{bmatrix} -3 & 1 \end{bmatrix}$
(c) $x = \begin{bmatrix} 1 & -3 \end{bmatrix}$
(d) $x = \begin{bmatrix} 1 & -2 \end{bmatrix}$
(e) More than one of the above
(f) None of the above

13. **True or False:** Since $xA = 0$ can be rewritten as $A^T x^T = 0$, we can think of the left null space as the null space of A^T.

14. Which line in the graph below represents the left null space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?



(a) line A
(b) line B
(c) line C
(d) line D
(e) None of the above

15. Let $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 2 & -1 & 1 & 1 \end{bmatrix}$. Which of the following vectors are in the nullspace of A?

(a) $\begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 3 \\ -1 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 3 \\ -1 \\ 3 \\ 2 \end{bmatrix}$

16. Let $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 2 & -1 & 1 & 1 \end{bmatrix}$. How many vectors are in the nullspace of A?

(a) Only one
(b) Probably more than one, but it's hard to say how many
(c) An infinite number

17. If A is an $m \times n$ matrix, then the column space of A is

(a) A subset of $\Re^m$ that may not include the origin.
(b) A subset of $\Re^m$ that includes the origin.
(c) A subset of $\Re^n$ that may not include the origin.
(d) A subset of $\Re^n$ that includes the origin.
(e) None of the above

18. If A is an $m \times n$ matrix, then the row space of A is

(a) A subset of $\Re^m$ that may not include the origin.
(b) A subset of $\Re^m$ that includes the origin.
(c) A subset of $\Re^n$ that may not include the origin.
(d) A subset of $\Re^n$ that includes the origin.
(e) None of the above

19. If A is an $m \times n$ matrix, then the null space of A is

(a) A subset of $\Re^m$ that may not include the origin.
(b) A subset of $\Re^m$ that includes the origin.
(c) A subset of $\Re^n$ that may not include the origin.
(d) A subset of $\Re^n$ that includes the origin.
(e) None of the above

20. If A is an $m \times n$ matrix, then the left null space of A is

(a) A subset of $\Re^m$ that may not include the origin.
(b) A subset of $\Re^m$ that includes the origin.
(c) A subset of $\Re^n$ that may not include the origin.
(d) A subset of $\Re^n$ that includes the origin.
(e) None of the above

21. Two vector spaces, V and W are *orthogonal complements* if and only if V is the set of all vectors which are orthogonal to every vector in W. Recall that for the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ the null space consists of all multiples of the vector $(-2, 1)$ and the left null space consists of all multiples of the vector $(-3, 1)$. Which of the following are true?

(a) The column space and null space are orthogonal complements.
(b) The column space and row space are orthogonal complements.
(c) The column space and left null space are orthogonal complements.
(d) None of the above

22. Recall that for the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ the null space consists of all multiples of the vector $(-2, 1)$ and the left null space consists of all multiples of the vector $(-3, 1)$. Which of the following vector subspaces are orthogonal complements?

(a) The row space and null space are orthogonal complements.
(b) The row space and column space are orthogonal complements.
(c) The row space and left null space are orthogonal complements.
(d) None of the above