MathQuest: Linear Algebra

Linearly Independent Sets

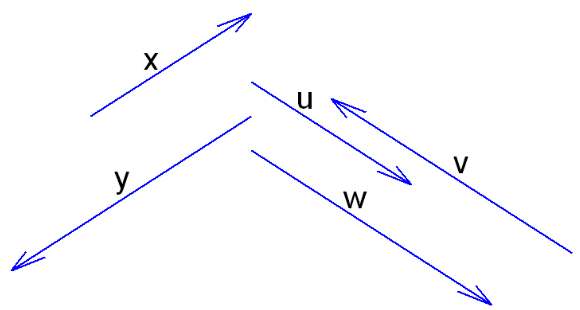
1. To determine whether a set S of vectors is linearly independent, you form a matrix which has those vectors as columns, and you calculate its reduced row echelon form. Suppose the resulting form is $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. How many linearly independent vectors are in S?

   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

2. To determine whether a set S of vectors is linearly independent, you form a matrix which has those vectors as columns, and you calculate its reduced row echelon form. Suppose the resulting form is $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Which of the following subsets of S are linearly independent?

   (a) The first, second, and third vectors
   (b) The first, second, and fourth vectors
   (c) The first, third, and fourth vectors
   (d) The second, third, and fourth vectors
   (e) All of the above
   (f) More than one, but not all, of the above

3. Consider the vectors x, y, u, v, and w in $\Re^2$ plotted below and form a matrix M which has these vectors as columns. What is the rank of this matrix?



(a) $\text{rank}(M) = 1$
(b) $\text{rank}(M) = 2$
(c) $\text{rank}(M) = 3$
(d) $\text{rank}(M) = 4$
(e) $\text{rank}(M) = 5$

4. To determine whether a set of vectors is linearly independent, you form a matrix which has those vectors as columns. If the matrix is square and its determinant is zero, what do you conclude?

   (a) The vectors are linearly independent.
   (b) The vectors are not linearly independent.
   (c) This test is inconclusive, and further work must be done.

5. Which of the following expressions is a linear combination of the functions $f(t)$ and $g(t)$?

   (a) $2f(t) + 3g(t) + 4$
   (b) $f(t) - 2g(t) + t$
   (c) $2f(t)g(t) - 3f(t)$
   (d) $f(t) - g(t)$
   (e) All of the above
   (f) None of the above
   (g) Some of the above

6. **True or False** The function $h(t) = 4 + 3t$ is a linear combination of the functions $f(t) = (1+t)^2$ and $g(t) = 2 - t - 2t^2$.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

7. **True or False** The function $h(t) = \sin(t+2)$ is a linear combination of the functions $f(t) = \sin t$ and $g(t) = \cos t$.

   (a) True, and I am very confident

(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

8. **True or False** $h(t) = t^2$ is a linear combination of $f(t) = (1-t)^2$ and $g(t) = (1+t)^2$.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

9. Let $y_1(t) = \sin(2t)$. For which of the following functions $y_2(t)$ will $\{y_1(t), y_2(t)\}$ be a linearly independent set?

   (a) $y_2(t) = \sin(t)\cos(t)$
   (b) $y_2(t) = 2\sin(2t)$
   (c) $y_2(t) = \cos(2t - \pi/2)$
   (d) $y_2(t) = \sin(-2t)$
   (e) All of the above
   (f) None of the above

10. Let $y_1(t) = e^{2t}$. For which of the following functions $y_2(t)$ will $\{y_1(t), y_2(t)\}$ be a linearly independent set?

   (a) $y_2(t) = e^{-2t}$
   (b) $y_2(t) = te^{2t}$
   (c) $y_2(t) = 1$
   (d) $y_2(t) = e^{3t}$
   (e) All of the above
   (f) None of the above

11. The functions $y_1(t)$ and $y_2(t)$ are linearly independent on the interval $a < t < b$ if

   (a) for some constant k, $y_1(t) = ky_2(t)$ for $a < t < b$.
   (b) there exists some $t_0 \in (a, b)$ and some constants c_1 and c_2 such that $c_1y_1(t_0) + c_2y_2(t_0) \neq 0$.

(c) the equation $c_1y_1(t) + c_2y_2(t) = 0$ holds for all $t \in (a, b)$ only if $c_1 = c_2 = 0$.

(d) the ratio $y_1(t)/y_2(t)$ is a constant function.

(e) All of the above

(f) None of the above

12. The functions $y_1(t)$ and $y_2(t)$ are linearly dependent on the interval $a < t < b$ if

(a) there exist two constants c_1 and c_2 such that $c_1y_1(t)+c_2y_2(t) = 0$ for all $a < t < b$.

(b) there exist two constants c_1 and c_2, not both 0, such that $c_1y_1(t)+c_2y_2(t) = 0$ for all $a < t < b$.

(c) for each t in (a, b), there exists constants c_1 and c_2 such that $c_1y_1(t)+c_2y_2(t) = 0$.

(d) for some $a < t_0 < b$, the equation $c_1y_1(t_0) + c_2y_2(t_0) = 0$ can only be true if $c_1 = c_2 = 0$.

(e) All of the above

(f) None of the above