

Linearly Independent Sets

1. To determine whether a set S of vectors is linearly independent, you form a matrix which has those vectors as columns, and you calculate its reduced row echelon form.

Suppose the resulting form is $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. How many linearly independent vectors are in S ?

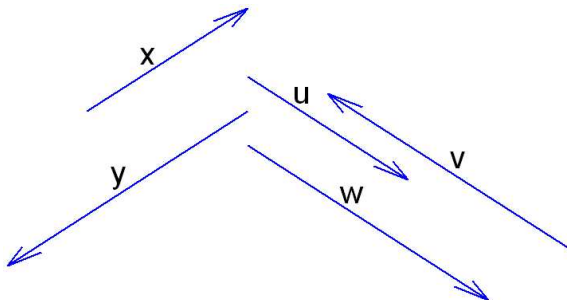
- (a) 0
- (b) 1
- (c) 2
- (d) 3
- (e) 4

2. To determine whether a set S of vectors is linearly independent, you form a matrix which has those vectors as columns, and you calculate its reduced row echelon form.

Suppose the resulting form is $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Which of the following subsets of S are linearly independent?

- (a) The first, second, and third vectors
- (b) The first, second, and fourth vectors
- (c) The first, third, and fourth vectors
- (d) The second, third, and fourth vectors
- (e) All of the above
- (f) More than one, but not all, of the above

3. Consider the vectors $x, y, u, v,$ and w in \mathbb{R}^2 plotted below and form a matrix M which has these vectors as columns. What is the rank of this matrix?



- (a) $\text{rank}(M) = 1$
 - (b) $\text{rank}(M) = 2$
 - (c) $\text{rank}(M) = 3$
 - (d) $\text{rank}(M) = 4$
 - (e) $\text{rank}(M) = 5$
4. To determine whether a set of vectors is linearly independent, you form a matrix which has those vectors as columns. If the matrix is square and its determinant is zero, what do you conclude?
- (a) The vectors are linearly independent.
 - (b) The vectors are not linearly independent.
 - (c) This test is inconclusive, and further work must be done.
5. Which of the following expressions is a linear combination of the functions $f(t)$ and $g(t)$?
- (a) $2f(t) + 3g(t) + 4$
 - (b) $f(t) - 2g(t) + t$
 - (c) $2f(t)g(t) - 3f(t)$
 - (d) $f(t) - g(t)$
 - (e) All of the above
 - (f) None of the above
6. **True or False** The function $h(t) = 4 + 3t$ is a linear combination of the functions $f(t) = (1 + t)^2$ and $g(t) = 2 - t - 2t^2$.
7. **True or False** The function $h(t) = \sin(t + 2)$ is a linear combination of the functions $f(t) = \sin t$ and $g(t) = \cos t$.
8. **True or False** $h(t) = t^2$ is a linear combination of $f(t) = (1 - t)^2$ and $g(t) = (1 + t)^2$.
9. Let $y_1(t) = \sin(2t)$. For which of the following functions $y_2(t)$ will $\{y_1(t), y_2(t)\}$ be a linearly independent set?
- (a) $y_2(t) = \sin(t) \cos(t)$
 - (b) $y_2(t) = 2 \sin(2t)$

- (c) $y_2(t) = \cos(2t - \pi/2)$
- (d) $y_2(t) = \sin(-2t)$
- (e) All of the above
- (f) None of the above
10. Let $y_1(t) = e^{2t}$. For which of the following functions $y_2(t)$ will $\{y_1(t), y_2(t)\}$ be a linearly independent set?
- (a) $y_2(t) = e^{-2t}$
- (b) $y_2(t) = te^{2t}$
- (c) $y_2(t) = 1$
- (d) $y_2(t) = e^{3t}$
- (e) All of the above
- (f) None of the above
11. The functions $y_1(t)$ and $y_2(t)$ are linearly independent on the interval $a < t < b$ if
- (a) for some constant k , $y_1(t) = ky_2(t)$ for $a < t < b$.
- (b) there exists some $t_0 \in (a, b)$ and some constants c_1 and c_2 such that $c_1y_1(t_0) + c_2y_2(t_0) \neq 0$.
- (c) the equation $c_1y_1(t) + c_2y_2(t) = 0$ holds for all $t \in (a, b)$ only if $c_1 = c_2 = 0$.
- (d) the ratio $y_1(t)/y_2(t)$ is a constant function.
- (e) All of the above
- (f) None of the above
12. The functions $y_1(t)$ and $y_2(t)$ are linearly dependent on the interval $a < t < b$ if
- (a) there exist two constants c_1 and c_2 such that $c_1y_1(t) + c_2y_2(t) = 0$ for all $a < t < b$.
- (b) there exist two constants c_1 and c_2 , not both 0, such that $c_1y_1(t) + c_2y_2(t) = 0$ for all $a < t < b$.
- (c) for each t in (a, b) , there exists constants c_1 and c_2 such that $c_1y_1(t) + c_2y_2(t) = 0$.
- (d) for some $a < t_0 < b$, the equation $c_1y_1(t_0) + c_2y_2(t_0) = 0$ can only be true if $c_1 = c_2 = 0$.
- (e) All of the above
- (f) None of the above