MathQuest: Linear Algebra

Spanning Sets, Bases, and Dimension

1. Write $d = (3, -5, 10)$ as a linear combination of the vectors $a = (-1, 0, 3), b = (0, 1, 5)$, and $c = (4, -2, 0)$.

   (a) $d = -3a - 5b + c$

   (b) $d = 5a - b + 2c$

   (c) $d = (10/3)a + (5/2)c$

   (d) d cannot be written as a linear combination of a, b, and c.

2. Which of the following sets of vectors spans $\Re^3$?

   i. $\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}$

   ii. $\begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

   iii. $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ -2 \end{bmatrix}$

   iv. $\begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 12 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$

   (a) i, ii, iii, and iv

   (b) ii, iii, and iv only

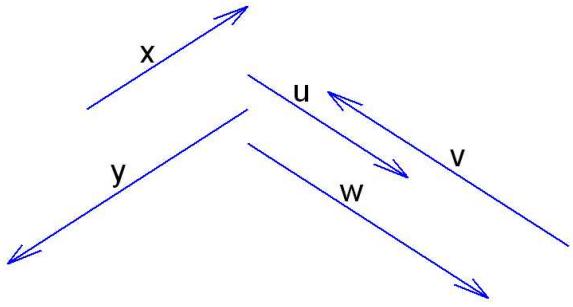
   (c) ii and iii only

   (d) i, ii and iii only

   (e) iii and iv only

   (f) ii only

3. Which of the following sets of vectors spans $\Re^2$?



   i. x, y

   ii. u, v, w

   iii. x, v

   iv. y, u, w

(a) i, ii, iii, and iv

(b) ii, iii, and iv only

(c) ii and iii only

(d) ii and iv only

(e) iii and iv only

(f) ii only

4. Which of the following sets of vectors forms a basis for $\Re^3$?

i. $\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}$

ii. $\begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

iii. $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ -2 \end{bmatrix}$

iv. $\begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 12 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$

(a) i, ii, iii, and iv

(b) ii, iii, and iv only

(c) ii and iii only

(d) i, ii and iii only

(e) iii and iv only

(f) ii only

5. Which of the following describes the subspace of $\Re^3$ spanned by the vectors $\begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 12 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$?

(a) A line

(b) A plane

(c) $\Re^2$

(d) All of $\Re^3$

(e) Both (b) and (c)

6. Which of the following describes a basis for a subspace V?

(a) A basis is a linearly independent spanning set for V.

(b) A basis is a minimal spanning set for V.

(c) A basis is a largest possible set of linearly independent vectors in V.

(d) All of the above

(e) Some of the above

(f) None of the above

7. Let $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 3 & -1 & 0 & 2 \end{bmatrix}$. The reduced row echelon form of A is $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.
What is the dimension of the column space of A?

(a) 1

(b) 2

(c) 3

(d) 4

8. Let $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 3 & -1 & 0 & 2 \end{bmatrix}$. The reduced row echelon form of A is $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.
Which columns would form a basis for the column space of A?

(a) All four

(b) The first three

(c) Any three

(d) Any two

9. Let $B = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$. Which of the following describes the column space of B?

(a) The column space of B is all of $\Re^3$.

(b) The column space of B is a proper subset of $\Re^3$.

(c) The column space of B is $\Re^4$.

(d) The column space of B is a proper subset of $\Re^4$.

(e) None of the above

10. Let $B = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$. What is the dimension of the column space of B?

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

(f) Infinite

11. Let $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 2 & -1 & 1 & 1 \end{bmatrix}$. What is the dimension of the nullspace of A?

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

(f) Infinite

12. Let A be an $n \times n$ matrix. If A is an invertible matrix, what else must be true?

(a) The columns of A form a basis of $\Re^n$.

(b) The rank of A is n.

(c) The dimension of the column space of A is n.

(d) The dimension of the null space of A is 0.

(e) All of the above must be true.

(f) More than one, but not all, of the above have to be true.

13. Howard's store sells three blends of flour: standard, extra wheat, and extra soy. Each is a blend of whole wheat flour and soy flour, and the table below shows how many pounds of each type of flour is needed to make one pound of each blend.

Standard Blend	Extra Wheat	Extra Soy	
0.5	0.8	0.3	whole wheat flour
0.5	0.2	0.7	soy flour

Do the column vectors in this table span $\Re^2$? Do they form a basis for $\Re^2$?

(a) Yes, they span $\Re^2$, and they form a basis.

(b) They do span $\Re^2$, but they do not form a basis.

(c) They do not span $\Re^2$, but they do form a basis for $\Re^2$.

(d) They do not span $\Re^2$, nor do they form a basis.

14. Howard's store sells three blends of flour: standard, extra wheat, and extra soy. Each is a blend of whole wheat flour and soy flour, and the table below shows how many pounds of each type of flour is needed to make one pound of each blend.

Standard Blend	Extra Wheat	Extra Soy	
0.5	0.8	0.3	whole wheat flour
0.5	0.2	0.7	soy flour

To save rent money, the store will be moving to a smaller space and will need to cut back on inventory. If possible, the manager would like to only stock two of these blends, and make the third from those as necessary. Which blends can be made from the others?

(a) Standard Blend can be made from Extra Wheat Blend and Extra Soy Blend.

(b) Extra Wheat Blend can be made from Standard Blend and Extra Soy Blend.

(c) Extra Soy Blend can be made from Standard Blend and Extra Wheat Blend.

(d) Any one blend can be made from the other two.

15. Howard's store sells three blends of flour: standard, extra wheat, and extra soy. Each is a blend of whole wheat flour and soy flour, and the table below shows how many pounds of each type of flour is needed to make one pound of each blend.

Standard Blend	Extra Wheat	Extra Soy	
0.5	0.8	0.3	whole wheat flour
0.5	0.2	0.7	soy flour

If the store continues to stock all three of these blends, which special-request blends could be made from these three?

(a) Any special request could be accomodated by mixing the right combination of these three blends.

(b) It would be possible to make any blend that is between 30% and 80% whole wheat.

(c) It would be possible to make a broader range of blends than what is described in answer (b), but there are still some blends that would not be possible.

(d) It would be possible to satisfy some special requests, but not all of the ones described in answer (b).