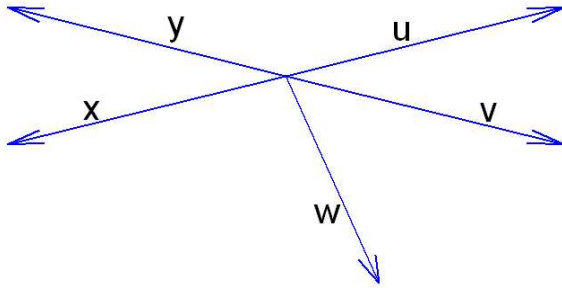


Linear Transformations and Projections

1. Define $T(v) = Av$, where $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Then $T(v)$

- (a) reflects v about the x_2 -axis.
- (b) reflects v about the x_1 -axis.
- (c) rotates v clockwise $\pi/2$ radians about the origin.
- (d) rotates v counterclockwise $\pi/2$ radians about the origin.
- (e) None of the above

2. Define $T(u) = Au$, where $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$. Using the vectors from \mathfrak{R}^2 plotted below, this means that



- (a) $T(u) = v$.
- (b) $T(u) = w$.
- (c) $T(u) = x$.
- (d) $T(u) = y$.
- (e) None of the above

3. If the linear transformation $T(v) = Av$ rotates the vectors $v_1 = (-1, 0)$ and $v_2 = (0, 1)$ clockwise $\pi/2$ radians, the resulting vectors are

- (a) $T(v_1) = (-\sqrt{2}/2, \sqrt{2}/2)$ and $T(v_2) = (\sqrt{2}/2, \sqrt{2}/2)$
- (b) $T(v_1) = (-\sqrt{2}/2, -\sqrt{2}/2)$ and $T(v_2) = (-\sqrt{2}/2, \sqrt{2}/2)$
- (c) $T(v_1) = (0, -1)$ and $T(v_2) = (-1, 0)$
- (d) $T(v_1) = (0, 1)$ and $T(v_2) = (1, 0)$

(e) None of the above

4. If the linear transformation $T(v) = Av$ rotates the vectors $(-1, 0)$ and $(0, 1)$ clockwise $\pi/2$ radians then

(a) $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(d) $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(e) None of the above

5. If the linear transformation $T(v) = Av$ rotates the vectors $v_1 = (-1, 0)$ and $v_2 = (0, 1)$ clockwise π radians, the resulting vectors are

(a) $T(v_1) = (1, 0)$ and $T(v_2) = (0, -1)$

(b) $T(v_1) = (-1, 0)$ and $T(v_2) = (0, 1)$

(c) $T(v_1) = (0, 1)$ and $T(v_2) = (1, 0)$

(d) $T(v_1) = (0, -1)$ and $T(v_2) = (-1, 0)$

(e) None of the above

6. If the linear transformation $T(v) = Av$ rotates the vectors $(-1, 0)$ and $(0, 1)$ π radians clockwise then

(a) $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $A = \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

(d) $A = \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$

(e) None of the above

7. If the linear transformation $T(v) = Av$ rotates the vector v θ radians clockwise, then

(a) $A = \begin{bmatrix} \sin \theta & \cos \theta \\ \cos \theta & -\sin \theta \end{bmatrix}$

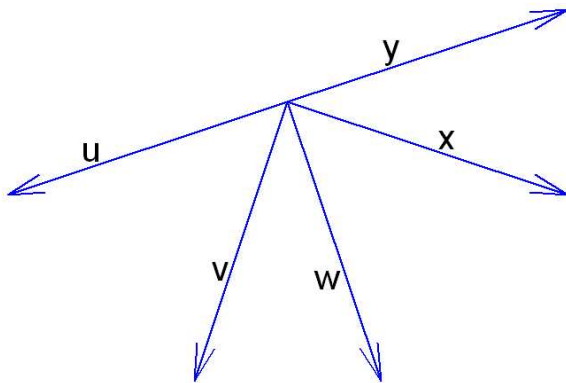
(b) $A = \begin{bmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{bmatrix}$

(c) $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$

(d) $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$

(e) None of the above

8. The linear transformation $T(v) = Av$ produces $T(u) = w$, $T(v) = x$ and $T(w) = y$, as shown below. Which of the following could be the matrix A ?



(a) $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(d) $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(e) None of the above

9. The linear transformation $T(x, y) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$, can be written as

(a) $T(x, y) = (x, y)$

(b) $T(x, y) = (y, x)$

(c) $T(x, y) = (-x, y)$

- (d) $T(x, y) = (-y, x)$
- (e) None of the above

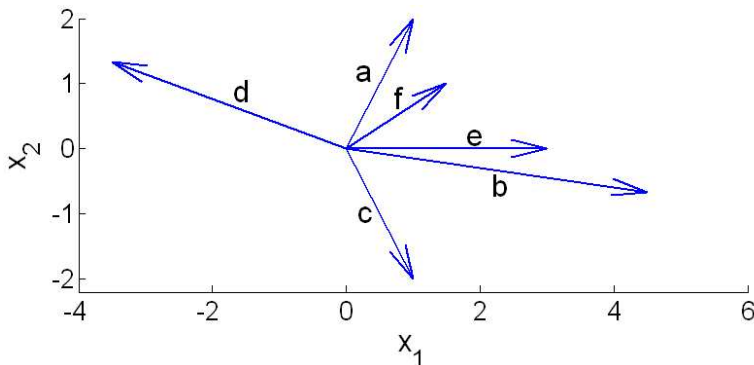
10. The linear transformation $T(x, y) = (x + 2y, x - 2y)$, can be written as a matrix transformation $T(x, y) = A \begin{bmatrix} x \\ y \end{bmatrix}$ where

- (a) $A = \begin{bmatrix} x & 2y \\ x & -2y \end{bmatrix}$
- (b) $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$
- (c) $A = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$
- (d) It can't be written in matrix form

11. Which of the following is not a linear transformation?

- (a) $T(x, y) = (x, y + 1)$
- (b) $T(x, y) = (x - 2y, x)$
- (c) $T(x, y) = (4y, x - 2y)$
- (d) $T(x, y) = (x, 0)$
- (e) All are linear transformations
- (f) More than one, but not all are linear transforms

12. **True or False** If a transformation produces $T(a) = b$, $T(c) = d$, and $T(e) = f$, for the vectors plotted below, then this transformation must be nonlinear.

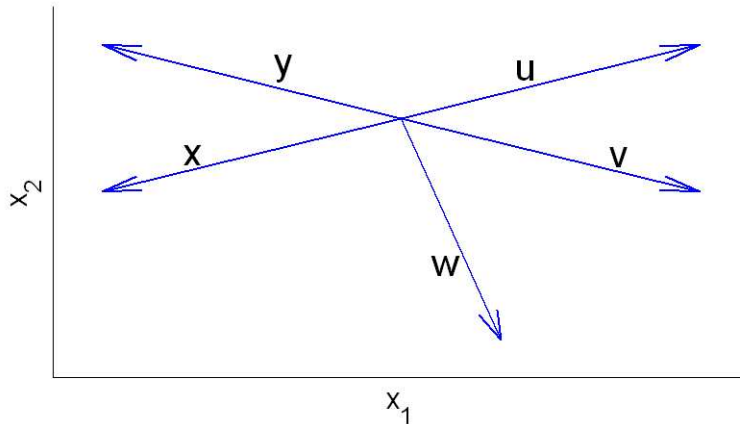


13. Is the transformation $T(x, y, z) = (x, y, 0)$ linear?

- (a) No, it is not linear because all z components map to 0.

- (b) No, it is not linear because it does not satisfy the scalar multiplication property.
- (c) No, it is not linear because it does not satisfy the vector addition property.
- (d) No, it is not linear for a reason not listed here.
- (e) Yes, it is linear.

14. **True or False** If a transformation produces $T(x) = y$, $T(y) = u$, $T(u) = v$, and $T(v) = w$ for the vectors plotted below, then this transformation must be nonlinear.



15. Is the transformation $T(f) = f'$ linear?
- (a) No, it is not linear because it does not satisfy the scalar multiplication property.
 - (b) No, it is not linear because it does not satisfy the vector addition property.
 - (c) No, it is not linear for a reason not listed here.
 - (d) Yes, it is linear.

16. What is the range of $T(v) = Av$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 0 \end{bmatrix}$?

- (a) All of \mathbb{R}^3
- (b) All of \mathbb{R}^2
- (c) A line in \mathbb{R}^2
- (d) A plane in \mathbb{R}^3
- (e) A line in \mathbb{R}^3

17. When we map w to Aw and w is an eigenvector of A , what is the geometric effect?

- (a) Aw is a rotation of w .
- (b) Aw is a reflection of w in the x -axis.
- (c) Aw is a reflection of w in the y -axis.
- (d) Aw is parallel to w but may have a different length.