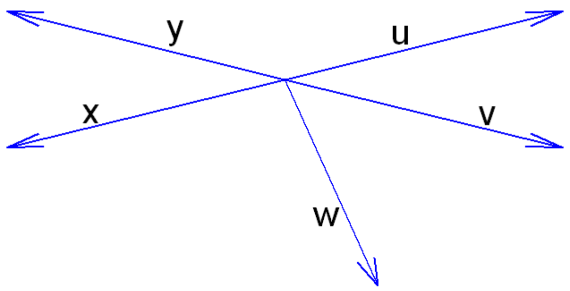
Linear Transformations and Projections

1. Define $T(v) = Av$, where $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Then $T(v)$

   (a) reflects v about the x_2-axis.
   (b) reflects v about the x_1-axis.
   (c) rotates v clockwise $\pi/2$ radians about the origin.
   (d) rotates v counterclockwise $\pi/2$ radians about the origin.
   (e) None of the above

2. Define $T(u) = Au$, where $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$. Using the vectors from $\Re^2$ plotted below, this means that



   (a) $T(u) = v$.
   (b) $T(u) = w$.
   (c) $T(u) = x$.
   (d) $T(u) = y$.
   (e) None of the above

3. If the linear transformation $T(v) = Av$ rotates the vectors $v_1 = (-1, 0)$ and $v_2 = (0, 1)$ clockwise $\pi/2$ radians, the resulting vectors are

   (a) $T(v_1) = \left(-\sqrt{2}/2, \sqrt{2}/2\right)$ and $T(v_2) = \left(\sqrt{2}/2, \sqrt{2}/2\right)$
   (b) $T(v_1) = \left(-\sqrt{2}/2, -\sqrt{2}/2\right)$ and $T(v_2) = \left(-\sqrt{2}/2, \sqrt{2}/2\right)$
   (c) $T(v_1) = (0, -1)$ and $T(v_2) = (-1, 0)$
   (d) $T(v_1) = (0, 1)$ and $T(v_2) = (1, 0)$

(e) None of the above

4. If the linear transformation $T(v) = Av$ rotates the vectors $(-1, 0)$ and $(0, 1)$ clockwise $\pi/2$ radians then

   (a) $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

   (d) $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

   (e) None of the above

5. If the linear transformation $T(v) = Av$ rotates the vectors $v_1 = (-1, 0)$ and $v_2 = (0, 1)$ clockwise π radians, the resulting vectors are

   (a) $T(v_1) = (1, 0)$ and $T(v_2) = (0, -1)$

   (b) $T(v_1) = (-1, 0)$ and $T(v_2) = (0, 1)$

   (c) $T(v_1) = (0, 1)$ and $T(v_2) = (1, 0)$

   (d) $T(v_1) = (0, -1)$ and $T(v_2) = (-1, 0)$

   (e) None of the above

6. If the linear transformation $T(v) = Av$ rotates the vectors $(-1, 0)$ and $(0, 1)$ π radians clockwise then

   (a) $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

   (d) $A = \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$

   (e) None of the above

7. If the linear transformation $T(v) = Av$ rotates the vector v θ radians clockwise, then

(a) $A = \begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix}$

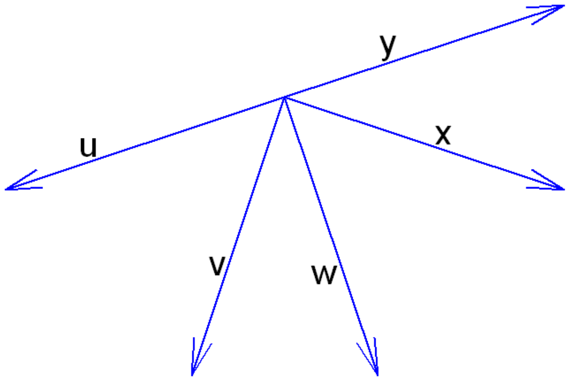
(b) $A = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$

(c) $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

(d) $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

(e) None of the above

8. The linear transformation $T(v) = Av$ produces $T(u) = w$, $T(v) = x$ and $T(w) = y$, as shown below. Which of the following could be the matrix A?



(a) $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(d) $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(e) None of the above

9. The linear transformation $T(x, y) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$, can be written as

(a) $T(x, y) = (x, y)$

(b) $T(x, y) = (y, x)$

(c) $T(x, y) = (-x, y)$

(d) $T(x, y) = (-y, x)$

(e) None of the above

10. The linear transformation $T(x, y) = (x + 2y, x - 2y)$, can be written as a matrix transformation $T(x, y) = A \begin{bmatrix} x \\ y \end{bmatrix}$ where

(a) $A = \begin{bmatrix} x & 2y \\ x & -2y \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$

(d) It can't be written in matrix form

11. Which of the following is not a linear transformation?

(a) $T(x, y) = (x, y + 1)$

(b) $T(x, y) = (x - 2y, x)$

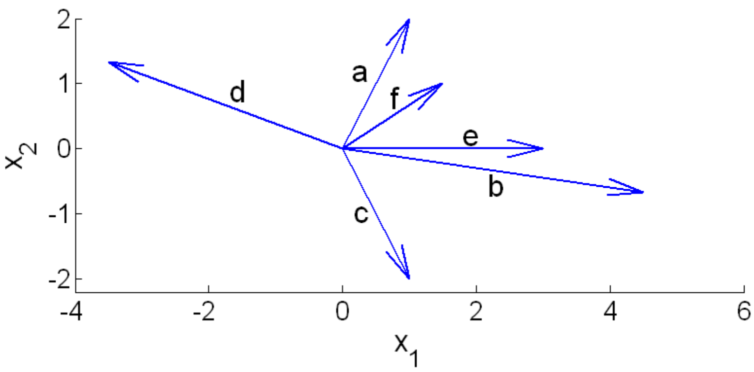
(c) $T(x, y) = (4y, x - 2y)$

(d) $T(x, y) = (x, 0)$

(e) All are linear transformations

(f) More than one of these are not linear transforms

12. **True or False** If a transformation produces $T(a) = b$, $T(c) = d$, and $T(e) = f$, for the vectors plotted below, then this transformation must be nonlinear.



(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

13. Is the transformation $T(x, y, z) = (x, y, 0)$ linear?

    (a) No, it is not linear because all z components map to 0.

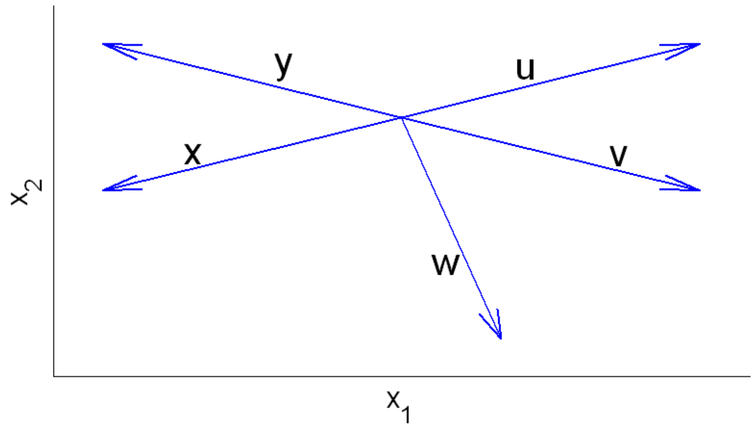
    (b) No, it is not linear because it does not satisfy the scalar multiplication property.

    (c) No, it is not linear because it does not satisfy the vector addition property.

    (d) No, it is not linear for a reason not listed here.

    (e) Yes, it is linear.

14. **True or False** If a transformation produces $T(x) = y$, $T(y) = u$, $T(u) = v$, and $T(v) = w$ for the vectors plotted below, then this transformation must be nonlinear.



    (a) True, and I am very confident

    (b) True, but I am not very confident

    (c) False, but I am not very confident

    (d) False, and I am very confident

15. If f is a function, is the transformation $T(f) = f'$ linear?

    (a) No, it is not linear because it does not satisfy the scalar multiplication property.

    (b) No, it is not linear because it does not satisfy the vector addition property.

    (c) No, it is not linear for a reason not listed here.

    (d) Yes, it is linear.

16. What is the range of $T(v) = Av$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 0 \end{bmatrix}$?

(a) All of $\Re^3$

(b) All of $\Re^2$

(c) A line in $\Re^2$

(d) A plane in $\Re^3$

(e) A line in $\Re^3$

17. When we map w to Aw and w is an eigenvector of A, what is the geometric effect?

(a) Aw is a rotation of w.

(b) Aw is a reflection of w in the x-axis.

(c) Aw is a reflection of w in the y-axis.

(d) Aw is parallel to w but may have a different length.