

MathQuest: Linear Algebra

Dot Products

1. What is the dot product of $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix}$?

(a) $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$

(b) 5

(c) 0

(d) The dot product cannot be computed for these vectors.

2. What is the dot product of $\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$?

(a) $\begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}$

(b) -4

(c) 0

(d) The dot product cannot be computed for these vectors.

3. The magnitude of a vector v is defined to be its dot product with itself $v \cdot v$. What is the magnitude of the vector $(2, -1, -1)$?

(a) 0

(b) 2

(c) 4

(d) 6

4. It is possible for a vector to have a negative magnitude?

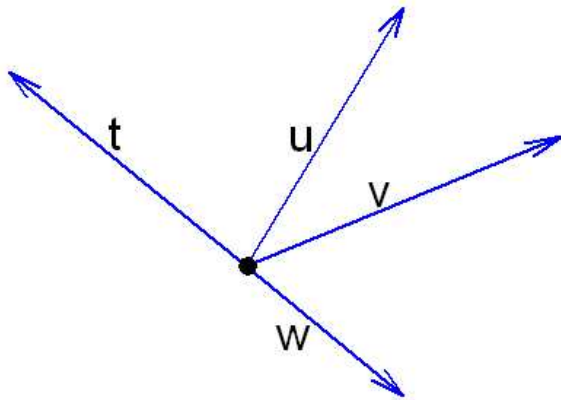
(a) Yes

- (b) No
- (c) Not enough information is given

5. What can we say about two vectors whose dot product is negative?

- (a) The vectors are orthogonal.
- (b) The angle between the two vectors is less than 90° .
- (c) The angle between the two vectors is greater than 90° .

6. Rank the dot products $u \cdot v$, $u \cdot t$ and $u \cdot w$.



- (a) $u \cdot v > u \cdot w > u \cdot t$
- (b) $u \cdot v > u \cdot t > u \cdot w$
- (c) $u \cdot w > u \cdot t > u \cdot v$
- (d) $u \cdot w > u \cdot v > u \cdot t$

7. **True or False** If x and y are $n \times 1$ vectors, then $x^T y = y^T x$.

8. **True or False** If x and y are $n \times 1$ vectors, then xy^T is an $n \times n$ matrix.

9. **True or False** If x and y are $n \times 1$ vectors, then $xy^T = yx^T$.

10. **True or False** If x and y are $n \times 1$ nonzero vectors, then xy^T is an $n \times n$ matrix with rank 1.

11. If A and B are matrices which can be multiplied, then the (i, j) -entry of AB is

- (a) [the i^{th} row of A] \cdot [the j^{th} column of B]
- (b) [the i^{th} column of A] \cdot [the j^{th} row of B]
- (c) None of the above

12. When we are in the vector space of real valued functions, it is often useful to have the equivalent of a dot product, which we call an inner product: We define the inner product of two functions $f(x)$ and $g(x)$ as $\langle f, g \rangle \equiv \int_a^b f(x)g(x)dx$. Consider the functions $f(x) = \sin 2\pi x$ on the interval $(a, b) = (0, 1)$. What is the inner product of this function with itself $\langle f, f \rangle$?

- (a) 0
- (b) $\frac{1}{2}$
- (c) 1
- (d) 2
- (e) This is not a meaningful statement.

13. $\langle \sin(2\pi x), \sin(4\pi x) \rangle \equiv \int_0^1 \sin(2\pi x) \sin(4\pi x)dx = 0$. What does this mean?

- (a) These are parallel functions.
- (b) These are orthogonal functions.
- (c) These are acute functions.
- (d) These are obtuse functions.