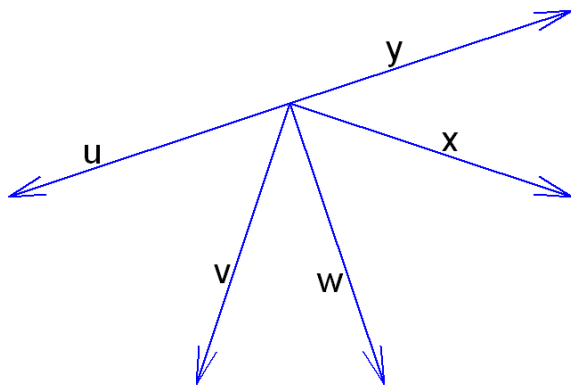


## Orthogonal Sets

1. Which of the following sets of vectors is *not* an orthogonal set?

- (a)  $(1, 1, 1), (1, 0, -1)$
- (b)  $(2, 3), (-6, 4)$
- (c)  $(3, 0, 0, 2), (0, 1, 0, 1)$
- (d)  $(0, 2, 0), (-1, 0, 3)$
- (e)  $(\cos \theta, \sin \theta), (\sin \theta, -\cos \theta)$

2. Which of the following sets of vectors is *not* an orthogonal set?



- (a)  $u, w$
- (b)  $x, v$
- (c)  $v, y$
- (d)  $u, w, y$
- (e) More than one of the above
- (f) None of the above

3. Let  $A$  be a square matrix whose columns are mutually orthogonal, nonzero vectors. Which of the following are true?

- (a) The dot product of any two different column vectors is zero.
- (b) The set of column vectors is linearly independent.
- (c)  $\det(A) \neq 0$ .

- (d) For any  $b$ , there is a unique solution to  $Ax = b$ .
- (e) All of the above.

4. **True or False** If two vectors are linearly independent, they must be orthogonal.

- (a) True, and I am very confident
- (b) True, but I am not very confident
- (c) False, but I am not very confident
- (d) False, and I am very confident

5. **True or False** Any orthogonal set of nonzero vectors that spans a vector space must be a basis for that space.

- (a) True, and I am very confident
- (b) True, but I am not very confident
- (c) False, but I am not very confident
- (d) False, and I am very confident

6. Let  $A$  be any matrix. Which of the following are true?

- (a) The row space of  $A$  and the nullspace of  $A$  are orthogonal to each other.
- (b) The column space of  $A$  and the row space of  $A$  are orthogonal to each other.
- (c) The column space of  $A$  and the nullspace of  $A$  are orthogonal to each other.
- (d) Exactly two of (a), (b), and (c) are true.
- (e) All of (a), (b), and (c) are true.

7. Let  $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ . Which of the following vectors is orthogonal to the row space of  $A$ ?

- (a)  $(1, 1, -1)$
- (b)  $(1, 4, 2)$
- (c)  $(0, 0, 5)$
- (d)  $(-1, 0, 1)$

8. Let  $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ . Which of the following vectors is orthogonal to the column space of  $A$ ?

- (a)  $(1, 1, -1)$
- (b)  $(1, 4, 2)$
- (c)  $(0, 1, -2)$
- (d)  $(2, 0, 2)$

9. Let  $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$ . Which of the following vectors is orthogonal to the nullspace of  $A$ ?

- (a)  $(1, 1, -1)$
- (b)  $(1, 4, 2)$
- (c)  $(0, 1, -2)$
- (d)  $(2, 0, 2)$

10. Which of the following sets of vectors is an orthonormal set?

- (a)  $(1, 1, 1), (1, 0, -1)$
- (b)  $(2, 3), (-6, 4)$
- (c)  $(0, 2, 0), (-1, 0, 3)$
- (d)  $(\cos \theta, \sin \theta), (\sin \theta, -\cos \theta)$

11. Let  $A$  be a matrix whose columns are mutually orthogonal. Which of the following must be true? Try several examples of matrices with mutually orthogonal columns to build your intuition, then try to provide a proof.

- (a)  $A$  is symmetric.
- (b)  $A^{-1} = A^T$ .
- (c)  $A^T A$  is diagonal.
- (d)  $\det(A) \neq 0$ .
- (e) All of the above must be true.
- (f) More than one, but not all, of the above must be true.

12. Let  $M$  be any matrix. **True or False** The columns of  $M$  are orthonormal if and only if  $M^T M$  is an identity matrix.
- (a) True, and I am very confident
  - (b) True, but I am not very confident
  - (c) False, but I am not very confident
  - (d) False, and I am very confident
13. Let  $Q$  be a square matrix with orthonormal columns. **True or False**  $Q^{-1} = Q^T$ .
- (a) True, and I am very confident
  - (b) True, but I am not very confident
  - (c) False, but I am not very confident
  - (d) False, and I am very confident
14. **True or False** Any set of nonzero orthogonal vectors must also be linearly independent.
- (a) True, and I am very confident
  - (b) True, but I am not very confident
  - (c) False, but I am not very confident
  - (d) False, and I am very confident
15. **True or False** The only orthonormal basis for  $\mathbb{R}^2$  is  $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ .
- (a) True, and I am very confident
  - (b) True, but I am not very confident
  - (c) False, but I am not very confident
  - (d) False, and I am very confident