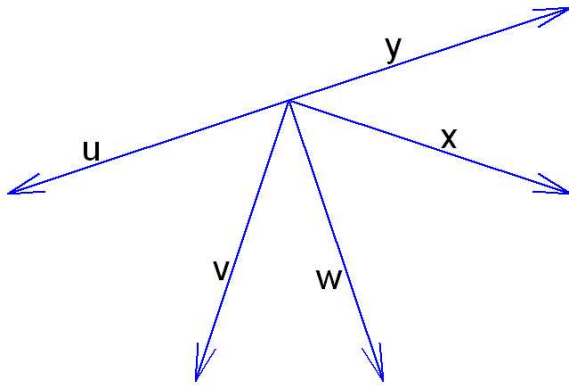


Orthogonal Sets

1. Which of the following sets of vectors is *not* an orthogonal set?

- (a) $(1, 1, 1), (1, 0, -1)$
- (b) $(2, 3), (-6, 4)$
- (c) $(3, 0, 0, 2), (0, 1, 0, 1)$
- (d) $(0, 2, 0), (-1, 0, 3)$
- (e) $(\cos \theta, \sin \theta), (\sin \theta, -\cos \theta)$

2. Which of the following sets of vectors is *not* an orthogonal set?



- (a) u, w
- (b) x, v
- (c) v, y
- (d) u, w, y
- (e) More than one of the above
- (f) None of the above

3. Let A be a square matrix whose columns are mutually orthogonal, nonzero vectors. Which of the following are true?

- (a) The dot product of any two different column vectors is zero.
- (b) The set of column vectors is linearly independent.
- (c) $\det(A) \neq 0$.

- (d) For any b , there is a unique solution to $Ax = b$.
- (e) All of the above.

4. **True or False** If two vectors are linearly independent, they must be orthogonal.
5. **True or False** Any orthogonal set of nonzero vectors that spans a vector space must be a basis for that space.
6. Let A be any matrix. Which of the following are true?
- (a) The row space of A and the nullspace of A are orthogonal to each other.
 - (b) The column space of A and the row space of A are orthogonal to each other.
 - (c) The column space of A and the nullspace of A are orthogonal to each other.
 - (d) Exactly two of (a), (b), and (c) are true.
 - (e) All of (a), (b), and (c) are true.

7. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$. Which of the following vectors is orthogonal to the row space of A ?

- (a) $(1, 1, -1)$
- (b) $(1, 4, 2)$
- (c) $(0, 0, 5)$
- (d) $(-1, 0, 1)$

8. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$. Which of the following vectors is orthogonal to the column space of A ?

- (a) $(1, 1, -1)$
- (b) $(1, 4, 2)$
- (c) $(0, 1, -2)$
- (d) $(2, 0, 2)$

9. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$. Which of the following vectors is orthogonal to the nullspace of A ?
- (a) $(1, 1, -1)$
 - (b) $(1, 4, 2)$
 - (c) $(0, 1, -2)$
 - (d) $(2, 0, 2)$
10. Which of the following sets of vectors is an orthonormal set?
- (a) $(1, 1, 1), (1, 0, -1)$
 - (b) $(2, 3), (-6, 4)$
 - (c) $(0, 2, 0), (-1, 0, 3)$
 - (d) $(\cos \theta, \sin \theta), (\sin \theta, -\cos \theta)$
11. Let A be a matrix whose columns are mutually orthogonal. Which of the following must be true? Try several examples of matrices with mutually orthogonal columns to build your intuition, then try to provide a proof.
- (a) A is symmetric.
 - (b) $A^{-1} = A^T$.
 - (c) $A^T A$ is diagonal.
 - (d) $\det(A) \neq 0$.
 - (e) All of the above must be true.
 - (f) More than one, but not all, of the above must be true.
12. Let M be any matrix. **True or False** The columns of M are orthonormal if and only if $M^T M$ is an identity matrix.
13. Let Q be a square matrix with orthonormal columns. **True or False** $Q^{-1} = Q^T$.
14. **True or False** Any set of nonzero orthogonal vectors must also be linearly independent.
15. **True or False** The only orthonormal basis for \mathfrak{R}^2 is $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$.