Orthogonal Projections

1. If \( b = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \) and \( y = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \), then the orthogonal projection of \( b \) onto \( y \) is

   (a) \( \begin{bmatrix} 2 \\ 1 \end{bmatrix} \)
   (b) \( \begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix} \)
   (c) \( \begin{bmatrix} 10 \\ 5 \end{bmatrix} \)
   (d) \( \begin{bmatrix} 1/10 \\ 3/10 \end{bmatrix} \)

2. If \( b = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \) and \( l \) is the line \( y = \frac{1}{2}x \), then the orthogonal projection of \( b \) onto \( l \) is

   (a) \( \begin{bmatrix} 2 \\ 1 \end{bmatrix} \)
   (b) \( \begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix} \)
   (c) \( \begin{bmatrix} 10 \\ 5 \end{bmatrix} \)
   (d) \( \begin{bmatrix} 1/10 \\ 3/10 \end{bmatrix} \)

3. If \( l \) is the line \( y = 3x \), \( b \in \mathbb{R}^2 \), and \( z \) is the orthogonal projection of \( b \) on \( l \), then which of the following are true?

   (a) \( b - z \) is perpendicular to \( l \).
   (b) \( b - z \) is a point on \( l \).
   (c) \( z \) is of the form \((c, 3c)\)
   (d) Exactly two of the statements are true.
   (e) None of the above are true.
4. Let $A$ be an $n \times p$ matrix. Let $W$ be the column space of $A$, so $W$ is a subspace of $\mathbb{R}^n$. Let $b \in \mathbb{R}^n$ and let $z$ be an orthogonal projection of $b$ on $W$. Then which of the following is not true?

(a) $A^T(b - z) = 0$.
(b) $z$ is orthogonal to $W$.
(c) $b - z$ is orthogonal to $W$.
(d) $z$ is the vector in $W$ closest to $b$.

5. Let $v_1 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$, and $v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Let $z$ be the orthogonal projection of $v$ on the span of $\{v_1, v_2\}$, and let $A = \begin{bmatrix} v_1 & v_2 \end{bmatrix}$. Which of the following are true?

(a) $z = Ax$ for some $x$.
(b) $z$ is a linear combination of $v_1$ and $v_2$.
(c) $z = -\frac{4}{5} \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + \frac{7}{50} \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$

(d) All of the above statements are true.
(e) Exactly two of the above statements are true.
(f) None of the above statements are true.