

Orthogonal Projections

1. If $b = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ and $y = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, then the orthogonal projection of b onto y is
- (a) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$
 - (b) $\begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix}$
 - (c) $\begin{bmatrix} 10 \\ 5 \end{bmatrix}$
 - (d) $\begin{bmatrix} 1/10 \\ 3/10 \end{bmatrix}$
2. If $b = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ and l is the line $y = \frac{1}{2}x$, then the orthogonal projection of b onto l is
- (a) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$
 - (b) $\begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix}$
 - (c) $\begin{bmatrix} 10 \\ 5 \end{bmatrix}$
 - (d) $\begin{bmatrix} 1/10 \\ 3/10 \end{bmatrix}$
3. If l is the line $y = 3x$, $b \in \mathbb{R}^2$, and z is the orthogonal projection of b on l , then which of the following are true?
- (a) $b - z$ is perpendicular to l .
 - (b) $b - z$ is a point on l .
 - (c) z is of the form $(c, 3c)$
 - (d) Exactly two of the statements are true.
 - (e) None of the above are true.

4. Let A be an $n \times p$ matrix. Let W be the column space of A , so W is a subspace of \mathbb{R}^n . Let $b \in \mathbb{R}^n$ and let z be an orthogonal projection of b on W . Then which of the following is *not* true?

- (a) $A^T(b - z) = 0$.
- (b) z is orthogonal to W .
- (c) $b - z$ is orthogonal to W .
- (d) z is the vector in W closest to b .

5. Let $v_1 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$, and $v = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$. Let z be the orthogonal projection of v on the span of $\{v_1, v_2\}$, and let $A = [v_1 \ v_2]$. Which of the following are true?

- (a) $z = Ax$ for some x .
- (b) z is a linear combination of v_1 and v_2 .
- (c) $z = -\frac{1}{5} \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + \frac{7}{30} \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$
- (d) All of the above statements are true.
- (e) Exactly two of the above statements are true.
- (f) None of the above statements are true.