Gram-Schmidt Orthogonalization

1. Let A be an $m \times n$ matrix with linearly independent columns $x_1, x_2, \ldots, x_n$. Applying the Gram-Schmidt process to $x_1, x_2, \ldots, x_n$ will produce

   (a) an orthogonal basis for A.

   (b) an orthogonal basis for the column space of A.

   (c) an orthogonal basis for the row space of A.

   (d) an orthogonal basis for the null space of A.

2. Let $v_1 = (2, -1, 0)$ and $v_2 = (1, 1, 1)$. The Gram-Schmidt process, when applied to these vectors, produces $\{v_1', v_2'\}$ where

   (a) $v_1' = (2, -1, 0)$ and $v_2' = (-1, 2, 1)$.

   (b) $v_1' = (2, -1, 0)$ and $v_2' = (3/5, 6/5, 1)$.

   (c) $v_1' = (2, -1, 0)$ and $v_2' = (2/5, -1/5, 0)$.

   (d) $v_1' = (2, -1, 0)$ and $v_2' = (7/5, 6/5, 1)$.

   (e) $v_1' = (2, -1, 0)$ and $v_2' = (3/2, 3, 1)$.

3. **True or False** If $W = \mathit{span}\left\{ \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$, then $S = \left\{ \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 5 \end{bmatrix} \right\}$ is an orthogonal basis for W.