Markov Chains

1. A vector with nonnegative entries that add up to one is called a probability vector. Which of the following vectors is a probability vector?

   (a) $\begin{bmatrix} 0.2 \\ 0.5 \\ 0.1 \end{bmatrix}$

   (b) $\begin{bmatrix} -2.1 \\ 2.8 \\ 0.3 \end{bmatrix}$

   (c) $\begin{bmatrix} 0.1 \\ 0 \\ 0.9 \end{bmatrix}$

   (d) More than one of the given vectors are probability vectors.

   (e) None of the given vectors are probability vectors.

2. A stochastic matrix is a square matrix whose columns are probability vectors. Which of the following matrices is a stochastic matrix?

   (a) $\begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}$

   (b) $\begin{bmatrix} 0.2 & 0.6 \\ 0.8 & 0.4 \end{bmatrix}$

   (c) $\begin{bmatrix} 0.2 & 0.8 \\ 0.3 & 0.7 \end{bmatrix}$

   (d) Both (a) and (b) are stochastic matrices.

   (e) Both (a) and (c) are stocahstic matrices.

   (f) All three are stochastic matrices.

3. A small, isolated town has two grocery stores, Mike's Market and Sharon's Shoppe. While some customers are completely loyal to one store or another, there is another group of customers who change their shopping habits each month. Of the shoppers who favor Mike's Market one month, only 70% will still shop there the following month, while Sharon's Shoppe retains 78% of its customer base each month. Everyone in the town shops at one of the two stores, and no one from out of town ever shops at either store. If Mike's Market currently has 2500 customers and Sharon's Shoppe has 1900 customers, how many customers will Mike's Market have next month?

(a) 418
(b) 1750
(c) 2168
(d) 3080

4. Referring to the scenario in the previous question, what will the product

$$\begin{bmatrix} 0.70 & 0.22 \\ 0.30 & 0.78 \end{bmatrix} \begin{bmatrix} 2500 \\ 1900 \end{bmatrix}$$

tell us?

(a) This product will tell us the percentage of customers that will switch from one store to the other store next month.
(b) This product will tell us the number of customers who will shop at each store next month.
(c) This product will tell us the total number of customers who switched stores this month.
(d) This product doesn't have any meaning.

5. Continuing the scenario from the previous questions, what does the (2, 1)-entry of the matrix $\begin{bmatrix} 0.70 & 0.22 \\ 0.30 & 0.78 \end{bmatrix}^3$ represent?

(a) This represents the probability that a customer will switch from Mike's Market to Sharon's Shoppe between months 3 and 4.
(b) This represents the probability that a customer will switch from Sharon's Shoppe to Mike's Market between months 3 and 4.
(c) This represents the probability that a customer who currently shops at Mike's Market will be shopping at Sharon's Shoppe three months from now.
(d) This represents the probability that a customer who currently shops at Sharon's Shoppe will be shopping at Mike's Market three months from now.

6. A steady-state (or equilibrium) vector for a stochastic matrix P is a probability vector x such that $Px = x$. Which of the following is a steady-state vector for $\begin{bmatrix} 0.70 & 0.22 \\ 0.30 & 0.78 \end{bmatrix}$?

(a) $\begin{bmatrix} 22 \\ 30 \end{bmatrix}$

(b) $\begin{bmatrix} 11/26 \\ 15/26 \end{bmatrix}$

(c) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(d) All of the above

7. What does the steady-state vector mean in the context of Mike's Market and Sharon's Shoppe?

   (a) In the long-run, the probability of staying at Mike's Market will be 11/26 and the probability of switching to Sharon's Shoppe will be 15/26.

   (b) In the long-run, the probability of staying at Mike's Market will be 11/26 and the probability of staying at Sharon's Shoppe will be 15/26.

   (c) In the long-run, Mike's Market will approach 11/26 of the market share, while Sharon's Shoppe will approach 15/26 of the market share.

8. **True or False** A stochastic matrix will always have a steady-state vector.

   (a) True, and I am very confident

   (b) True, but I am not very confident

   (c) False, but I am not very confident

   (d) False, and I am very confident