MathQuest: Linear Algebra

Systems of Equations

1. What is the solution to the following system of equations?

$$\begin{aligned} 2x + y &= 3 \\ 3x - y &= 7 \end{aligned}$$

   (a) $x = 4$ and $y = -5$

   (b) $x = 4$ and $y = 5$

   (c) $x = 2$ and $y = -1$

   (d) $x = 2$ and $y = 1/2$

   (e) There are an infinite number of solutions to this system.

   (f) There are no solutions to this system.

   Answer: (c). If we add these equations together, the resulting equation is $5x = 10$. This gives us $x = 2$. Substituting back into the first equation, we obtain $4 + y = 3$, which gives us $y = -1$. Students may solve this a variety of ways - by testing the proposed solutions, by adding to eliminate a variable, by solving for one variable and substituting into the other equation, or by graphing. Try to draw out each of these methods for a full coverage of possible solution techniques.

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   CC HZ MA117 S07: 9/0/**67**/0/9/15
   CC HZ MA232 S08: 14/0/**86**/0/0/0 time 1:50
   HHS JG MA232 S08: 0/0/**100**/0/0/0
   CHS DP MA232 S08: 0/0/**100**/0/0/0
   CC JS MA232 S09: 11/0/**89**/0/0/0

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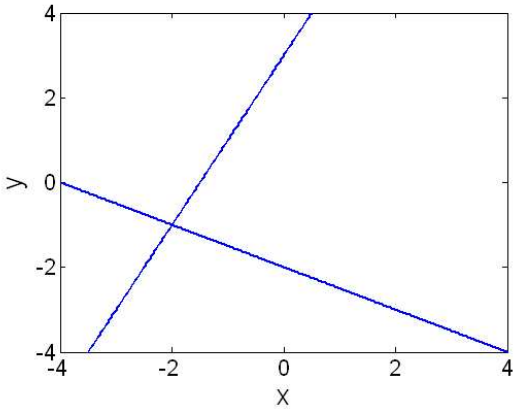
   LA.00.01.010

2. Which of the following systems of equations could be represented in the graph below?



(a) $3x + 3y = -6$, $x + 2y = 3$

(b) $x - y = -5$, $2x + y = 4$

(c) $-8x + 4y = 12$, $2x + 4y = -8$

(d) $-x + 3y = 9$, $2x - y = 4$

Answer: (c). This is a good problem to emphasize the many different approaches that can be used. One approach is to observe that on the graph, the solution is approximately (-2,-1) and to test this in the different sets of equations. Another approach is to put the equations into slope-intercept form, recognizing that we have slopes of 2 and $-\frac{1}{2}$ along with intercepts of 3 and -2. Still another approach is to solve the different sets of equations to find which gives us the point (-2,-1).

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3. What is the solution to the following system of equations?

$$\begin{aligned} 2x + y &= 3 \\ 4x + 2y &= 6 \end{aligned}$$

(a) $x = 0$ and $y = 0$

(b) $x = 2$ and $y = -1$

(c) $x = 0$ and $y = 1$

(d) $x = 0$ and $y = 3$

(e) There are an infinite number of solutions to this system.

(f) There are no solutions to this system.

Answer: (e). The second equation is two times the first equation, so these are really representing the same line. Note that answer (d) is a point on this line, but it is not the only solution. The answers to this question were modified after the spring 07 statistics.

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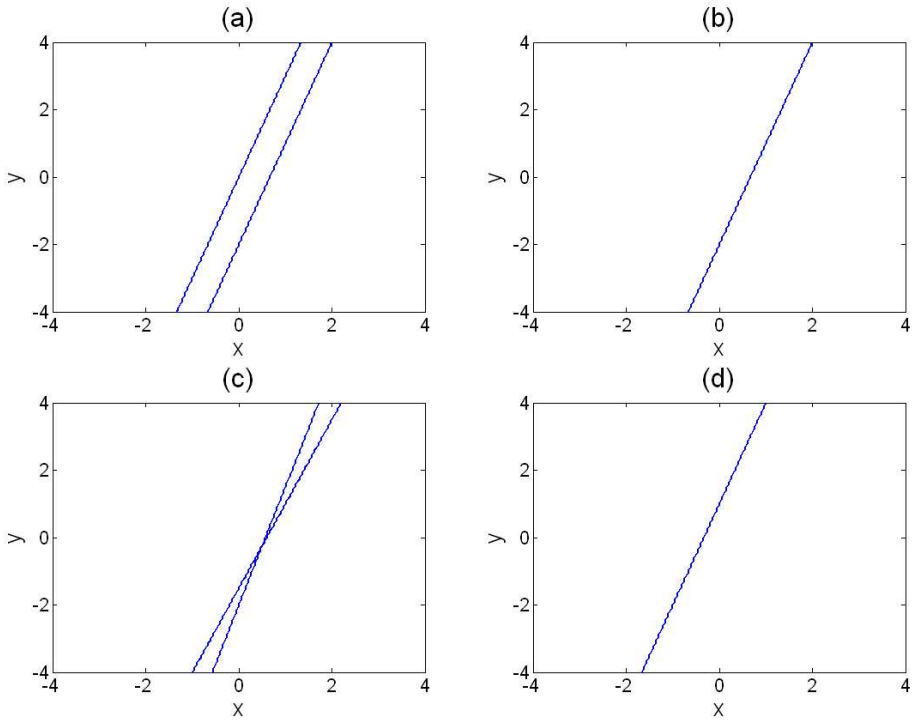
4. Which of the graphs below could represent the following linear system?

$$\begin{aligned} 3x - y &= 2 \\ -9x + 3y &= -6 \end{aligned}$$



Answer: (b). Here we have two equations for the same line, because the second equation is simply the first equation multiplied by -3. This rules out (a) and (c) which both show two distinct lines. From the equations, we can see that the y intercept of the line should be -2, which matches (b) but not (d), which has a positive intercept.

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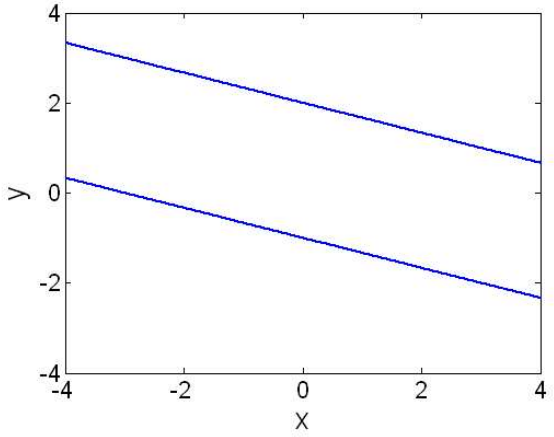
5. Which of the following systems of equations could be represented in the graph below?



(a) $-x + 3y = 6$, $2x + 6y = -6$

(b) $-x + 3y = 6$, $2x + 6y = 12$

(c) $x + 3y = 6$, $2x + 6y = 12$

(d) $x + 3y = 6$, $x + 3y = -3$

Answer: (d). Options (a) and (b) are not parallel lines, because the coefficients of x and y in the first equation are not multiples of the coefficients in the second equation. Option (c) gives two equations for the same line. Thus, the answer must be (d).

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6. What is the solution to the following system of equations?

$$\begin{aligned} -3x + 2y &= 4 \\ 12x - 8y &= 10 \end{aligned}$$

(a) $x = -4/3$ and $y = 0$

(b) $x = 1/2$ and $y = -1/2$

(c) $x = 0$ and $y = 2$

(d) $x = 1/3$ and $y = 5/2$

(e) There are an infinite number of solutions to this system.

(f) There are no solutions to this system.

Answer: (f). When we add four times the first equation to the second equation we obtain $0 = 26$, so there are no solutions to this system. Ask the students what their geometric interpretation is, or have someone sketch this system. They should see that these are parallel lines.

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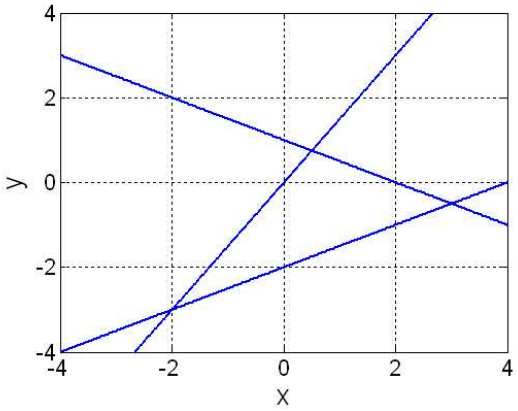
LA.00.01.030

7. We have a system of three linear equations with two unknowns, as plotted in the graph below. How many solutions does this system have?



(a) 0

(b) 1

(c) 2

(d) 3

(e) Infinite

Answer: (a). These three lines do not share a common point of intersection, thus there is no point (x, y) that can satisfy all three equations, so the system has no solutions.

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HHS JG MA232 S08: **50**/0/0/50/0
CHS DP MA232 S08: **92**/0/0/0/8
CC JS MA232 S09: **28**/11/6/56/0
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LA.00.01.035

8. A system of linear equations could *not* have exactly _____ solutions.

   (a) 0

   (b) 1

   (c) 2

   (d) infinite

   (e) All of these are possible numbers of solutions to a system of linear equations.

   Answer: (c). A system of linear equations may have one solution, an infinite number of solutions, or no solutions. If it hasn't already come up, this would be a good time to discuss what each of these cases looks like graphically when we have two equations in two variables.

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   CC HZ MA117 S07: 0/5/**79**/0/11 time 1:15
   CC HZ MA232 S08: 0/4/**78**/0/18 time 1:15
   HHS JG MA232 S08: 0/0/**33**/25/42
   CHS DP MA232 S08: 0/0/**92**/0/8
   CC JS MA232 S09: 0/5/**80**/15/0
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9. The system

$$\begin{aligned} x + y &= 2 \\ 2x + 2y &= 4 \end{aligned}$$

   has an infinite number of solutions. Which of the following describes the set of solutions to this system?

   (a) $x = 1$ and $y = 1$

   (b) $x = 2 - t$ and $y = t$

   (c) x and y could each be anything.

   (d) None of the above

   Answer: (b). If $y = t$, then $x = 2 - y = 2 - t$. This is intended to be used before parameterization of solutions has been discussed, and this question should lead the students into that discussion.

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   CC HZ MA232 S08: 0/**78**/18/4 time 2:50
   HHS JG MA232 S08: 0/**67**/0/33
   CC JS MA232 S09: 0/**90**/5/5

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10. Which of the following options describes the set of solutions to the system below?

$$\begin{aligned} x + y &= 1 \\ x - y &= 0 \\ 2x + y &= 3 \end{aligned}$$

(a) $x = 1 - t$ and $y = t$

(b) $x = 1$ and $y = 1$

(c) no solution exists

(d) None of the above

Answer: (c). We get $y = x$ from the 2nd equation, and thus $x = y = 1/2$ from the first equation. This solution does not satisfy the third equation.

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11. Which of the following options describes the set of solutions to the system below?

$$\begin{aligned} x + y &= 2 \\ 2x - y &= -2 \\ x - 2y &= -4 \end{aligned}$$

(a) $x = t$ and $y = 2 - t$

(b) $x = 0$ and $y = 2$

(c) no solution exists

(d) None of the above

Answer: (b). There is only 1 solution to this system. As students struggle with parameterization, it is good to remind them that not all systems have an infinite number of solutions.

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12. $x = 3 - 2t$ and $y = t$ represent the set of solutions to a system of equations. What line in $\Re^2$ does this set of solutions represent?

(a) $x + 2y = 3$

(b) $x - 2y = 3$

(c) $x + y = 3 - t$

(d) It is impossible to answer this question with the information given.

Answer: (a). We can substitute $y = t$ into the equation for x, giving us $x = 3 - 2y$. Students have a lot of trouble with the idea of parameterizing the solutions to a system, so this question is an attempt to tackle that idea from a different angle.

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HHS JG MA232 S08: **100**/0/0/0

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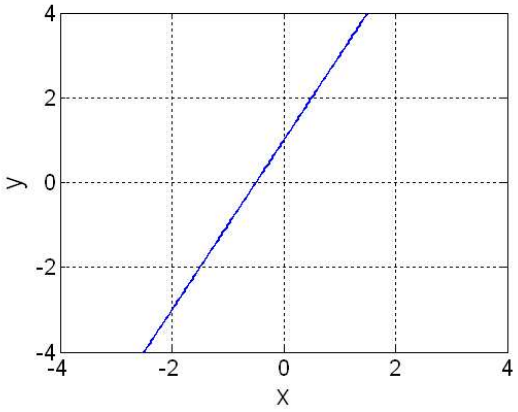
13. The set of solutions to a system of linear equations is plotted below. Which of the following parameterizations represents this solution set?



(a) $x = 2t$ and $y = 4t + 1$

(b) $x = \frac{1}{2}t - \frac{1}{2}$ and $y = t$

(c) $x = t - 1$ and $y = 2t - 1$

(d) $x = t$ and $y = 2t + 1$

(e) All of the above

Answer: (e). All of these parameterizations represent the line $y = 2x+1$, which is plotted. Many students struggle with the idea that there are many ways to parameterize a given line.

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14. A certain mini-golf course does not list their prices. I paid \$26.25 for 3 children and 4 adults. The group in front of me had paid \$25.50 for 6 children and 2 adults. Which system of equations would allow us to determine the prices for children and adults?

(a)

$$\begin{aligned} 3x + 6y &= 26.25 \\ 4x + 2y &= 25.50 \end{aligned}$$

(b)

$$\begin{aligned} 3x + 4y &= 26.25 \\ 6x + 2y &= 25.50 \end{aligned}$$

(c)

$$\begin{aligned} 26.25x + 25.50y &= 51.75 \\ 9x + 6y &= 15 \end{aligned}$$

(d)

$$\begin{aligned} (26.25/3)x + (26.25/4)y &= 0 \\ (25.50/6)x + (25.50/6)y &= 0 \end{aligned}$$

Answer: (b). Here x is the admission price for children, and y is the admission price for adults.

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HHS JG MA232 S08: 0/**100**/0/0

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LA.00.01.070

15. A system of 3 linear equations with 3 variables could not have exactly _____ solutions.

(a) 0

(b) 1

(c) 2

(d) 3

(e) More than one of (a)-(d) are impossible.

(f) All of (a)-(d) are possible numbers of solutions.

Answer: (e). It is not possible to have either 2 or 3 solutions.

CC KC MA232 S07: 0/24/14/0/**33**/29 time 2:30

HC AS MA339 F07: 0/0/9/0/**65**/26

HHS JG MA232 S08: 0/0/0/33/**67**/0

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LA.00.01.080

16. A linear equation with two variables can be geometrically represented as a line in $\Re^2$. How can we best represent a linear equation with three variables?

    (a) As a line in $\Re^2$

    (b) As a line in $\Re^3$

    (c) As a plane in $\Re^3$

    (d) As a volume in $\Re^3$

    Answer: (c). A linear equation in three variables is best represented by a plane in $\Re^3$.

    CHS DP MA232 S08: 0/15/**69**/15
    HHS JG MA232 S08: 17/0/**83**/0

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17. We find that a system of three linear equations in three variables has an infinite number of solutions. How could this happen?

    (a) We have three equations for the same plane.

    (b) At least two of the equations must represent the same plane.

    (c) The three planes intersect along a line.

    (d) The planes represented are parallel.

    (e) More than one of the above are possible.

    Answer: (e). Both (a) and (c) are possible. To help students visualize (c), point out that the planes could be like pages of a book that all intersect at a line along the spine of the book.

    HC AS MA339 F07: 0/0/17/0/**83**

    CC HZ MA232 S08: 57/0/11/0/**32** time 2:00
    HHS JG MA232 S08: 17/0/17/0/**67**

CHS DP MA232 S08: 0/0/23/0/**77**

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LA.00.01.085

18. We consider a system of three linear equations in three variables, and visualize the graph of each equation as a plane in $\Re^3$. Suppose no solutions exist to this system. This means that

    (a) all three planes must be parallel.

    (b) at least two of the planes must be parallel.

    (c) at least two of the equations represent the same plane.

    (d) none of these planes ever intersects with another.

    (e) None of the above

    Answer: (e). If there is no solution, then there are no points where all three planes intersect simultaneously, and this does not require any of the planes to be parallel. For example, students could visualize a triangular prism of the shape often used to mail posters: Each plane intersects with each of the other two planes along a line, but there is no point common to all three planes. Or visualize two planes whose intersection is a line, and another plane parallel to that line but offset.

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    HHS JG MA232 S08: 17/50/0/17/**17**
    CHS DP MA232 S08: 15/69/0/0/**15**

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    LA.00.01.086

19. We have a system of four linear equations in four variables. We can think about the graph of each equation as a 3-dimensional volume in $\Re^4$. Which of the following could geometrically represent the solutions to this system?

    (a) A point in $\Re^4$

    (b) A line in $\Re^4$

    (c) A plane in $\Re^4$

    (d) A three dimensional volume in $\Re^4$

    (e) All of the above

    (f) None of the above

Answer: (e). Just as a system of three linear equations in three variables can have solutions geometrically represented by a point, a line, or a plane in $\Re^3$, a system of four linear equations in four variables can have solutions geometrically represented by a point, a line, a plane, or a three dimensional volume in $\Re^4$.

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LA.00.01.087

20. How can we geometrically represent the parametric equations $x = 2t$, $y = -t + 1$, and $z = t$?

    (a) A line in $\Re^2$
    (b) A line in $\Re^3$
    (c) A plane in $\Re^3$
    (d) A volume in $\Re^3$

    Answer: (b). Each value of t gives us a different point in $\Re^3$, and as t changes, each of the coordinates (x, y, z) vary in a linear manner, thus these equations can be represented as a line in $\Re^3$.

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21. A system of 5 linear equations and 7 variables could not have exactly _____ solutions.

    (a) 0
    (b) 1
    (c) infinite
    (d) More than one of these is impossible.
    (e) All of these are possible numbers of solutions.

    Answer: (b). It is not possible to have one unique solution if you have more variables than equations. There is either no solution, or there are infinite solutions.

    CC KC MA232 S07: 14/**24**/0/14/48 time 3:00

    HHS JG MA232 S08: 0/**0**/0/0/100
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22. A system of 8 linear equations and 6 variables could not have exactly _____ solutions.

(a) 0

(b) 1

(c) infinite

(d) More than one of these is impossible.

(e) All of these are possible numbers of solutions.

Answer: (e). If you have more equations than unknowns, then it is possible that there is no solution, and it is possible that there is one solution, and it is possible that there are infinite solutions.

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23. Howard's store sells three blends of flour: standard, extra wheat, and extra soy. Each is a blend of whole wheat flour and soy flour, and the table below shows how many pounds of each type of flour is needed to make one pound of each blend.

whole wheat flour	soy flour	
0.5	0.5	Standard Blend
0.8	0.2	Extra Wheat
0.3	0.7	Extra Soy

A customer comes in who wants one pound of a blend that is 60% wheat and 40% soy. Which system of equations below would allow us to solve for the amount of each blend needed to fulfill this special request?

(a)

$$\begin{aligned} 0.5x_1 + 0.5x_2 &= 1 \\ 0.8x_1 + 0.2x_2 &= 1 \\ 0.3x_1 + 0.7x_2 &= 1 \end{aligned}$$

(b)

$$\begin{aligned} 0.5x_1 + 0.5x_2 &= 0.6 \\ 0.8x_1 + 0.2x_2 &= 0.4 \\ 0.3x_1 + 0.7x_2 &= 0 \end{aligned}$$

(c)

$$\begin{aligned} 0.5x_1 + 0.8x_2 + 0.3x_3 &= 1 \\ 0.5x_1 + 0.2x_2 + 0.7x_3 &= 1 \end{aligned}$$

(d)

$$\begin{aligned} 0.5x_1 + 0.8x_2 + 0.3x_3 &= 0.6 \\ 0.5x_1 + 0.2x_2 + 0.7x_3 &= 0.4 \end{aligned}$$

Answer: (d). Since we wish to know how much of each blend we should use, we need three variables here: one for each blend. In answer (d), x_1 represents the amount of Standard Blend to use (in pounds), x_2 represents the amount of Extra Wheat Blend, and x_3 represents the amount of Extra Soy Blend. The first equation represents the amount of whole wheat flour in the new mixture, while the second equation represents the amount of soy flour in the mixture. This is the first in a series of two questions. Note: although it may at first appear that we need the equation $x_1 + x_2 + x_3 = 1$ to assure that we end up with one pound of the requested blend, this equation is not necessary because the requirement of one pound is used to compute the right-hand-sides of the equations. That is, we need 60% of one pound, or 0.6 pounds, to be whole wheat flour. A later question follows up on this point by asking about the units of the 0.6.

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24. In the previous question you set up a system of equations so that you could find the amount of each blend needed to make a new mixture. How many solutions must this system have? (You do not need to solve the system.)

    (a) 0

    (b) 1

    (c) 2

    (d) 3

    (e) Infinite

    Answer: (e). Since there are more variables than equations, this system either has an infinite number of solutions or no solution. Since the planes described by the two equations are not parallel, the system must have an infinite number of solutions. This could be followed up with a question to the class about how we might decide which solution to use (maybe the store is out of one blend, or maybe the prices vary substantially between blends). That particular point will be followed up in detail with questions in the Gaussian Elimination section. This is the second in a series of two questions.

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25. The previous two questions dealt with the system

$$\begin{aligned} 0.5x_1 + 0.8x_2 + 0.3x_3 &= 0.6 \\ 0.5x_1 + 0.2x_2 + 0.7x_3 &= 0.4 \end{aligned}$$

In the context given, what quantity or unit does 0.6 represent?

(a) pounds

(b) %

(c) pounds^2

(d) pounds per %

(e) 0.6 does not have units

Answer: (a). 0.6 comes from taking 60% of the desired one pound of new blend, so has units of pounds.

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