Gaussian Elimination

1. Which of the following operations on an augmented matrix could change the solution set of a system?

   (a) Interchanging two rows
   (b) Multiplying one row by any constant
   (c) Adding one row to an other
   (d) Adding a multiple of one row to an other
   (e) None of the above
   (f) More than one of the above (which ones?)

   Answer: (b). If we multiply a row by zero, then we may change the solution set.

   by Mathilde Lahaye-Hitier

   LA.00.03.005

2. Which of the following matrices is NOT row equivalent to the one below? In other words, which matrix could you NOT get from the matrix below through elementary row operations?

$$\begin{bmatrix} 2 & 1 & 3 & 1 \\ 0 & 1 & 3 & 4 \\ 1 & 2 & 0 & 4 \end{bmatrix}$$

   (a)
$$\begin{bmatrix} 2 & 4 & 0 & 8 \\ 0 & 1 & 3 & 4 \\ 2 & 1 & 3 & 1 \end{bmatrix}$$

   (b)
$$\begin{bmatrix} 2 & 1 & 3 & 1 \\ 0 & 1 & 3 & 4 \\ 1 & 3 & 3 & 8 \end{bmatrix}$$

   (c)
$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 0 & 3 & 4 \\ 2 & 1 & 0 & 4 \end{bmatrix}$$

(d) More than one of the above

(e) All are possible through elementary row operations.

Answer: (c). Here we have interchanged columns, which is not an elementary row operation. As a follow up question, ask how (a) and (b) were generated from the original matrix.

by Carroll College MathQuest

LA.00.03.010

CC KC MA232 S07: 10/10/**40**/25/15 time 4:00
HC AS MA339 F07: 0/0/**77**/18/5
MCC KS MAT210-60 F10: 7/0/**7**/71/14 n = 14
MCC KS MAT210-01 F10: 12/0/**50**/38/0 n = 8 shared clickers
HC AS MA339 F11: 0/16.67/**50**/27.78/5.56/0 ,

3. Which of the following matrices is row equivalent to the one below? In other words, which matrix could you get from the matrix below through elementary row operations?

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a)

$$\begin{bmatrix} 2 & 5 & 7 \\ 0 & 1 & 3 \\ 4 & 8 & 1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 3 & 1 & 5 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} -3 & 1 & 3 \\ -2 & 1 & 0 \\ 3 & 9 & 2 \end{bmatrix}$$

(d) More than one of the above

(e) All are possible through elementary row operations.

Answer: (e). This is intended to be asked before we have introduced the identity matrix, to help them discover some of its properties: In this case, any of these matrices can be created from the identity matrix through elementary row operations.

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LA.00.03.020

CC KC MA232 S07: 5/0/0/15/**75** time 3:30
CC TM MA117 S12: 18/14/0/5/**64**
CC TM MA117 F11: 18/14/0/5/**64**

4. Which of the following matrices is NOT row equivalent to the one below? In other words, which matrix could you NOT get from the matrix below through elementary row operations?

$$\begin{bmatrix} 6 & 0 & 4 & 7 \\ 2 & 0 & 1 & 9 \\ 5 & 0 & 3 & 5 \end{bmatrix}$$

(a)
$$\begin{bmatrix} 12 & 0 & 8 & 14 \\ 2 & 0 & 1 & 9 \\ 1 & 0 & 1 & 2 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 12 & 0 & 8 & 14 \\ 0 & 0 & 1 & -20 \\ 2 & 1 & 3 & 0 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 6 & 0 & 4 & 7 \\ 2 & 0 & 1 & 9 \\ 7 & 0 & 4 & 14 \end{bmatrix}$$

(d) All are possible through elementary row operations.

Answer: (b). The original matrix has a column of zeros, so we could never get any nonzero entry in this column through elementary row operations, thus making matrix (b) impossible.

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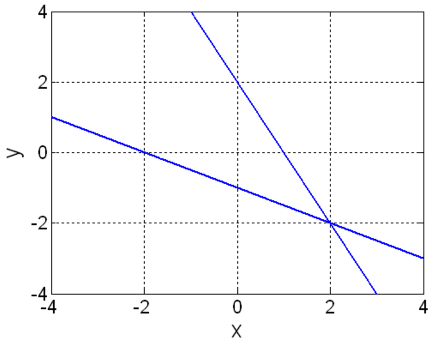
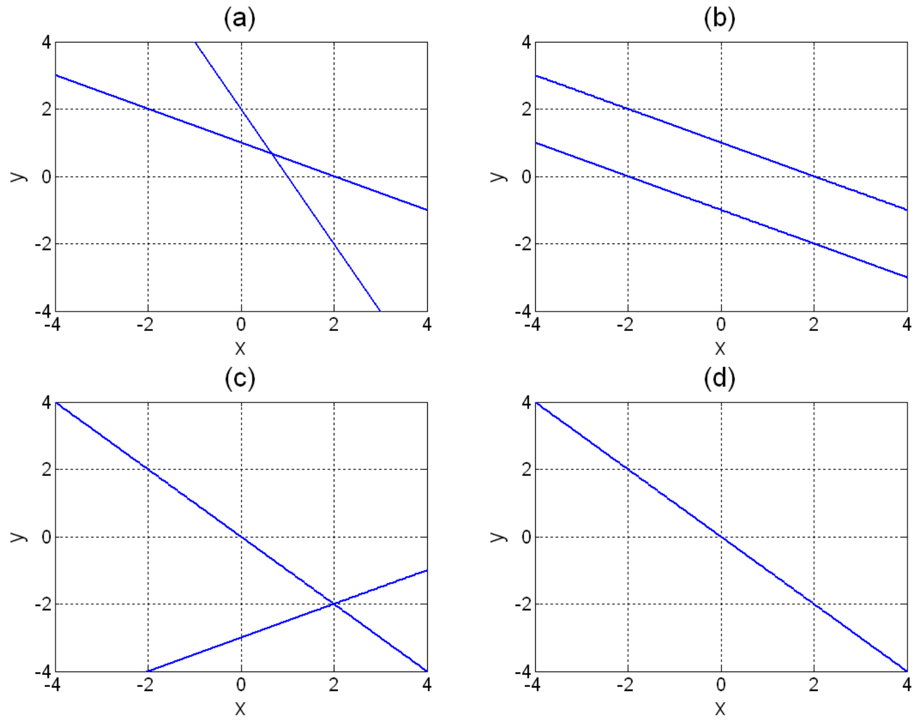
LA.00.03.030

CC KC MA232 S07: 5/**80**/10/5 time 5:00
CC TM MA117 S12: 5/**64**/27/5
CC TM MA117 F11: 0/**75**/15/10

5. A linear system of equations is plotted below. We create an augmented matrix to represent this linear system, then perform a series of elementary row operations. Which of the following graphs could represent the result of these row operations?





Answer: (c). When we perform elementary row operations on an augmented matrix, the solution to the linear system it represents does not change. Thus, when we graph any other row equivalent system, the lines must cross at the same point.

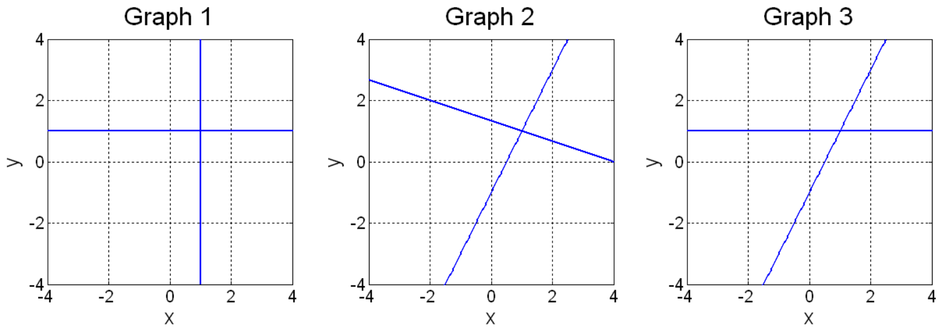
by Carroll College MathQuest

LA.00.03.035

CC HZ MA232 S08: 0/4/**59**/33 time 2:40
CC HZ MA232 S10: 17/3/**69**/10 time 4:00
CC HZ MA232 S11: 0/0/**83**/17 time 2:30
CC HZ MA232 S12: 0/0/**90**/10 time 3:00
CC HZ MA232 S13: 8/0/**64**/28 time 2:45

6. We have a system of two linear equations and two unknowns which we solve by performing Gaussian elimination on an augmented matrix. Along the way we create the

graphs below, showing geometrical representations of the initial system, the system at an intermediate step in the row reduction process, and the system after it has been put into reduced row echelon form. Put these graphs in order, starting with the initial system and ending with the system in reduced row echelon form.



(a) Graph 2, Graph 3, Graph 1

(b) Graph 1, Graph 3, Graph 2

(c) Graph 1, Graph 2, Graph 3

(d) Graph 2, Graph 1, Graph 3

(e) Graph 3, Graph 2, Graph 1

Answer: (a). When we perform the Gaussian elimination process on a linear system with one solution, the lines represented by the equations pivot around their point of intersection to make them parallel to the principal axes. Thus Graph 2 must be our starting system, Graph 3 is our intermediate system, and Graph 1 represents the final system in reduced row echelon form.

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LA.00.03.037

CC HZ MA232 S08: **100**/0/0/0/0 time 2:15
CC HZ MA232 S10: **86**/11/0/4/0 time 2:00
CC HZ MA232 S11: **83**/0/6/6/0 time 1:30
KC MS MA224 F09: **100**/0/0/0/0 time 2:20
KC MS MA224 F10: **100**/0/0/0/0 time 2:00
CC HZ MA232 S12: **100**/0/0/0/0 time 2:30
CC HZ MA232 S13: **88**/8/0/0/0 time 1:15

7. What is the value of a so that the linear system represented by the following matrix would have infinitely many solutions?

$$\begin{bmatrix} 2 & 6 & 8 \\ 1 & a & 4 \end{bmatrix}$$

(a) $a = 0$

(b) $a = 2$
(c) $a = 3$
(d) $a = 4$
(e) This is not possible.
(f) More than one of the above

Answer: (c). If we choose $a = 3$ then the rows are just multiples of each other, so these represent two equations for the same line, meaning that the linear system would have infinite solutions.

by Carroll College MathQuest

LA.00.03.040
CC TM MA117 S12: 4/4/**74**/7/4/7
CC HZ MA232 S13: 4/0/**96**/0/0/0 Review
CC TM MA117 F11: 4/0/**96**/0/0/0

8. We start with a system of two linear equations in two variables and we translate this system into an augmented matrix M. After performing Gaussian elimination, putting this matrix into reduced row echelon form, we get the matrix R which tells us that this system has no solution. How could we geometrically represent the linear equations contained in the rows of the augmented matrix R?

   (a) We can represent the equations of R as two parallel lines.
   (b) We can represent the equations of R as two lines that may not be parallel.
   (c) We can represent the equations of R as a single line.
   (d) The equations of R cannot be represented geometrically.

   Answer: (d). When take an augmented matrix representing a linear system with no solution and we put it into reduced row echelon form, we get a row with zeros in the coefficient columns and a non-zero entry in the final column. This equation has no solution, so it cannot be graphed or represented geometrically: It is the empty set.

   by Carroll College MathQuest

   LA.00.03.045

   SFCC GG MA220 Su10: 83/0/0/**17** "review, one clicker per pair"

9. We start with a system of three linear equations in three variables and we translate this system into an augmented matrix M. After performing Gaussian elimination, putting this matrix into reduced row echelon form, we get the matrix R which tells us that this system has no solution. How could we best geometrically represent the linear equations contained in the rows of the augmented matrix M?

(a) We can represent the equations of M as three parallel lines.

(b) We can represent the equations of M as three parallel planes.

(c) We can represent the equations of M as three planes, where at least two must be parallel.

(d) We can represent the equations of M as three planes, where none of the planes ever intersects with another.

(e) We can represent the equations of M as three planes, which do not share any points in common.

(f) The equations of M cannot be represented geometrically.

Answer: (e). Because this system has no solution, we know that the original system of equations represented by M must not share any points in common, and this does not require any of the planes to be parallel.

by Carroll College MathQuest

LA.00.03.047

KC MS MA224 F09: 0/0/32/8/**60** time 5:00 "Gaussian elimination is referred to in the Poole text as Gauss-Jordan Elimination! (Poole uses ""Gaussian Elim"" to refer to a weaker version of the process.)"
KC MS MA224 F10: 0/14/73/0/**14** time 3:00

10. Howard's store sells three blends of flour: standard, extra wheat, and extra soy. Each is a blend of whole wheat flour and soy flour, and the table below shows how many pounds of each type of flour is needed to make one pound of each blend.

whole wheat flour	soy flour	
0.5	0.5	Standard Blend
0.8	0.2	Extra Wheat
0.3	0.7	Extra Soy

A customer comes in who wants one pound of a blend that is 60% wheat and 40% soy. We can solve the following system of equations to determine the amount of Standard Blend (x_1), Extra Wheat Blend (x_2), and Extra Soy Blend (x_3) needed to create this special mixture.

$$\begin{aligned} 0.5x_1 + 0.8x_2 + 0.3x_3 &= 0.6 \\ 0.5x_1 + 0.2x_2 + 0.7x_3 &= 0.4 \end{aligned}$$

If we form an augmented matrix for this system, the reduced row echelon form is $R = \begin{bmatrix} 1 & 0 & 5/3 & 2/3 \\ 0 & 1 & -2/3 & 1/3 \end{bmatrix}$.

If the store is out of Extra Soy Blend, how much of each of the other blends is needed?

(a) 2/3 pound of Standard Blend and 1/3 pound of Extra Wheat Blend
(b) 5/3 pound of Standard Blend and 2/3 pound of Extra Wheat Blend
(c) There are an infinite number of options for the amounts of Standard and Extra Wheat Blend.
(d) It is not possible to create this mixture without Extra Soy Blend.

Answer: (a). If the store is out of Extra Soy Blend, we can set $x_3 = 0$. Effectively, then, we can ignore the third column of the matrix R, so we now easily read off the unique solution of 2/3 pound of Standard Blend and 1/3 pound of Extra Wheat Blend. This question is a follow-up to a series of questions in the Systems of Equations section, and it is the first in a series of three questions in this section.

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LA.00.03.050

CC TM MA117 S12: **62**/8/4/27

11. Referring to the previous question, if the store is out of Extra Wheat Blend (x_2), how much of each of the other blends is needed to make the special mixture?

(a) 2/3 pound of Standard Blend and 1/3 pound of Extra Soy Blend
(b) 1/6 pound of Standard Blend and 1/2 pound of Extra Soy Blend
(c) There are an infinite number of options for the amounts of Standard Blend and Extra Wheat Blend.
(d) It is not possible to create this mixture without Extra Wheat Blend.

Answer: (d). Working from the matrix R, we see that with $x_2 = 0$ we must have $(-2/3x_3 = 1/3)$, or $x_3 = -1/2$. Since this is the unique solution and it is infeasible when we put it in context, we conclude that it is not possible to create this mixture without Extra Wheat Blend. Alternatively, we can go back to the original table describing the blends and determine that we cannot create a mixture that has 60% soy from two mixtures whose highest percentage of soy is 50%. This is the second in a series of three questions.

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LA.00.03.060

CC TM MA117 S12: 48/28/4/**20**

12. Referring to the previous two questions, what values are realistic for x_3 in this context?

(a) x_3 can be any value.

(b) $x_3 \geq 0$

(c) $\frac{2}{3} \leq x_3 \leq \frac{5}{3}$

(d) $-\frac{1}{2} \leq x_3 \leq \frac{2}{5}$

(e) $0 \leq x_3 \leq \frac{2}{5}$

Answer: (e). Writing out the equations represented by matrix R we have $x_1 = (2/3) - (5/3)x_3$ and $x_2 = (1/3) + (2/3)x_3$. We know a negative value for a variable would not be realistic, so the equation for x_1 tells us that $(2/3) - (5/3)x_3 \geq 0$. Solving this for x_3 we obtain $x_3 \leq 2/5$, and we also know we need $x_3 \geq 0$. This could be followed up by asking the students to define constraints on the other variables. This is the third in a series of three questions.

by Carroll College MathQuest

LA.00.03.070

13. Let R be the reduced row echelon form of a $n \times n$ matrix A. Then

 (a) R is the identity.

 (b) R has at least one row of zeros.

 (c) None of the above.

 (d) All of the above are possible but there exist also other possibilities.

 (e) The two possibilities above are the only ones.

 (f) We can't tell without having the matrix A.

Answer: (e).

by Mathilde Lahaye-Hitier

LA.00.03.080