

Linear Combinations

1. If $u = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ and $v = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$, what is $2u - 3v$?

(a) $\begin{bmatrix} -4 \\ 4 \\ 23 \end{bmatrix}$

(b) $\begin{bmatrix} 8 \\ 4 \\ -7 \end{bmatrix}$

(c) $\begin{bmatrix} 8 \\ 4 \\ 23 \end{bmatrix}$

(d) $\begin{bmatrix} 7 \\ 6 \\ 2 \end{bmatrix}$

Answer: (b). $2u - 3v = \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix} - \begin{bmatrix} -6 \\ 0 \\ 15 \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ -7 \end{bmatrix}$. This should be a simple question, but it can be used to introduce the idea of linear combinations.

by Carroll College MathQuest

LA.00.05.010

CC HZ MA232 S08: 25/**71**/4/0 time 2:05

CC KC MA334 S08: 9/**91**/0/0 time 1:30

CC KC MA334 S09: 0/**92**/8/0 time 1:30

CC HZ MA232 S10: 7/**93**/0/0 time 2:00

CC KC MA334 S10: 15/**85**/0/0

SFCC GG MA220 Su10: 0/**88**/12/0 "warm-up, one clicker per pair"

CC HZ MA232 S11: 0/**94**/0/0 time 2:00

WW JD MA289 W11: 10/**86**/3/0

CC KC MA334 S12: 8/**92**/0/0

CC HZ MA232 S13: 4/**96**/0/0 time 1:15

CC KC MA334 S13: 0/**100**/0/0

2. Write $z = \begin{bmatrix} -5 \\ 3 \\ 16 \end{bmatrix}$ as a linear combination of $x = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$ and $y = \begin{bmatrix} -3 \\ 2 \\ 6 \end{bmatrix}$.

- (a) $z = -5x$
- (b) $z = -2x + y$
- (c) $z = x + 2y$
- (d) $z = 2x + y$
- (e) z cannot be written as a linear combination of x and y .
- (f) None of the above

Answer: (c). $x + 2y = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix} + \begin{bmatrix} -6 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 16 \end{bmatrix}$. Be sure to ask students how they came up with their answers on this one. Most will likely test the answers given to find the right one. Ask how they could do this if it wasn't multiple choice.

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LA.00.05.020

CC HZ MA232 S08: 0/7/**89**/0/0/4 time 2:05

CC KC MA334 S08: 0/0/**91**/0/9/0 time 2:20

CC KC MA334 S09: 0/0/**100**/0/0/0 time 1:30

CC JS MA232 S09: 0/5/**90**/0/5/0

CC HZ MA232 S10: 0/7/**83**/0/7/3 time 2:40

CC KC MA334 S10: 0/0/**100**/0/0/0 time 1:30

SFCC GG MA220 Su10: 0/0/**100**/0/0/0 "warm-up, one clicker per pair"

CC HZ MA232 S11: 0/0/**94**/6/0/0 time 2:45

WW JD MA289 W11: 4/4/**75**/4/11/4

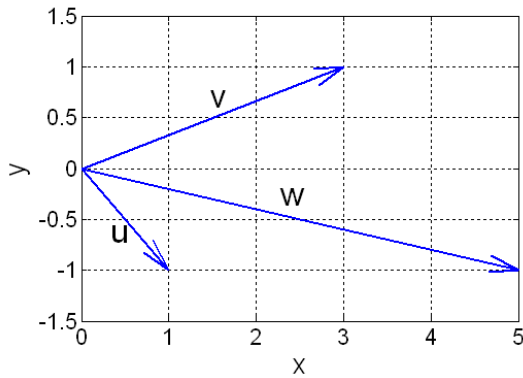
CC HZ MA232 S12: 0/5/**95**/0/0/0 time 3:00

CC KC MA334 S12: 0/15/**85**/0/0/0

CC HZ MA232 S13: 0/4/**92**/0/4/0 time 2:00

CC KC MA334 S13: 0/8/**88**/0/0/4 time 1:30

3. Write the vector w as a linear combination of u and v .



- (a) $w = 2u + v$
- (b) $w = u + v$
- (c) $w = -u + v$
- (d) $w = u - v$
- (e) w cannot be written as a linear combination of u and v .

Answer: (a). $u = (1, -1)$, $v = (3, 1)$, and $w = (5, -1)$, so $w = 2u + v$. We can see that graphically, if we start with vector v , then put down two copies of vector u head to tail, starting at the end of v , we will reach the point at the head of vector w .

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LA.00.05.025

HC AS MA339 F07: **91**/0/0/0/9

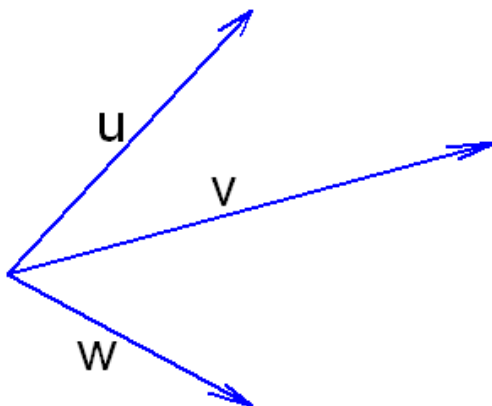
SFCC GG MA220 Su10: **100**/0/0/0/0 "review, one clicker per pair"

WW JD MA289 W11: **86**/3/0/7/3

HC AS MA339 F11: **76.47**/11.76/5.88/0/5.88/0 ,

CC KC MA334 S13: **100**/0/0/0/0/0 time 1:25

4. Write the vector w as a linear combination of u and v .



- (a) $w = 2u + v$
- (b) $w = u + v$
- (c) $w = -u + v$
- (d) $w = u - v$
- (e) w cannot be written as a linear combination of u and v .

Answer: (c). If we start with the vector v , and add the vector $-u$ to it, head to tail, we find that we have reached the point at the head of the vector w .

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LA.00.05.027

CC JS MA232 S09: 0/5/**95**/0/0

5. Write $z = \begin{bmatrix} 2 \\ -2 \\ 6 \end{bmatrix}$ as a linear combination of $x = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$ and $y = \begin{bmatrix} 4 \\ -5 \\ 7 \end{bmatrix}$.

- (a) $z = x + y$
- (b) $z = -x + y$
- (c) $z = 3x + 2y$
- (d) $z = -3x + y$
- (e) z cannot be written as a linear combination of x and y .
- (f) None of the above

Answer: (e). If students set up a system of equations to find the coefficients for the linear combination, they should find that the system has no solutions. Alternatively students may notice that, except for the third component, $z = x + y$, but we cannot get the third component this way. This lays the foundation for later discussing linear independence and spanning sets.

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LA.00.05.030

CC HZ MA232 S08: 11/0/0/7/**68**/14 time 2:40

CC KC MA334 S08: 27/0/0/0/**73**/0 time 2:00

CC KC MA334 S09: 21/0/0/7/**50**/21 time 2:00

CC HZ MA232 S10: 10/3/0/0/**83**/3 time 3:00

CC KC MA334 S10: 0/0/0/0/**85**/15 time 2:30

CC HZ MA232 S11: 19/0/0/6/**69**/6 time 3:00

CC HZ MA232 S12: 0/0/0/0/**90**/10 time 4:30

CC HZ MA232 S13: 36/0/0/4/**56**/4 time 3:45 Review

CC KC MA334 S13: 12/4/0/0/**76**/8 time 3:00

6. Suppose we have the vectors $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$. Which of the following is *not* a linear combination of these?

(a) $\begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix}$

(b) $\begin{bmatrix} 8 \\ 0 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 8 \\ 1 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$

(e) $\begin{bmatrix} 40 \\ 5 \\ 15 \end{bmatrix}$

(f) More than one of the above is not a linear combination of the given vectors.

Answer: (b). The vector in answer (b) is not a linear combination of the two given vectors because its second component is zero, implying that it must be a multiple of the second vector. Since it is not such a multiple, it cannot be a linear combination of the given vectors. Be sure to ask students for their thought process on this one, as coming up with different ways of looking at linear combinations is part of the intent of this question. If we call the vectors in the problem x and y , then answer (a) is $2x$, answer (c) is $x + y$, answer (d) is $y - x$, and answer (e) is $5(x + y)$.

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LA.00.05.040

CC KC MA334 S08: 0/**64**/0/0/0/36 time 2:30

CC KC MA334 S09: 0/**100**/0/0/0/0 time 3:00

CC KC MA334 S10: 0/**100**/0/0/0/0 time 2:30

WW JD MA289 W11: 4/**71**/4/4/0/18

CC KC MA334 S12: 0/**85**/0/0/0/15

CC KC MA334 S13: 0/**88**/0/0/0/12 time 3:00

7. Suppose we have the vectors $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$. Which of the following is true?

- (a) Every vector in $\mathfrak{R}^3$ can be written as a linear combination of these vectors.
- (b) Some, but not all, vectors in $\mathfrak{R}^3$ can be written as a linear combination of these vectors.
- (c) Every vector in $\mathfrak{R}^2$ can be written as a linear combination of these vectors.
- (d) More than one of the above is true.
- (e) None of the above are true.

Answer: (b). The previous question has given us a vector which cannot be written as a linear combination of these two, as well as several vectors which can be written as a linear combination of these. However, students may think that since we have two vectors, we can obtain any vector in $\mathfrak{R}^2$.

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LA.00.05.050

HC AS MA339 F07: 13/**83**/0/4/0

CC KC MA334 S08: 0/**64**/18/9/9 time 3:00

CC KC MA334 S09: 0/**64**/14/22/0 time 2:30

CC JS MA232 S09 19/**67**/5/9/0

CC KC MA334 S10: 0/**77**/0/23/0 time 2:30

CC KC MA334 S12: 0/**77**/15/8/0

CC KC MA334 S13: 0/**92**/0/4/4

8. Which of the following vectors can be written as a linear combination of the vectors $(1, 0)$ and $(0, 1)$?
- (a) $(2, 0)$
 - (b) $(-3, 1)$
 - (c) $(0.4, 3.7)$
 - (d) All of the above

Answer: (d). This question gives students an early look at the idea of spanning sets. Follow this up either by asking students if there is any vector in $\mathfrak{R}^2$ that *cannot* be written as a linear combination of these two vectors, or by asking the next question.

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LA.00.05.060

CC KC MA334 S08: 0/0/0/**100** time 1:15

CC KC MA334 S12: 0/8/0/**92**

9. How do you describe the set of all linear combinations of the vectors $(1, 0)$ and $(0, 1)$?

- (a) A point
- (b) A line segment
- (c) A line
- (d) $\mathbb{R}^2$
- (e) $\mathbb{R}^3$

Answer: (d). This question gives students an early look at the idea of spanning sets, using geometric thinking. This is also a good lead-in to a discussion of affine and convex combinations, for those who wish to go that direction.

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LA.00.05.070

SFCC GG MA220 Su10: 0/12/0/88/0 "warm-up, one clicker per pair"

10. Which of the following vectors can be written as a linear combination of the vectors $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$?

- (a) $(0, 2, 0)$
- (b) $(-3, 0, 1)$
- (c) $(0.4, 3.7, -1.5)$
- (d) All of the above

Answer: (d). This question gives students an early look at the idea of spanning sets. Follow this up either by asking students if there is any vector in $\mathbb{R}^3$ that *cannot* be written as a linear combination of these two vectors, or by using the next question.

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LA.00.05.080

CC KC MA334 S09: 0/7/0/93 time 1:30

SFCC GG MA220 Su10: 0/0/0/100 "review, one clicker per pair"

11. How do you describe the set of all linear combinations of the vectors $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$?

- (a) A point
- (b) A line segment
- (c) A line

- (d) $\mathbb{R}^2$
- (e) $\mathbb{R}^3$

Answer: (e). This question gives students an early look at the idea of spanning sets, using geometric thinking. This is also a good lead-in to a discussion of affine and convex combinations, for those who wish to go that direction.

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LA.00.05.090

HC AS MA339 F07: 0/0/0/0/**100**

CC KC MA334 S09: 12/16/12/0/**60** time 2:00

CC KC MA334 S10: 8/8/8/0/**76** time 1:30

WW JD MA289 W11: 17/3/7/3/**69**

CC KC MA334 S13: 8/0/0/0/**92** time 2:30

12. How do you describe the set of all linear combinations of the vectors $(1, 2, 0)$ and $(-1, 1, 0)$?
- (a) A point
 - (b) A line
 - (c) A plane
 - (d) $\mathbb{R}^2$
 - (e) $\mathbb{R}^3$

Answer: (c). All vectors in $\mathbb{R}^3$ cannot be written as a linear combination of only two vectors. Instead the set of linear combinations of these two vectors describes a plane in $\mathbb{R}^3$.

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LA.00.05.095

CC KC MA334 S09: 0/50/**21**/0/28 time 2:00

CC KC MA334 S10: 15/23/**62** time 1:45

SFCC GG MA220 Su10: 0/0/**75**/25/0 "warm-up, one clicker per pair"

CC KC MA334 S12: 8/15/**69**/0/8

CC KC MA334 S13: 0/38/**54**/8/0 time 2:30

13. Let z be any vector from $\mathbb{R}^3$. If we have a set V of unknown vectors from $\mathbb{R}^3$, how many vectors must be in V to guarantee that z can be written as a linear combination of the vectors in V ?
- (a) 2

- (b) 3
- (c) 4
- (d) It is not possible to make such a guarantee.

Answer: (d). The vectors of V could all be multiples of one another, and if z is not a multiple of those vectors, then no number of vectors in V will allow us to write z as a linear combination of vectors from V . This question leads into the ideas of linear independence, and later, bases.

by Carroll College MathQuest

LA.00.05.100

CC KC MA334 S08: 0/46/9/**46** time 2:00

CC KC MA334 S09: 0/20/7/**73** time 2:40

CC KC MA334 S10: 38/38/8/**16** time 2:00

SFCC GG MA220 Su10: 0/38/12/**50** "warm-up, one clicker per pair"

WW JD MA289 W11: 0/0/0/**100**

CC KC MA334 S12: 15/62/8/**15**

HC AS MA339 F11: 0/22.22/22.22/**55.56**

CC KC MA334 S13: 8/73/15/**4** time 2:45

14. Suppose y and z are both solutions to $Ax = b$. **True or False** All linear combinations of y and z also solve $Ax = b$. (You should be prepared to support your answer with either a proof or a counterexample.)
- (a) True, and I am very confident
 - (b) True, but I am not very confident
 - (c) False, but I am not very confident
 - (d) False, and I am very confident

Answer: False. A counterexample will demonstrate this. Suppose we have the very simple system $2x_1 + x_2 = 4$. (Add a multiple of that equation to the system if the students prefer to have two equations for their two unknowns.) The vectors $(2, 0)$ and $(0, 4)$ both solve this equation, but if we add them together we find that $(2, 4)$ is not a solution to the system.

by Carroll College MathQuest

LA.00.05.110

CC KC MA334 S09: 33/**67** time 2:40

WW JD MA289 W11: 13/43/30/**13**

15. Suppose y and z are both solutions to $Ax = 0$. **True or False** All linear combinations of y and z also solve $Ax = 0$. (You should be prepared to support your answer with either a proof or a counterexample.)
- (a) True, and I am very confident
 - (b) True, but I am not very confident
 - (c) False, but I am not very confident
 - (d) False, and I am very confident

Answer: True. Proof: $A(\lambda_1 y + \lambda_2 z) = \lambda_1 Ay + \lambda_2 Az = 0$.

by Carroll College MathQuest

LA.00.05.120

CC KC MA334 S08: **64**/36 time 2:30

CC KC MA334 S09: **88**/12 time 2:00

WW JD MA289 W11: **57**/30/7/7

16. Lucinda owns two ice cream parlors. The first ice cream shop sells 5 gallons of vanilla ice cream and 8 gallons of chocolate ice cream each day. The daily sales at the second store are 6 gallons of vanilla ice cream and 10 gallons of chocolate ice cream. The daily sales at stores one and two can be represented by the vectors $s_1 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ and $s_2 = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$, respectively. In this context, what interpretation can be given to the vector $15s_1$?
- (a) $15s_1$ shows the number of people that can be served with 15 gallons of vanilla ice cream.
 - (b) $15s_1$ shows the gallons of vanilla and chocolate ice cream sold by store 1 in 15 days.
 - (c) $15s_1$ gives the total revenue from selling 15 gallons of ice cream at store 1.
 - (d) $15s_1$ represents the number of days it will take to sell 15 gallons of ice cream at store 1.

Answer: (b). Although probably not very difficult, this question puts linear combinations into an applied setting.

by Carroll College MathQuest, based on Exercise 27, section 1.3 of Lay's Linear Algebra, 3rd edition

LA.00.05.130

17. Lucinda owns two ice cream parlors. The first ice cream shop sells 5 gallons of vanilla ice cream and 8 gallons of chocolate ice cream each day. The daily sales at the second store are 6 gallons of vanilla ice cream and 10 gallons of chocolate ice cream. The daily sales at stores one and two can be represented by the vectors $s_1 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ and $s_2 = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$, respectively. The stores are run by different managers, and they are not always able to be open the same number of days in a month. If store 1 is open for c_1 days in March, and store 2 is open for c_2 days in March, which of the following represents the total sales of each flavor of ice cream between the two stores?

- (a) $c_1 s_1 + c_2 s_2$
- (b) $\begin{bmatrix} 5 & 6 \\ 8 & 10 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$
- (c) $\begin{bmatrix} 5c_1 \\ 8c_1 \end{bmatrix} + \begin{bmatrix} 6c_2 \\ 10c_2 \end{bmatrix}$
- (d) All of the above
- (e) None of the above

Answer: (d). Here students see an application of linear combinations, and they have to equate three different ways of expressing a linear combination.

by Carroll College MathQuest, based on Exercise 27, section 1.3 of Lay's Linear Algebra, 3rd edition

LA.00.05.140

HC AS MA339 F07: 0/0/0/0/100

WW JD MA289 W11: 13/10/0/73/3

18. Lucinda owns two ice cream parlors. The first ice cream shop sells 5 gallons of vanilla ice cream and 8 gallons of chocolate ice cream each day. The daily sales at the second store are 6 gallons of vanilla ice cream and 10 gallons of chocolate ice cream. The daily sales at stores one and two can be represented by the vectors $s_1 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ and $s_2 = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$, respectively. Lucinda is getting ready to close her ice cream parlors for the winter. She has a total of 39 gallons of vanilla ice cream in her warehouse, and 64 gallons of chocolate ice cream. She would like to distribute the ice cream to the two stores so that it is used up before the stores close for the winter. How much ice cream should she take to each store? The stores may stay open for different number of days, but no store may run out of ice cream before the end of the day on which it closes.
- (a) Lucinda should take 3 gallons of each kind of ice cream to store 1 and 4 gallons of each kind to store 2.

- (b) Lucinda should take 3 gallons of vanilla to each store and 4 gallons of chocolate to each store.
- (c) Lucinda should take 15 gallons of vanilla and 24 gallons of chocolate to store 1, and she should take 24 gallons of vanilla and 40 gallons of chocolate to store 2.
- (d) Lucinda should take 15 gallons of vanilla and 32 gallons of chocolate to store 1, and she should take 18 gallons of vanilla and 40 gallons of chocolate to store 2.
- (e) This cannot be done unless ice cream is thrown out or a store runs out of ice cream before the end of the day.

Answer: (c). A calculator may be helpful for this problem. We need to find a linear combination of s_1 and s_2 that will result in the vector $(39, 64)$. The coefficients for the linear combination turn out to be 3 and 4, and these represent the number of days that each store should stay open. If store 1 stays open 3 more days, it will need $3(5) = 15$ gallons of vanilla ice cream and $3(8) = 24$ gallons of chocolate ice cream, while store 2 will need $4(6) = 24$ gallons of vanilla ice cream and $4(10) = 40$ gallons of chocolate ice cream to stay open for 4 more days.

by Carroll College MathQuest, based on Exercise 27, section 1.3 of Lay's Linear Algebra, 3rd edition

LA.00.05.150