MathQuest: Linear Algebra

Matrix Operations

1. What size is this matrix?

$$\begin{bmatrix} 6 & 11 & -2 \\ 23 & 31 & 5 \end{bmatrix}$$

   (a) 2x3

   (b) 3x2

   (c) 6

   Answer: (a). This is a quick check to see if students can determine the size of a matrix.

   CC HZ MA117 S07: **100**/0/0

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2. Let $A = \begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$

   What is $A + B$?

   (a) 71

   (b)
   $$\begin{bmatrix} 6 & 9 \\ 7 & 11 \end{bmatrix}$$

   (c)
   $$\begin{bmatrix} 6 & 11 \\ 23 & 31 \end{bmatrix}$$

   (d)
   $$\begin{bmatrix} 26 & 62 \\ 112 & 268 \end{bmatrix}$$

   (e)
   $$\begin{bmatrix} 4 & 6 & 2 & 5 \\ 20 & 24 & 3 & 7 \end{bmatrix}$$

Answer: (c). This is a quick check to see if students know how to add matrices. This question can be used before introducing the concept to see what knowledge students bring to the course. Answer (a) comes from adding all of the entries of A and B. Answer (d) is the product AB.

CC HZ MA117 S07: 8/0/**92**/0/0

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3. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$ what is A^T?

   (a) $A^T = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$

   (b) $A^T = \begin{bmatrix} 2 & 0 & -2 \\ 3 & -1 & 0 \\ 1 & 3 & 4 \end{bmatrix}$

   (c) $A^T = \begin{bmatrix} -2 & 0 & 4 \\ 0 & -1 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

   (d) $A^T = \begin{bmatrix} 1 & 3 & 4 \\ 3 & -1 & 0 \\ 2 & 0 & -2 \end{bmatrix}$

   Answer: (b). This is a quick check to see if students can find the transpose of a matrix.

   CC HZ MA117 S07: 0/**92**/8/0 time 0:35

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4. If $A = \begin{bmatrix} 4 & 6 \\ 20 & 7 \end{bmatrix}$ what is $5A$?

   (a) $5A = \begin{bmatrix} 9 & 6 \\ 20 & 7 \end{bmatrix}$

   (b) $5A = \begin{bmatrix} 9 & 11 \\ 25 & 12 \end{bmatrix}$

   (c) $5A = \begin{bmatrix} 20 & 6 \\ 20 & 7 \end{bmatrix}$

   (d) $5A = \begin{bmatrix} 20 & 30 \\ 100 & 35 \end{bmatrix}$

Answer: (d). This is a quick check to see if students can multiply a matrix by a scalar.
CC HZ MA117 S07: 0/4/0/**96**

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5. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ then calculate the product AB.

   (a) $AB = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$
   (b) $AB = \begin{bmatrix} 10 & 7 \end{bmatrix}$
   (c) $AB = \begin{bmatrix} 8 & 4 \\ -3 & -2 \end{bmatrix}$
   (d) $AB = \begin{bmatrix} 7 \\ 10 \end{bmatrix}$
   (e) None of the above.
   (f) This matrix multiplication is impossible.

   Answer: (d). This is a quick check of the matrix multiplication process.
   CC HZ MA117 S07: 0/8/40/**4**/0/48 (before intro to multiplication)

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6. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ then calculate the product AB.

   (a) $AB = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$
   (b) $AB = \begin{bmatrix} 10 & 7 \end{bmatrix}$
   (c) $AB = \begin{bmatrix} 8 & 4 \\ -3 & -2 \end{bmatrix}$
   (d) $AB = \begin{bmatrix} 7 \\ 10 \end{bmatrix}$
   (e) None of the above.
   (f) This matrix multiplication is impossible.

   Answer: (d). This is a quick check of the matrix multiplication process.
   CC HZ MA117 S07: 0/8/40/**4**/0/48

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7. Calculate $\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix}$.

   (a) $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & -2 \\ 2 & 5 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 & 0 \\ -6 & 2 \end{bmatrix}$

   (d) None of the above.

   (e) This matrix multiplication is impossible.

   Answer: (b). This is a quick check of the matrix multiplication process.

   CC KC MA232 S07: 0/**77**/0/24 time 2:00

   CC HZ MA117 S07: 0/**80**/5/15/0 time 3:00

   CC HZ MA232 S08: 4/**54**/14/25/4 time 2:40
   HHS JG MA232 S08: 0/**100**/0/0/0

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8. Calculate $\begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$.

   (a) $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & -2 \\ 2 & 5 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 & 0 \\ -6 & 2 \end{bmatrix}$

   (d) None of the above.

   (e) This matrix multiplication is impossible.

   Answer: (a). With the previous problem, this demonstrates the noncommutative nature of matrix multiplication.

   CC KC MA232 S07: **100**/0/0/0/0 time 1:30

   CC HZ MA117 S07: **95**/0/0/5/0 time 1:15

   HHS JG MA232 S08: **100**/0/0/0/0

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9. **True or False** If A and B are square matrices with the same dimensions, then $(A + B) \times (A + B) = A^2 + 2AB + B^2$.

   Answer: (False). When we expand this expression we get $A^2 + AB + BA + B^2$. Good follow-up questions are to ask for an example where the "identity" is not true, then an example where the "identity" is true. They will need to choose matrices where $AB = BA$. This is done easily by choosing one of the matrices to be an identity matrix.

   CC KC MA232 S07: 28/**72** time 2:00

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10. If A and B are both 2x3 matrices, then which of the following is not defined?

    (a) $A + B$

    (b) $A^T B$

    (c) BA

    (d) AB^T

    (e) More than one of the above

    (f) All of these are defined.

    Answer: (c). In order to make matrix multiplication possible the number of columns of the first matrix must equal the number of rows of the second matrix.

    CC KC MA232 S07: 0/0/**83**/0/17/0 time 1:30

    CC HZ MA117 S07: 0/16/**37**/0/32/10 time 1:15

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11. If A is a 2x3 matrix and B is a 3x6 matrix, what size is AB?

    (a) 2x6

    (b) 6x2

    (c) 3x3

    (d) 2x3

    (e) 3x6

(f) This matrix multiplication is impossible.

Answer: (a). This is a quick check on final size of a matrix product. Note that this is a different concept than knowing when the multiplication is possible.

CC HZ MA117 S07: **84**/0/0/16/0/0 time 1:00

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12. In order to compute the matrix product AB, what must be true about the sizes of A and B?

(a) A and B must have the same number of rows.

(b) A and B must have the same number of columns.

(c) A must have as many rows as B has columns.

(d) A must have as many columns as B has rows.

Answer: (d). This question is intended to extend students' conceptualization of matrix multiplication so that they really think of it as a row times a column. This is one level deeper than memorizing that if A is mxn and B is rxs, then we must have $n = r$.

CC HZ MA117 S07: 0/0/16/**84** time 1:10

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13. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 & 2 \\ 1 & 2 & -1 \\ 3 & 1 & 0 \end{bmatrix}$ what is the (3,2)-entry of AB? (You should be able to determine this without computing the entire matrix product.)

(a) 1

(b) 3

(c) 4

(d) 8

Answer: (c). We compute -2*0 + 0*2 + 4*1 = 4. This question, as the previous one, is intended to extend students' conceptualization of matrix multiplication so that they really think of it as a row times a column.

CC HZ MA117 S07: 32/21/**47**/0 time 2:30

CC HZ MA232 S08: 14/4/**75**/7 time 2:30
HHS JG MA232 S08: 0/0/**100**/0

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14. You have a business that sells tables and chairs. You have brown tables and white tables, and corresponding chairs. Your May sales are 4 brown tables, 6 white tables, 20 brown chairs, and 24 white chairs, which is represented by the matrix $M = \begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$. where the first row is tables, the second row is chairs, the first column is brown items, and the second column is white items. If your October sales are 50% more than your May sales, which of the following would represent your October sales?

    (a) $M + 50$

    (b) $0.5M$

    (c) $1.5M$

    (d) $M^{.5}$

    Answer: (c). If the October sales are 50% more than the May sales, then the October sales must be $M + 0.5M = 1.5M$. This is a quick application of multiplying a matrix by a scalar.

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15. You have a business that sells tables and chairs. You have brown tables and white tables, and corresponding chairs. Your May sales are 4 brown tables, 6 white tables, 20 brown chairs, and 24 white chairs, which is represented by the matrix $M = \begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$. where the first row is tables, the second row is chairs, the first column is brown items, and the second column is white items. Your June sales are given by the analogous matrix J, where $J = \begin{bmatrix} 6 & 8 \\ 22 & 32 \end{bmatrix}$. Which of the following matrix operations would make sense in this scenario? Be prepared to explain what the result tells you.

    (a) $M + J$

    (b) $M - J$

    (c) $1.2J$

    (d) MJ

    (e) All of the above make sense.

    (f) More than one, but not all, of the above make sense.

Answer: (f). (a) tells us the combined sales for May and June. (b) tells us the difference in sales between May and June. (c) represents sales that are 20% higher than June's sales. Only (d) does not make sense. The multiplication is defined (the matrices are the right sizes), but it does not make sense in this context because we end up multiplying tables by chairs.

CC HZ MA232 S08: 36/0/0/4/0/**57** time 2:30
HHS JG MA232 S08: 9/0/0/9/0/**82**

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LA.00.08.240

16. You have a business that sells tables and chairs. You have brown tables and white tables, and corresponding chairs. Your May sales are 4 brown tables, 6 white tables, 20 brown chairs, and 24 white chairs, which is represented by the matrix $M = \begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$. where the first row is tables, the second row is chairs, the first column is brown items, and the second column is white items. All tables cost \$350 and all chairs cost \$125, which we represent with the cost vector $C = \begin{bmatrix} 350 \\ 125 \end{bmatrix}$. Which of the following matrix operations could be useful in this scenario? Be prepared to explain what the result tells you.

    (a) MC
    (b) CM
    (c) $C^T M$
    (d) MC^T

    Answer: (c). Operations (b) and (d) are not defined. (a) is defined, but it does not make sense in this context, multiplying the number of white tables by the per item chair cost. (c) is defined and makes sense. Ask students to interpret the meaning of this result: The product is a vector containing the revenue from brown items and from white items.

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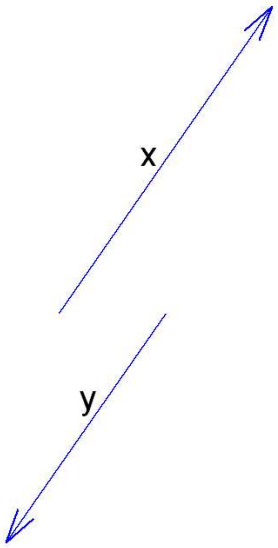
17. **True or False** Given the vectors x and y plotted below and some matrix A, if we know that $Ax = 0$, this means that $Ay = 0$ as well.



Answer: True. The vectors x and y are parallel, meaning that they are multiples of each other. If x is a solution to the homogeneous equation $Ax = 0$, then any multiple of x must be a solution as well, thus we know that $Ay = 0$.

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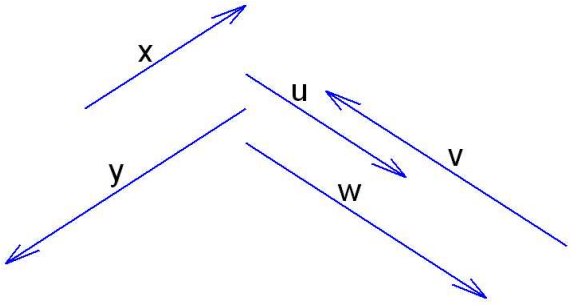
18. Given the vectors x, y, u, v, and w plotted below and some matrix A, if we know that $Ax = u$, what does this tell us about the product Ay?



(a) $Ay = u$

(b) $Ay = v$

(c) $Ay = w$

(d) We cannot say anything about Ay without knowing more about A.

Answer: (b). The vectors x and y are parallel, meaning that they are multiples of each other: In this case $y = -1.5x$, because it is 50% longer than x and points in the opposite direction. If we know that $Ax = u$ then $Ay = -1.5Ax = -1.5u$. Thus Ay must be a vector which points in the opposite direction as u and is 50% longer than u, which is v.

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