

## Matrix Inverses

1. Which of the following matrices does not have an inverse?

(a)  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(b)  $\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}$

(c)  $\begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$

(d)  $\begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$

(e) More than one of the above do not have inverses.

(f) All have inverses.

*Answer: (b).* The key here is to recognize that the reason (b) has no inverse is because its rows are multiples of each other, thus its reduced row echelon form will not be equal to the identity matrix.

by Carroll College MathQuest

LA.00.09.010

CC KC MA232 S07: 0/**68**/0/18/0/14 time 3:00

HC AS MA339 F07: 0/**79**/0/5/16/0

CC JS MA232 S09: 0/**75**/5/20/0/0

WW JD MA289 W11: 0/**88**/0/4/4/4

CC TM MA117 S12: 5/**30**/10/0/0/55

CC KC MA117 F11: 0/**88**/8/0/4/0

HC AS MA339 F11: 0/**81.25**/12.5/0/0/6.25 ,

CC TM MA117 F11: 4/**79**/4/8/0/0

2. When we put a matrix  $A$  into reduced row echelon form, we get the matrix  $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ .

This means that

(a) Matrix  $A$  has no inverse.

(b) The matrix we have found is the inverse of matrix  $A$ .

- (c) Matrix  $A$  has an inverse, but this isn't it.  
 (d) This tells us nothing about whether  $A$  has an inverse.

*Answer: (a).* If we put a matrix into reduced row echelon form and we do not get the identity matrix, then the original matrix does not have an inverse.

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LA.00.09.020

CC KC MA232 S07: **73**/5/9/14 time 2:00

CC HZ MA232 S08: **22**/7/26/44 time 2:45

HHS JG MA232 S08: **82**/0/0/18

CC JS MA232 S09: **80**/10/10/0

CC HZ MA232 S10: **30**/7/10/50 time 3:20

CC HZ MA232 S11: **31**/6/13/50 time 2:15

KC MS MA224 F10: **91**/0/0/9 time 2:00

3. Let  $A = \begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$ . What is  $A^{-1}$ ?

(a)  $\begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$ .

(b)  $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$ .

(c)  $\begin{bmatrix} 0 & 1/4 \\ 1/2 & 0 \end{bmatrix}$ .

(d)  $\begin{bmatrix} 0 & 1/2 \\ 1/4 & 0 \end{bmatrix}$ .

*Answer: (d).*  $A^{-1} = \frac{1}{-8} \begin{bmatrix} 0 & -4 \\ -2 & 0 \end{bmatrix}$ . This question could be used as a quick computational check after the formula for the inverse of a  $2 \times 2$  matrix has been introduced.

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LA.00.09.025

WW JD MA289 W11: 0/12/4/**85**

CC TM MA117 S12: 0/0/4/**96**

CC KC MA117 F11: 0/0/40/**60**

HC AS MA339 F11: 6.25/0/0/**93.75**/0/0 ,

CC TM MA117 F11: 12/8/4/**71**

4. We find that for a square coefficient matrix  $A$ , the homogeneous matrix equation  $AX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ , has only the trivial solution  $X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ . This means that
- (a) Matrix  $A$  has no inverse.
  - (b) Matrix  $A$  has an inverse.
  - (c) This tells us nothing about whether  $A$  has an inverse.

*Answer: (b).* If this matrix equation has only the trivial solution, this means that when we put  $A$  into reduced row echelon form, we will get the identity matrix, and this means that  $A$  must have an inverse.

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LA.00.09.030

CC KC MA232 S07: 18/**68**/14 time 1:15

HC AS MA339 F07: 5/**63**/32

CC HZ MA232 S08: 7/**15**/78 time 2:55

HHS JG MA232 S08: 50/**17**/33

CC JS MA232 S09: 38/**24**/38

CC HZ MA232 S10: 21/**39**/39 time 4:00

CC HZ MA232 S11: 32/**32**/37 time 3:00 Revote (told to talk about RREF): 0/67/33

WW JD MA289 W11: 29/**42**/29

KC MS MA224 F10: 20/**30**/50 time 2:10

CC HZ MA232 S12: 21/**5**/74 time 3:00

CC HZ MA232 S13: 30/**60**/10 time 3:45

5. **True or False** If  $A$ ,  $B$ , and  $C$  are square matrices and we know that  $AB = AC$ , this means that matrix  $B$  is equal to matrix  $C$ .
- (a) True, and I am very confident
  - (b) True, but I am not very confident
  - (c) False, but I am not very confident
  - (d) False, and I am very confident

*Answer: (False).* This is only true if  $A$  has an inverse. Ask the students to give an example of matrices so that  $AB = AC$  but  $B \neq C$ . An easy example is to let  $A$  be a matrix of zeros. Ask students for a nonzero (singular)  $A$  where  $AB = AC$  and  $B \neq C$ .

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LA.00.09.040

CC KC MA232 S07: 81/**19** time 1:30  
CC HZ MA232 S08: 33/**67** time 2:25  
HHS JG MA232 S08: 50/**50**  
WW JD MA289 W11: 33/43/10/**14**  
CC HZ MA232 S12: 29/24/24/**24** time 2:30  
CC TM MA117 S12: 25/35/5/**35**  
CC KC MA117 F11: 50/23/12/**15**  
CC TM MA117 F11: 35/35/9/**13**

6. **True or False** Suppose that  $A$ ,  $B$ , and  $C$  are square matrices, and  $CA = B$ , and  $A$  is invertible. This means that  $C = A^{-1}B$ .

- (a) True, and I am very confident
- (b) True, but I am not very confident
- (c) False, but I am not very confident
- (d) False, and I am very confident

*Answer: (False).* In order to solve  $CA = B$  for  $C$ , we must multiply this identity by  $A^{-1}$  on the right, so we get  $C = BA^{-1}$ , which is not necessarily equal to  $A^{-1}B$ . Ask the students to find specific matrices to demonstrate that the given statement is false.

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LA.00.09.050

CC KC MA232 S07: 86/**14** time 1:00  
HC AS MA339 F07: 53/**47**  
HHS JG MA232 S08: 100/**0**  
WW JD MA289 W11: 23/27/35/**15**  
CC HZ MA232 S12: 30/30/20/**20** time 1:45

7. We know that  $(5A)^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ . What is matrix  $A$ ?

- (a)  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- (b)  $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$
- (c)  $\begin{bmatrix} 1/5 & 0 \\ 0 & 1/5 \end{bmatrix}$
- (d)  $\begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$

(e) There is no matrix  $A$  which solves this equation.

*Answer: (c).* The identity matrix is its own inverse, so we know that  $5A = I$ , and thus  $A = \frac{1}{5}I$ .

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LA.00.09.060

CC KC MA232 S07: 5/32/64/0/0 time 2:00

HHS JG MA232 S08: 0/0/100/0/0

WW JD MA289 W11: 4/0/96/0/0

8.  $A$  and  $B$  are invertible matrices. If  $AB = C$ , then what is the inverse of  $C$ ?

(a)  $C^{-1} = A^{-1}B^{-1}$

(b)  $C^{-1} = B^{-1}A^{-1}$

(c)  $C^{-1} = AB^{-1}$

(d)  $C^{-1} = BA^{-1}$

(e) More than one of the above is true.

(f) Just because  $A$  and  $B$  have inverses, this doesn't mean that  $C$  has an inverse.

*Answer: (b).*  $C = AB$  and  $A^{-1}$  and  $B^{-1}$  exist, so we multiply  $AB$  by  $B^{-1}A^{-1}$  on the right to get  $ABB^{-1}A^{-1} = AA^{-1} = I$ . Therefore,  $C^{-1} = B^{-1}A^{-1}$ .

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LA.00.09.070

CC KC MA232 S07: 9/5/0/0/18/68 time 3:00

CC JS MA232 S09: 20/70/0/0/5/5

CC TM MA117 S12: 40/5/25/0/30/0

9. Let  $A$  be a  $2 \times 2$  matrix. The inverse of  $3A$  is

(a)  $\frac{1}{9}A^{-1}$

(b)  $\frac{1}{3}A^{-1}$

(c)  $A^{-1}$

(d)  $3A^{-1}$

(e) Not enough information is given.

*Answer: (b).* The inverse of  $kA$  is  $\frac{1}{k}A^{-1}$ , when  $k \neq 0$ . This question could be used before this particular property has been introduced, as students could work it out for the  $2 \times 2$  case, and the instructor could then explain that this hold in general. The next question offers an alternative form of this question.

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LA.00.09.080

10. If  $A$  is an invertible matrix, what else must be true?

- (a) If  $AB = C$  then  $B = A^{-1}C$ .
- (b)  $A^2$  is invertible.
- (c)  $A^T$  is invertible.
- (d)  $5A$  is invertible.
- (e) The reduced row echelon form of  $A$  is  $I$ .
- (f) All of the above must be true.

*Answer: (f).* (a) should be straightforward. To show that (b) must be true, we can construct the inverse of  $A^2$ : start with  $AA^{-1} = I$ , multiply on the left by  $A$  and on the right by  $A^{-1}$ . We see that the inverse of  $A^2$  is  $(A^{-1})^2$ . Statement (e) should be immediate if students have seen that reducing the augmented matrix  $[ A \ I ]$  results in the matrix  $[ I \ A^{-1} ]$ , and the other two statements can be derived from this one.

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LA.00.09.090

WW JD MA289 W11: 0/0/4/4/0/93