Eigenvalues and Eigenvectors

1. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   (a) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 3 \\ 3 \end{bmatrix}$

   (c) $\begin{bmatrix} 3 & 3 \end{bmatrix}$

   (d) $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$

   (e) None of the above

   Answer: (b). This is a straight-forward matrix multiplication problem, to help students discover the concept of eigenvectors.

   by Carroll College MathQuest

   LA.00.11.010
   CC KC MA232 S07: 0/**100**/0/0/0 time 1:00
   CC HZ MA232 S08: 0/**96**/0/0/4 time 1:30
   HHS JG MA232 S08: 0/**100**/0/0/0
   CC HZ MA232 S10: 7/**70**/13/3/7
   CC HZ MA232 S11: 0/**78**/6/17/0 time 1:30
   WW JD MA289 W11: 23/**73**/0/0/5
   KC MS MA224 F09: 0/**90**/5/5/0 time 1:40
   KC MS MA224 F10: 0/**100**/0/0/0 time 1:20
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   HC AS MA339 F11: 0/**100**/0/0/0/0 ,
   CC HZ MA232 S13: 0/**82**/18/0/0 time 1:30

2. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^2 \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   (a) $\begin{bmatrix} 3 \\ 3 \end{bmatrix}$

   (b) $\begin{bmatrix} 6 \\ 6 \end{bmatrix}$

   (c) $\begin{bmatrix} 9 \\ 9 \end{bmatrix}$

(d) $\begin{bmatrix} 12 \\ 12 \end{bmatrix}$

(e) None of the above

(f) This matrix multiplication is impossible.

Answer: (c). We multiply a matrix to a power with an eigenvector, seeing the pattern emerge.

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LA.00.11.020
CC KC MA232 S07: 0/5/**95**/0 time 1:00
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HC AS MA339 F11: 0/12.5/**87.5**/0/0/0 ,
CC HZ MA232 S13: 0/4/**96**/0/0/0 time 1:45

3. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^4 \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(a) $\begin{bmatrix} 27 \\ 27 \end{bmatrix}$

(b) $\begin{bmatrix} 81 \\ 81 \end{bmatrix}$

(c) $\begin{bmatrix} 243 \\ 243 \end{bmatrix}$

(d) $\begin{bmatrix} 729 \\ 729 \end{bmatrix}$

(e) None of the above

Answer: (b).

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LA.00.11.030
CC HZ MA232 S08: 0/**96**/0/4/0 time 0:45
HHS JG MA232 S08: 0/**100**/0/0/0
CC HZ MA232 S10: 0/**100**/0/0/0
SFCC GG MA220 Su10: 0/**100**/0/0/0 one clicker per pair

CC HZ MA232 S11: 0/**100**/0/0/0 time 0:30
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HC AS MA339 F11: 0/**93.33**/6.67/0/0/0 ,
CC HZ MA232 S13: 0/**92**/4/0/0 time 0:45

4. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   (a) $\begin{bmatrix} 3n \\ 3n \end{bmatrix}$

   (b) $3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   (c) $n^3 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   (d) $3^n \begin{bmatrix} n \\ n \end{bmatrix}$

   (e) $\begin{bmatrix} 3 \\ 3 \end{bmatrix}^n$

   (f) More than one of the above

   Answer: (b). We generalize the pattern, recognizing that a matrix to the nth times an eigenvector is simply the scalar eigenvalue to the nth times this eigenvector.

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   LA.00.11.040
   CC KC MA232 S07: 0/**52**/0/0/48/0 time 1:30
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   CC HZ MA232 S11: 6/**78**/0/0/11/6 time 0:45
   WW JD MA289 W11: 0/**91**/0/4/4/0
   KC MS MA224 F09: 0/**100**/0/0/0 time 1:20
   KC MS MA224 F10: 14/**67**/0/5/14 time 1:30
   CC HZ MA232 S12: 0/**84**/0/0/16/0 time 1:30
   HC AS MA339 F11: 0/**52.94**/5.88/0/29.41/11.76 ,
   CC HZ MA232 S13: 4/**75**/0/0/21/0 time 1:45

5. Suppose A is an $n \times n$ matrix, c is a scalar, and x is an $n \times 1$ vector. If $Ax = cx$, what is A^2x?

   (a) $2cx$

   (b) c^2x

(c) cx

(d) None of the above

Answer: (b). A generalization of the result.

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LA.00.11.045

CC HZ MA232 S10: 6/**90**/0/3
CC HZ MA232 S11: 0/**92**/8/0 time 3:00
KC MS MA224 F09: 0/**100**/0/0 time 1:15
CC HZ MA232 S12: 5/**95**/0/0 time 2:30

6. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

   (a) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 3 \\ -3 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

   (d) $\begin{bmatrix} -3 \\ 3 \end{bmatrix}$

   (e) None of the above

Answer: (a). We look at another simple product to find another eigenvector.

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LA.00.11.050

CC KC MA232 S07: **100**/0/0/0/0 time 1:30
CC HZ MA232 S08: **88**/8/4/0/0
HHS JG MA232 S08: **100**/0/0/0/0
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CC HZ MA232 S11: **100**/0/0/0/0 time 0:45
WW JD MA289 W11: **96**/4/0/0/0
CC HZ MA232 S12: **84**/11/0/5/0 time 1:00

7. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

   (a) $(-1)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(b) $(-1)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(c) $(-3)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) $\begin{bmatrix} (-1)^n \\ (-1)^{n+1} \end{bmatrix}$

(e) None of the above

(f) More than one of the above

Answer: (f). Both (a) and (d) are equivalent ways of generalizing this matrix to a power times an eigenvector.

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LA.00.11.060

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CC HZ MA232 S10: 52/23/6/6/6/**6**
CC HZ MA232 S11: 60/20/13/0/7/**0** time 1:00
WW JD MA289 W11: 50/33/8/4/0/**4**
CC HZ MA232 S12: 83/11/0/0/0/**6** time 2:00
KC MS MA224 F10: 5/5/19/5/**67** time 1:15

8. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

(a) $3^n \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

(b) $2^n \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

(c) $6^n \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(d) $3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

Answer: (a). Here we see that a nonzero multiple of the first eigenvector that we found is also an eigenvector.

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LA.00.11.070
CC KC MA232 S07: **90**/0/5/0/0/5 time 2:00
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HHS JG MA232 S08: **100**/0/0/0/0/0
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CC HZ MA232 S11: **36**/0/57/0/7/0 time 2:30
CC HZ MA232 S12: **65**/0/5/10/0/20 time 2:00

9. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} -5 \\ 5 \end{bmatrix}$

   (a) $3^n \begin{bmatrix} -5 \\ 5 \end{bmatrix}$
   (b) $(-1)^n \begin{bmatrix} -5 \\ 5 \end{bmatrix}$
   (c) $(-5)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
   (d) $5 \begin{bmatrix} (-1)^n \\ (-1)^n \end{bmatrix}$
   (e) None of the above
   (f) More than one of the above

   Answer: (b). Another demonstration that a nonzero multiple of an eigenvector is also an eigenvector.

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   LA.00.11.080
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   CC HZ MA232 S11: 29/**71**/0/0/0/0 time 2:00
   CC HZ MA232 S12: 6/**83**/11/0/0/0 time 2:00

10. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

   (a) $\begin{bmatrix} 3 \\ 15 \end{bmatrix}$
   (b) $\begin{bmatrix} -1 \\ -5 \end{bmatrix}$
   (c) $\begin{bmatrix} 11 \\ 7 \end{bmatrix}$

(d) $\begin{bmatrix} 7 \\ 11 \end{bmatrix}$

(e) None of the above

Answer: (c). This is a straight-forward matrix/vector multiplication, but this time we see that it is not an eigenvector.

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LA.00.11.090
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CC HZ MA232 S08: 0/0/**100**/0/0 time 0:35
HHS JG MA232 S08: 0/0/**100**/0/0
CC HZ MA232 S10: 3/0/**97**/0/0
CC HZ MA232 S11: 0/0/**100**/0/0 time 1:30
WW JD MA289 W11: 4/0/**96**/0/0
CC HZ MA232 S12: 0/0/**100**/0/0 time 1:45
HC AS MA339 F11: 0/0/**93.33**/6.67/0/0

11. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(a) $11^n \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(b) $7^n \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(c) $\begin{bmatrix} 11^n \\ 7^n \end{bmatrix}$

(d) $\begin{bmatrix} 25 \\ 29 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

Answer: (e). We discover that there is no simple generalization of a matrix to a power times a non-eigenvector.

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LA.00.11.100

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CC HZ MA232 S10: 0/6/13/0/**77**/3
CC HZ MA232 S11: 0/8/0/8/**85** time 3:00

CC HZ MA232 S12: 0/0/0/5/**79**/16 time 2:00
HC AS MA339 F11: 0/0/0/0/**100**/0 ,

12. Write the vector $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

    (a) $\begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

    (b) $\begin{bmatrix} 1 \\ 5 \end{bmatrix} = \frac{1}{2}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$

    (c) $\begin{bmatrix} 1 \\ 5 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 1 \end{bmatrix} - 2\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

    (d) None of the above

    (e) More than one of the above

    Answer: (c). We write this vector as a linear combination of the eigenvectors of the matrix. Answer (a) is a correct identity, so students may be lead to vote for (e), thinking that (a) and (c) are both correct. However, (a) is not what is asked for.

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    LA.00.11.110
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    HHS JG MA232 S08: 0/0/**30**/0/70
    CC HZ MA232 S10: 6/0/**88**/0/6
    CC HZ MA232 S11: 0/0/**77**/8/15 time 2:30
    WW JD MA289 W11: 0/0/**96**/4/0
    CC HZ MA232 S12: 0/0/**65**/5/30 time 2:15
    HC AS MA339 F11: 0/7.69/**76.92**/0/15.38/0 ,

13. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

    (a) $-1 \times 3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 3 \times (-2)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

    (b) $3 \times (-1)^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + (-2) \times 3^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

    (c) $3 \times 3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + (-2) \times (-1)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

    (d) $3 \times 3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + (-1) \times (-2)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

    (e) None of the above

(f) More than one of the above

Answer: (c). Here we learn to calculate $A^n x$, when x is not an eigenvector, by first writing x as a linear combination of the eigenvectors of A.

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LA.00.11.120
CC KC MA232 S07: 0/0/**44**/25/13/6 time 5:00
CC HZ MA232 S08: 4/4/**68**/21/4/0 time 6:00
HHS JG MA232 S08: 0/0/**0**/70/30/0
CC HZ MA232 S10: 0/0/**97**/3/0/0
CC HZ MA232 S11: 0/0/**100**/0/0/0 time 3:15
WW JD MA289 W11: 12/0/**81**/8/0/0
CC HZ MA232 S12: 0/6/**78**/0/6/11 time 3:00
HC AS MA339 F11: 0/0/**76.92**/7.69/15.38/0

14. Which of the following is an eigenvector of the matrix $\begin{bmatrix} 2 & -1 \\ -4 & -1 \end{bmatrix}$?

(a) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ -4 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

Answer: (c). We can see which of these is an eigenvector simply by multiplying the matrix by each and examining the result.

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LA.00.11.130

CC KC MA232 S07: 0/0/**82**/6/6/6 time 3:00
MCC KS MAT210-60 F10: 0/0/**77**/0/23 time 3.5 min n=13
MCC KS MAT210-01 F10: 6/6/**59**/6/24 n=17

15. Which of the following is an eigenvector of the matrix $\begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$?

(a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} -3 \\ -3 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ -2/3 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

Answer: (f). All four are eigenvectors. We can see that (a) and (b) are eigenvectors corresponding to the eigenvalues 4 and -1, respectively, by multiplying the matrix by each and examining the result. We further notice that (c) and (d) are multiples of (a) and (b), so they are also eigenvectors.

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LA.00.11.140
HC AS MA339 F07: 0/0/0/0/5/**95**
CC KC MA334 S09: 8/15/4/8/4/**61** time 4:30
CC KC MA334 S10: 9/0/0/5/9/**77** time 6:00
CC KC MA334 S12: 8/46/0/4/4/**38**
CC HZ MA232 S13: 10/5/14/14/14/**43** time 5:45 Review

16. Suppose the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has an eigenvalue 1 with associated eigenvector $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. What is $A^{50}x$?

(a) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(b) $\begin{bmatrix} a^{50} & b^{50} \\ c^{50} & d^{50} \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} 2^{50} \\ 3^{50} \end{bmatrix}$

(e) Way too hard to compute.

Answer: (c). $A^{50}x = 1^{50}x = x$. This should be a quick check on students' understanding the computational significance of eigenvalues and eigenvectors.

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LA.00.11.150
HC AS MA339 F07: 5/0/**85**/5/5
CC KC MA334 S09: 0/15/**23**/42/0 time 2:30
CC KC MA334 S10: 0/0/**86**/14/0 time 3:30
SFCC GG MA220 Su10: 0/0/**67**/33/0 one clicker per pair
CC HZ MA232 S11: 0/0/**79**/14/7 time 2:00
CC HZ MA232 S12: 0/0/**100**/0 time 1:15

17. Vector x is an eigenvector of matrix A. If $x = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $Ax = \begin{bmatrix} 4 \\ 12 \end{bmatrix}$, then what is the associated eigenvalue?

    (a) 1
    (b) 3
    (c) 4
    (d) Not enough information is given.

    Answer: (c). $Ax = 4x$, so the associated eigenvalue is 4.

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    LA.00.11.155
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    SFCC GG MA220 Su10: 0/0/**100**/0 one clicker per pair
    CC HZ MA232 S11: 0/0/**94**/6 time 1:15
    CC HZ MA232 S12: 0/0/**89**/11 time 1:30

18. Which of the following is an eigenvector of $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$? (You should be able to answer this by checking the vectors given, rather than by finding the eigenvectors of A from scratch.)

    (a) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$
    (b) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$
    (c) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$
    (d) None of the above

Answer: (c). We see that $\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix} = -2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. We can also learn from this that the associated eigenvalue is -2, or that discussion can wait until after the next question.

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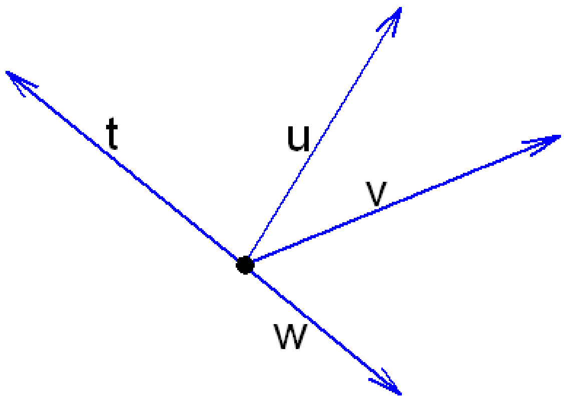
LA.00.11.160
CC HZ MA232 S12: 0/0/**88**/12 time 5:30

19. The vector t is an eigenvector of the matrix A. What could be the result of the product At?



(a) $At = u$

(b) $At = v$

(c) $At = w$

(d) None of the above

Answer: (c). The vector w is parallel to the vector t: it can be produced by multiplying vector t by a scalar.

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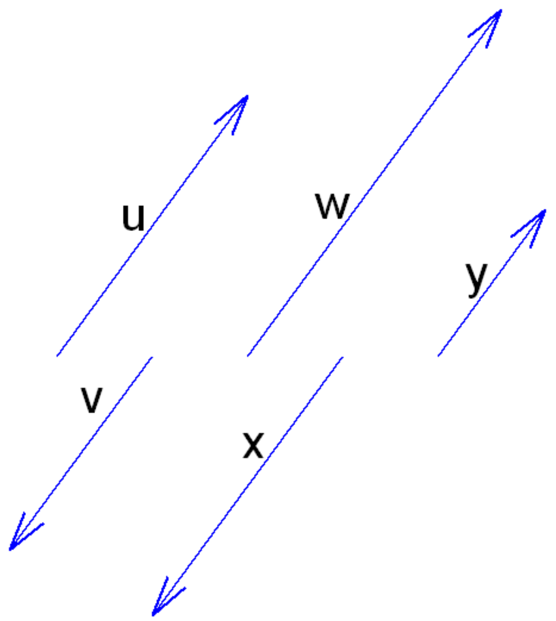
LA.00.11.165

HC AS MA339 F07: 0/0/**94**/6
CC KC MA334 S09: 4/4/**92**/0 time 2:00
CC HZ MA232 S10: 27/13/**43**/17
KC MS MA224 F09: 0/0/**100**/0 time 1:10
KC MS MA224 F10: 0/0/**91**/9 time 1:40
CC HZ MA232 S12: 6/0/**88**/6 time 3:45

20. The vector u is an eigenvector of the matrix A and $Au = v$, where the vectors u and v are shown below. What could be the result of the product Av?



(a) $Av = u$

(b) $Av = v$

(c) $Av = w$

(d) $Av = x$

(e) $Av = y$

Answer: (e). The vector v is shorter than u and points in the opposite direction which tells us that u must correspond to an eigenvalue between -1 and 0. If we then take the product Av, we will multiply by this eigenvalue again, so the result will be shorter than v and point in the opposite direction, thus it must be y: $Av = A(\lambda u) = \lambda Au = \lambda^2 u = y$.

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LA.00.11.167

CC KC MA334 S09: 12/0/15/8/**65** time 3:00
CC KC MA334 S10: 9/9/14/5/**64** time 3:00
KC MS MA224 F09: 0/0/6/0/**94** time 2:10
KC MS MA224 F10: 5/5/19/5/**67** time 1:15

21. $\begin{bmatrix} 4/3 \\ 1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$. What is the associated eigenvalue? (Think! Don't solve for all the eigenvalues and eigenvectors.)

(a) 4/3
(b) 5
(c) -2

Answer: (b). We see that $\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 4/3 \\ 1 \end{bmatrix} = \begin{bmatrix} 20/3 \\ 5 \end{bmatrix} = 5 \begin{bmatrix} 4/3 \\ 1 \end{bmatrix}$, so the associated eigenvalue is 5.

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LA.00.11.170

CC HZ MA232 S08: 11/**78**/11 time 5:40
CC KC MA334 S09: 20/**80**/0 time 3:00
CC HZ MA232 S10: 27/**57**/17
CC KC MA334 S10: 64/**36**/0 time 2:00
CC HZ MA232 S12: 44/**56**/0 time 3:30
CC KC MA334 S12: 4/**96**/0
CC KC MA334 S13: 0/**100**/0 time 4:00

22. The matrix $A = \begin{bmatrix} -1 & 4 \\ 3 & 0 \end{bmatrix}$ has an eigenvalue 3 with associated eigenvector $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Let $y = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. Which of the following statements is true?

(a) $Ax = 3x$
(b) $Ay = 3y$
(c) For any scalars c and d, $A(cx + dy) = 3(cx + dy)$
(d) All of the above are true.
(e) Only (a) and (b) are true.

Answer: (d). $y = 2x$, so y is also an eigenvector of A corresponding to the eigenvalue 3. Thus, both (a) and (b) are true. However, $A(cx + dy) = A(cx + 2dx) = A(c + 2d)x$. If we let $z = (c + 2d)x$, then z is also an eigenvector of A, so $Az = 3z = 3(cx + dy)$.

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LA.00.11.180
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CC KC MA334 S13: 0/0/0/**53**/47 time 3:30

23. The matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ has an eigenvalue 2 with associated eigenvectors $x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. Which of the following statements is true?

(a) $Ax = 2x$
(b) $Ay = 2y$
(c) For any scalars c and d, $A(cx + dy) = 2(cx + dy)$.
(d) For any nonzero scalars c and d, $cx + dy$ is an eigenvector of A corresponding to the eigenvalue 2.
(e) All of the above are true.
(f) Only (a) and (b) are true.

Answer: (e). We know that (a) and (b) are true because x and y are both eigenvectors of A corresponding to the eigenvalue 2. Statements (c) and (d) are equivalent, with (c) easily demonstrated with the given matrix and vectors.

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LA.00.11.190

SFCC GG MA220 Su10: 33/67/0/0/**0**/0 one clicker per pair
WW JD MA289 W11: 0/0/15/0/**65**/19

24. **True or False** Any nonzero linear combination of two eigenvectors of a matrix A is an eigenvector of A.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

Answer: False. This is not quite the correct generalization from the previous two questions. Ask the students to come up with the correct generalization: Any linear combination of two eigenvectors corresponding to the same eigenvalue of a matrix A is also an eigenvector of A corresponding to the same eigenvalue.

by Carroll College MathQuest

LA.00.11.200

KC MS MA224 F09: 10/**90** time 3:15
KC MS MA224 F10: 35/**65** time 3:20

25. If w is an eigenvector of A, how does the vector Aw compare geometrically to the vector w?

(a) Aw is a rotation of w.

(b) Aw is a reflection of w in the x-axis.

(c) Aw is a reflection of w in the y-axis.

(d) Aw is parallel to w but may have a different length.

Answer: (d). Since $Aw = \lambda w$, we see that Aw is just a multiple of the eigenvector. Geometrically this means that only the length of the vector has changed (unless the eigenvalue is one, in which case the length is the same).

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LA.00.11.210

26. What does it mean if 0 is an eigenvalue of a matrix A?

(a) The determinant of A is zero.

(b) The columns of A are linearly dependent.

(c) There are an infinite number of solutions to the system $Ax = 0$.

(d) All of the above

(e) None of the above

Answer: (d). Since λ is an eigenvalue of A when $\det(A - \lambda I) = 0$, a zero eigenvalue implies that $\det A = 0$. The other statements are equivalent to this one.

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LA.00.11.220

HC AS MA339 F07: 5/0/5/**55**/35
SFCC GG MA220 Su10: 0/0/0/**100** one clicker per pair

27. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 6 \\ 0 & 4 & 2 \end{bmatrix}$ and note that all of the rows sum to six. Which of the following is true?

(a) $w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector of A.

(b) 6 is an eigenvalue of A.

(c) Both statements are true.

(d) Neither statement is true.

Answer: (c). We can verify both statements by computing Aw. We will find that $Aw = 6w$, thus showing that w is an eigenvector of A corresponding to the eigenvalue 6. This question introduces students to the theorem that if the row sums of a real, square matrix are all equal, then that sum is an eigenvalue and the corresponding eigenvector has all ones. Ask students to explain why that theorem is true.

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LA.00.11.230