Classroom Voting Questions: Linear Algebra

Chapter 1: Linear Equations in Linear Algebra

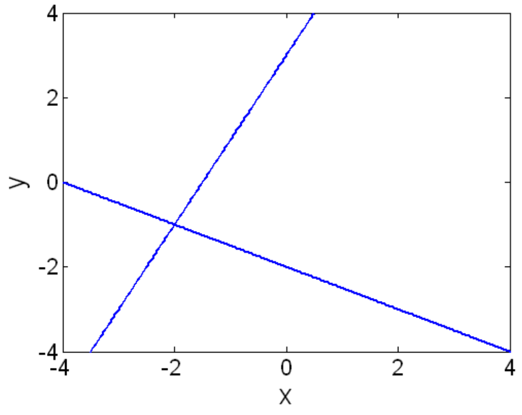
Systems of Equations

1. What is the solution to the following system of equations?

$$\begin{aligned} 2x + y &= 3 \\ 3x - y &= 7 \end{aligned}$$

   (a) $x = 4$ and $y = -5$
   (b) $x = 4$ and $y = 5$
   (c) $x = 2$ and $y = -1$
   (d) $x = 2$ and $y = 1/2$
   (e) There are an infinite number of solutions to this system.
   (f) There are no solutions to this system.

2. Which of the following systems of equations could be represented in the graph below?



   (a) $3x + 3y = -6$, $x + 2y = 3$
   (b) $x - y = -5$, $2x + y = 4$
   (c) $-8x + 4y = 12$, $2x + 4y = -8$
   (d) $-x + 3y = 9$, $2x - y = 4$

3. What is the solution to the following system of equations?

$$\begin{aligned} 2x + y &= 3 \\ 4x + 2y &= 6 \end{aligned}$$

(a) $x = 0$ and $y = 0$

(b) $x = 2$ and $y = -1$

(c) $x = 0$ and $y = 1$

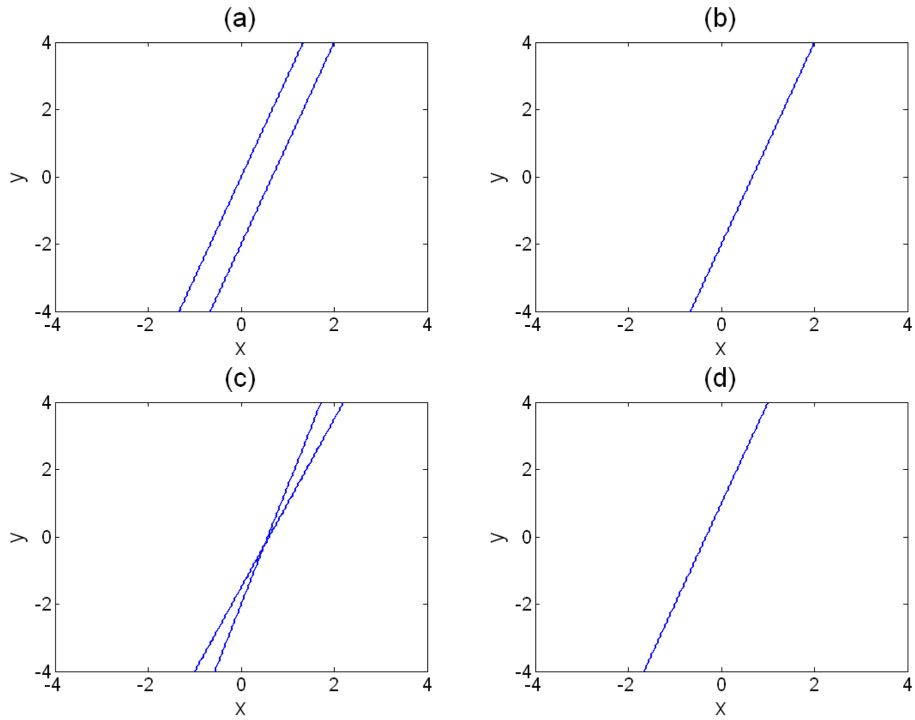
(d) $x = 0$ and $y = 3$

(e) There are an infinite number of solutions to this system.

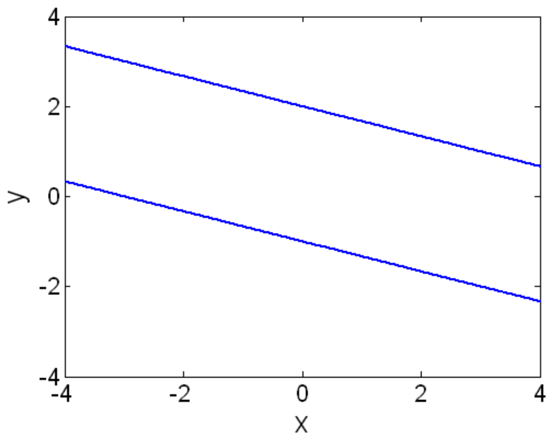
(f) There are no solutions to this system.

4. Which of the graphs below could represent the following linear system?

$$\begin{aligned} 3x - y &= 2 \\ -9x + 3y &= -6 \end{aligned}$$



5. Which of the following systems of equations could be represented in the graph below?



(a) $-x + 3y = 6,\ 2x + 6y = -6$

(b) $-x + 3y = 6,\ 2x + 6y = 12$

(c) $x + 3y = 6,\ 2x + 6y = 12$

(d) $x + 3y = 6,\ x + 3y = -3$

6. What is the solution to the following system of equations?

$$\begin{aligned} -3x + 2y &= 4 \\ 12x - 8y &= 10 \end{aligned}$$

(a) $x = -4/3$ and $y = 0$

(b) $x = 1/2$ and $y = -1/2$

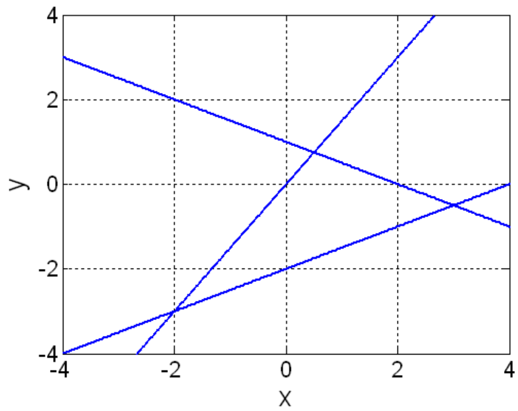
(c) $x = 0$ and $y = 2$

(d) $x = 1/3$ and $y = 5/2$

(e) There are an infinite number of solutions to this system.

(f) There are no solutions to this system.

7. We have a system of three linear equations with two unknowns, as plotted in the graph below. How many solutions does this system have?



(a) 0
(b) 1
(c) 2
(d) 3
(e) Infinite

8. A system of linear equations could *not* have exactly ________ solutions.

(a) 0
(b) 1
(c) 2
(d) infinite
(e) All of these are possible numbers of solutions to a system of linear equations.

9. The system

$$\begin{aligned} x + y &= 2 \\ 2x + 2y &= 4 \end{aligned}$$

has an infinite number of solutions. Which of the following describes the set of solutions to this system?

(a) $x = 1$ and $y = 1$
(b) $x = 2 - t$ and $y = t$
(c) x and y could each be anything.
(d) None of the above

10. Which of the following options describes the set of solutions to the system below?

$$\begin{aligned} x + y &= 1 \\ x - y &= 0 \\ 2x + y &= 3 \end{aligned}$$

(a) $x = 1 - t$ and $y = t$
(b) $x = 1$ and $y = 1$
(c) No solution exists
(d) None of the above

11. Which of the following options describes the set of solutions to the system below?

$$\begin{aligned} x + y &= 2 \\ 2x - y &= -2 \\ x - 2y &= -4 \end{aligned}$$

(a) $x = t$ and $y = 2 - t$

(b) $x = 0$ and $y = 2$

(c) no solution exists

(d) None of the above

12. $x = 3 - 2t$ and $y = t$ represent the set of solutions to a system of equations. What line in $\Re^2$ does this set of solutions represent?

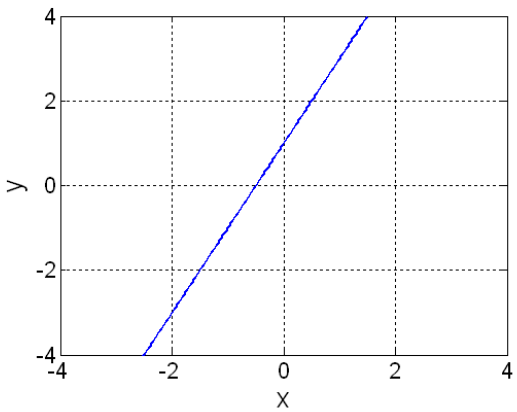
(a) $x + 2y = 3$

(b) $x - 2y = 3$

(c) $x + y = 3 - t$

(d) It is impossible to answer this question with the information given.

13. The set of solutions to a system of linear equations is plotted below. Which of the following parameterizations represents this solution set?



(a) $x = 2t$ and $y = 4t + 1$

(b) $x = \frac{1}{2}t - \frac{1}{2}$ and $y = t$

(c) $x = t - 1$ and $y = 2t - 1$

(d) $x = t$ and $y = 2t + 1$

(e) All of the above

14. A certain mini-golf course does not list their prices. I paid \$26.25 for 3 children and 4 adults. The group in front of me had paid \$25.50 for 6 children and 2 adults. Which system of equations would allow us to determine the prices for children and adults?

    (a)

    $$\begin{aligned} 3x + 6y &= 26.25 \\ 4x + 2y &= 25.50 \end{aligned}$$

    (b)

    $$\begin{aligned} 3x + 4y &= 26.25 \\ 6x + 2y &= 25.50 \end{aligned}$$

    (c)

    $$\begin{aligned} 26.25x + 25.50y &= 51.75 \\ 9x + 6y &= 15 \end{aligned}$$

    (d)

    $$\begin{aligned} (26.25/3)x + (26.25/4)y &= 0 \\ (25.50/6)x + (25.50/6)y &= 0 \end{aligned}$$

15. A system of 3 linear equations with 3 variables could not have exactly ____________ solutions.

    (a) 0
    (b) 1
    (c) 2
    (d) 3
    (e) More than one of (a)-(d) are impossible.
    (f) All of (a)-(d) are possible numbers of solutions.

16. A linear equation with two variables can be geometrically represented as a line in $\Re^2$. How can we best represent a linear equation with three variables?

    (a) As a line in $\Re^2$
    (b) As a line in $\Re^3$
    (c) As a plane in $\Re^3$
    (d) As a volume in $\Re^3$

17. We find that a system of three linear equations in three variables has an infinite number of solutions. How could this happen?

    (a) We have three equations for the same plane.
    (b) At least two of the equations must represent the same plane.
    (c) The three planes intersect along a line.
    (d) The planes represented are parallel.
    (e) More than one of the above are possible.

18. We consider a system of three linear equations in three variables, and visualize the graph of each equation as a plane in $\Re^3$. Suppose no solutions exist to this system. This means that

    (a) all three planes must be parallel.
    (b) at least two of the planes must be parallel.
    (c) at least two of the equations represent the same plane.
    (d) none of these planes ever intersects with another.
    (e) None of the above

19. We have a system of four linear equations in four variables. We can think about the graph of each equation as a 3-dimensional volume in $\Re^4$. Which of the following could geometrically represent the solutions to this system?

    (a) A point in $\Re^4$
    (b) A line in $\Re^4$
    (c) A plane in $\Re^4$
    (d) A three dimensional volume in $\Re^4$
    (e) All of the above
    (f) None of the above

20. How can we geometrically represent the parametric equations $x = 2t$, $y = -t + 1$, and $z = t$?

    (a) A line in $\Re^2$
    (b) A line in $\Re^3$
    (c) A plane in $\Re^3$
    (d) A volume in $\Re^3$

21. A system of 5 linear equations and 7 variables could not have exactly ____________ solutions.

   (a) 0
   (b) 1
   (c) infinite
   (d) More than one of these is impossible.
   (e) All of these are possible numbers of solutions.

22. A system of 8 linear equations and 6 variables could not have exactly ____________ solutions.

   (a) 0
   (b) 1
   (c) infinite
   (d) More than one of these is impossible.
   (e) All of these are possible numbers of solutions.

23. Howard's store sells three blends of flour: standard, extra wheat, and extra soy. Each is a blend of whole wheat flour and soy flour, and the table below shows how many pounds of each type of flour is needed to make one pound of each blend.

whole wheat flour	soy flour	
0.5	0.5	Standard Blend
0.8	0.2	Extra Wheat
0.3	0.7	Extra Soy

A customer comes in who wants one pound of a blend that is 60% wheat and 40% soy. Which system of equations below would allow us to solve for the amount of each blend needed to fulfill this special request?

   (a)

$$\begin{aligned} 0.5x_1 + 0.5x_2 &= 1 \\ 0.8x_1 + 0.2x_2 &= 1 \\ 0.3x_1 + 0.7x_2 &= 1 \end{aligned}$$

   (b)

$$\begin{aligned} 0.5x_1 + 0.5x_2 &= 0.6 \\ 0.8x_1 + 0.2x_2 &= 0.4 \\ 0.3x_1 + 0.7x_2 &= 0 \end{aligned}$$

(c)

$$\begin{aligned} 0.5x_1 + 0.8x_2 + 0.3x_3 &= 1 \\ 0.5x_1 + 0.2x_2 + 0.7x_3 &= 1 \end{aligned}$$

(d)

$$\begin{aligned} 0.5x_1 + 0.8x_2 + 0.3x_3 &= 0.6 \\ 0.5x_1 + 0.2x_2 + 0.7x_3 &= 0.4 \end{aligned}$$

24. In the previous question you set up a system of equations so that you could find the amount of each blend needed to make a new mixture. How many solutions must this system have? (You do not need to solve the system.)

    (a) 0
    (b) 1
    (c) 2
    (d) 3
    (e) Infinite

25. The previous two questions dealt with the system

$$\begin{aligned} 0.5x_1 + 0.8x_2 + 0.3x_3 &= 0.6 \\ 0.5x_1 + 0.2x_2 + 0.7x_3 &= 0.4 \end{aligned}$$

    In the context given, what quantity or unit does 0.6 represent?

    (a) pounds
    (b) %
    (c) pounds^2
    (d) pounds per %
    (e) 0.6 does not have units

Matrix Representations of Systems of Equations

26. You have a business that sells tables and chairs. You have brown tables and white tables, and corresponding chairs. Your current inventory is 4 brown tables, 6 white tables, 20 brown chairs, and 24 white chairs. Which matrix would best represent this information?

(a)

$$\begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 4 & 6 \\ 24 & 20 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 6 & 4 \\ 20 & 24 \end{bmatrix}$$

(d) They all represent the information equally well.

27. Which augmented matrix represents the following system of equations?

$$\begin{aligned} x + 2y &= 3 \\ 4y + 5x &= 6 \end{aligned}$$

(a)

$$\begin{bmatrix} 0 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

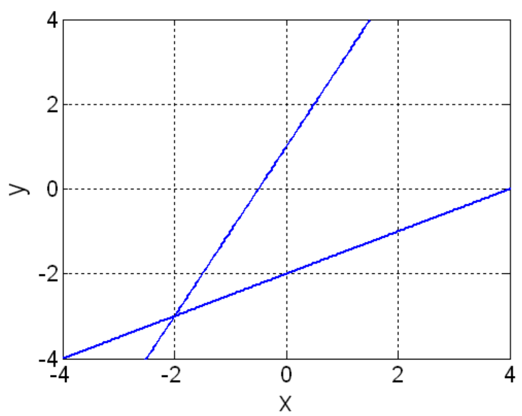
(c)

$$\begin{bmatrix} 1 & 2 & 3 \\ 5 & 4 & 6 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 0 & 2 & 3 \\ 5 & 4 & 6 \end{bmatrix}$$

28. The rows of which augmented matrix represent equations plotted below?



(a)

$$\begin{bmatrix} 1 & 0 & 3 \\ -3 & -3 & 4 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 2 & 2 & -3 \\ 0 & 1 & 4 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 2 & -4 \\ 3 & -2 & -2 \end{bmatrix}$$

(d)

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 4 \end{bmatrix}$$

29. Which matrix represents the following system of equations?

$$\begin{aligned} x &= 6 \\ y &= 3 \end{aligned}$$

(a)

$$\begin{bmatrix} 1 & 6 \\ 1 & 3 \end{bmatrix}$$

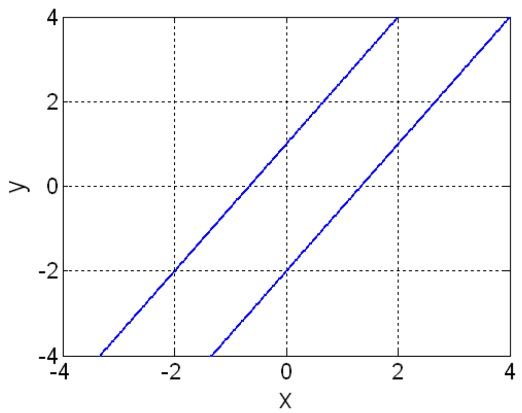
(b)

$$\begin{bmatrix} 1 & 1 & 9 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 3 \end{bmatrix}$$

30. The rows of which augmented matrix represent the equations plotted below?



(a)

$$\begin{bmatrix} 3 & -2 & 4 \\ 6 & -4 & 8 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 3 & -2 & -2 \\ 3 & -2 & 4 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 3 & 2 & -2 \\ 3 & -2 & 2 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 6 & -4 & 8 \\ 3 & 2 & -2 \end{bmatrix}$$

31. What is the solution to the system of equations represented with this augmented matrix?

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

(a) $x = 2$, $y = 3$, $z = 4$

(b) $x = -1$, $y = 1$, $z = 1$

(c) There are an infinite number of solutions.

(d) There is no solution.

(e) We can't tell without having the system of equations.

32. What is the solution to the system of equations represented with this augmented matrix?

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

(a) $x = 2$, $y = 3$, $z = 4$

(b) $x = -1$, $y = 1$, $z = 1$

(c) There are an infinite number of solutions.

(d) There is no solution.

(e) We can't tell without having the system of equations.

33. Suppose we want to graph the equations represented by the rows of the augmented matrix below. What will they look like?

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

(a) These equations represent two lines that intersect at $x = 2$ and $y = 3$.

(b) These equations represent three parallel planes.

(c) These equations represent three planes that are not parallel, but which do not share a common point of intersection.

(d) These equations cannot be represented geometrically.

34. What is the solution to the system of equations represented with this augmented matrix?

$$\begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) $x = 2$, $y = 3$, $z = 4$

(b) $x = -1$, $y = 1$, $z = 1$

(c) There are an infinite number of solutions.

(d) There is no solution.

(e) We can't tell without having the system of equations.

35. Suppose we want to graph the equations represented by the rows of the augmented matrix below. What will they look like?

$$\begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) These equations represent two equations for the same plane.

(b) These equations represent three equations for the same plane.

(c) These equations represent two planes that have a line of points in common.

(d) The intersection of these linear equations is represented by a plane in $\Re^3$.

(e) These equations cannot be represented geometrically.

Gaussian Elimination

36. Which of the following operations on an augmented matrix could change the solution set of a system?

    (a) Interchanging two rows
    (b) Multiplying one row by any constant
    (c) Adding one row to an other
    (d) Adding a multiple of one row to an other
    (e) None of the above
    (f) More than one of the above (which ones?)

37. Which of the following matrices is NOT row equivalent to the one below? In other words, which matrix could you NOT get from the matrix below through elementary row operations?

$$\begin{bmatrix} 2 & 1 & 3 & 1 \\ 0 & 1 & 3 & 4 \\ 1 & 2 & 0 & 4 \end{bmatrix}$$

    (a)
$$\begin{bmatrix} 2 & 4 & 0 & 8 \\ 0 & 1 & 3 & 4 \\ 2 & 1 & 3 & 1 \end{bmatrix}$$
    (b)
$$\begin{bmatrix} 2 & 1 & 3 & 1 \\ 0 & 1 & 3 & 4 \\ 1 & 3 & 3 & 8 \end{bmatrix}$$
    (c)
$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 0 & 3 & 4 \\ 2 & 1 & 0 & 4 \end{bmatrix}$$
    (d) More than one of the above
    (e) All are possible through elementary row operations.

38. Which of the following matrices is row equivalent to the one below? In other words, which matrix could you get from the matrix below through elementary row operations?

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a)

$$\begin{bmatrix} 2 & 5 & 7 \\ 0 & 1 & 3 \\ 4 & 8 & 1 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 3 & 1 & 5 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} -3 & 1 & 3 \\ -2 & 1 & 0 \\ 3 & 9 & 2 \end{bmatrix}$$

(d) More than one of the above

(e) All are possible through elementary row operations.

39. Which of the following matrices is NOT row equivalent to the one below? In other words, which matrix could you NOT get from the matrix below through elementary row operations?

$$\begin{bmatrix} 6 & 0 & 4 & 7 \\ 2 & 0 & 1 & 9 \\ 5 & 0 & 3 & 5 \end{bmatrix}$$

(a)

$$\begin{bmatrix} 12 & 0 & 8 & 14 \\ 2 & 0 & 1 & 9 \\ 1 & 0 & 1 & 2 \end{bmatrix}$$

(b)

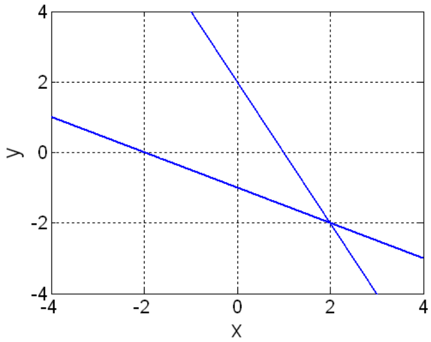
$$\begin{bmatrix} 12 & 0 & 8 & 14 \\ 0 & 0 & 1 & -20 \\ 2 & 1 & 3 & 0 \end{bmatrix}$$

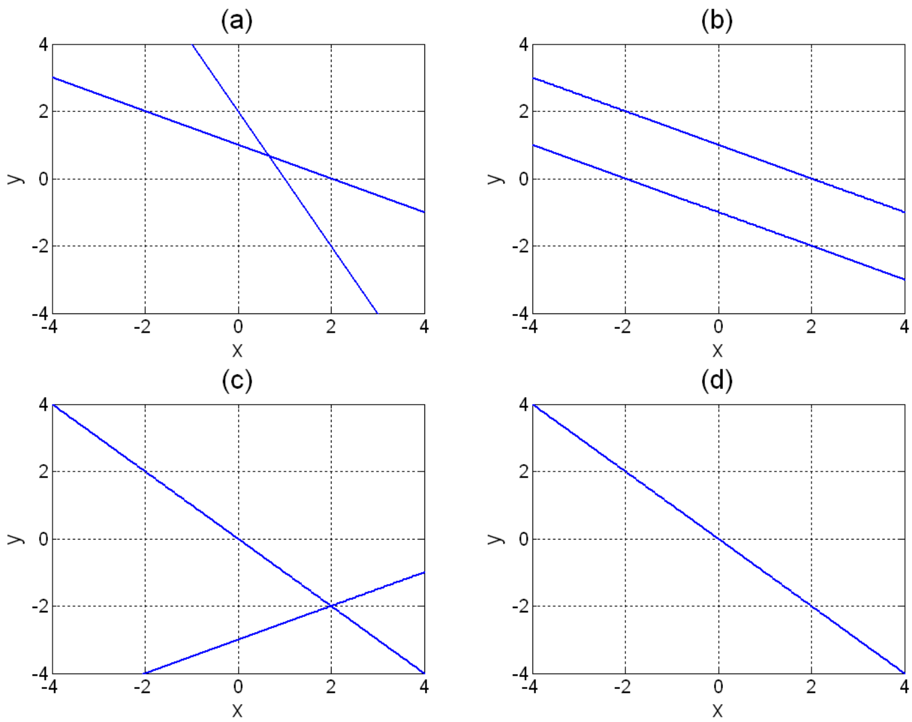
(c)

$$\begin{bmatrix} 6 & 0 & 4 & 7 \\ 2 & 0 & 1 & 9 \\ 7 & 0 & 4 & 14 \end{bmatrix}$$

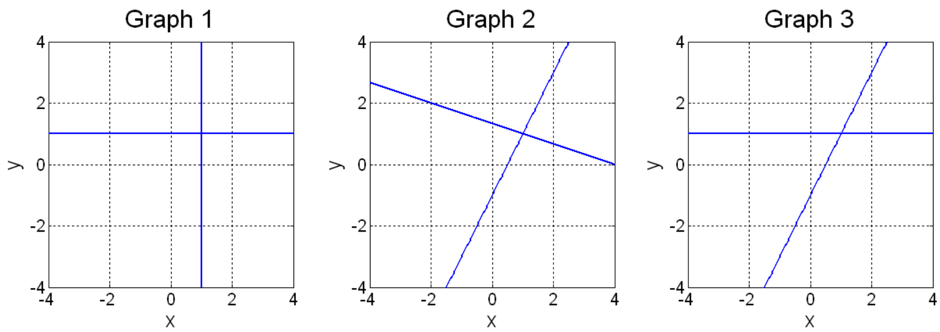
(d) All are possible through elementary row operations.

40. A linear system of equations is plotted below. We create an augmented matrix to represent this linear system, then perform a series of elementary row operations. Which of the following graphs could represent the result of these row operations?





41. We have a system of two linear equations and two unknowns which we solve by performing Gaussian elimination on an augmented matrix. Along the way we create the graphs below, showing geometrical representations of the initial system, the system at an intermediate step in the row reduction process, and the system after it has been put into reduced row echelon form. Put these graphs in order, starting with the initial system and ending with the system in reduced row echelon form.



(a) Graph 2, Graph 3, Graph 1
(b) Graph 1, Graph 3, Graph 2
(c) Graph 1, Graph 2, Graph 3
(d) Graph 2, Graph 1, Graph 3
(e) Graph 3, Graph 2, Graph 1

42. What is the value of a so that the linear system represented by the following matrix would have infinitely many solutions?

$$\begin{bmatrix} 2 & 6 & 8 \\ 1 & a & 4 \end{bmatrix}$$

(a) $a = 0$
(b) $a = 2$
(c) $a = 3$
(d) $a = 4$
(e) This is not possible.
(f) More than one of the above

43. We start with a system of two linear equations in two variables and we translate this system into an augmented matrix M. After performing Gaussian elimination, putting this matrix into reduced row echelon form, we get the matrix R which tells us that this system has no solution. How could we geometrically represent the linear equations contained in the rows of the augmented matrix R?

(a) We can represent the equations of R as two parallel lines.
(b) We can represent the equations of R as two lines that may not be parallel.
(c) We can represent the equations of R as a single line.
(d) The equations of R cannot be represented geometrically.

44. We start with a system of three linear equations in three variables and we translate this system into an augmented matrix M. After performing Gaussian elimination, putting this matrix into reduced row echelon form, we get the matrix R which tells us that this system has no solution. How could we best geometrically represent the linear equations contained in the rows of the augmented matrix M?

(a) We can represent the equations of M as three parallel lines.

(b) We can represent the equations of M as three parallel planes.

(c) We can represent the equations of M as three planes, where at least two must be parallel.

(d) We can represent the equations of M as three planes, where none of the planes ever intersects with another.

(e) We can represent the equations of M as three planes, which do not share any points in common.

(f) The equations of M cannot be represented geometrically.

45. Howard's store sells three blends of flour: standard, extra wheat, and extra soy. Each is a blend of whole wheat flour and soy flour, and the table below shows how many pounds of each type of flour is needed to make one pound of each blend.

whole wheat flour	soy flour	
0.5	0.5	Standard Blend
0.8	0.2	Extra Wheat
0.3	0.7	Extra Soy

A customer comes in who wants one pound of a blend that is 60% wheat and 40% soy. We can solve the following system of equations to determine the amount of Standard Blend (x_1), Extra Wheat Blend (x_2), and Extra Soy Blend (x_3) needed to create this special mixture.

$$\begin{aligned} 0.5x_1 + 0.8x_2 + 0.3x_3 &= 0.6 \\ 0.5x_1 + 0.2x_2 + 0.7x_3 &= 0.4 \end{aligned}$$

If we form an augmented matrix for this system, the reduced row echelon form is $R = \begin{bmatrix} 1 & 0 & 5/3 & 2/3 \\ 0 & 1 & -2/3 & 1/3 \end{bmatrix}$.

If the store is out of Extra Soy Blend, how much of each of the other blends is needed?

(a) 2/3 pound of Standard Blend and 1/3 pound of Extra Wheat Blend

(b) 5/3 pound of Standard Blend and 2/3 pound of Extra Wheat Blend

(c) There are an infinite number of options for the amounts of Standard and Extra Wheat Blend.

(d) It is not possible to create this mixture without Extra Soy Blend.

46. Referring to the previous question, if the store is out of Extra Wheat Blend (x_2), how much of each of the other blends is needed to make the special mixture?

    (a) 2/3 pound of Standard Blend and 1/3 pound of Extra Soy Blend
    (b) 1/6 pound of Standard Blend and 1/2 pound of Extra Soy Blend
    (c) There are an infinite number of options for the amounts of Standard Blend and Extra Wheat Blend.
    (d) It is not possible to create this mixture without Extra Wheat Blend.

47. Referring to the previous two questions, what values are realistic for x_3 in this context?

    (a) x_3 can be any value.
    (b) $x_3 \geq 0$
    (c) $\frac{2}{3} \leq x_3 \leq \frac{5}{3}$
    (d) $-\frac{1}{2} \leq x_3 \leq \frac{2}{5}$
    (e) $0 \leq x_3 \leq \frac{2}{5}$

48. Let R be the reduced row echelon form of a $n \times n$ matrix A. Then

    (a) R is the identity.
    (b) R has at least one row of zeros.
    (c) None of the above.
    (d) All of the above are possible but there exist also other possibilities.
    (e) The two possibilities above are the only ones.
    (f) We can't tell without having the matrix A.

Linear Combinations

49. If $u = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ and $v = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$, what is $2u - 3v$?

    (a) $\begin{bmatrix} -4 \\ 4 \\ 23 \end{bmatrix}$

(b) $\begin{bmatrix} 8 \\ 4 \\ -7 \end{bmatrix}$

(c) $\begin{bmatrix} 8 \\ 4 \\ 23 \end{bmatrix}$

(d) $\begin{bmatrix} 7 \\ 6 \\ 2 \end{bmatrix}$

50. Write $z = \begin{bmatrix} -5 \\ 3 \\ 16 \end{bmatrix}$ as a linear combination of $x = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$ and $y = \begin{bmatrix} -3 \\ 2 \\ 6 \end{bmatrix}$.

(a) $z = -5x$

(b) $z = -2x + y$

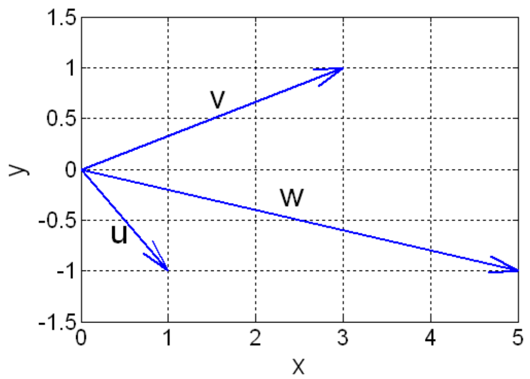
(c) $z = x + 2y$

(d) $z = 2x + y$

(e) z cannot be written as a linear combination of x and y.

(f) None of the above

51. Write the vector w as a linear combination of u and v.



(a) $w = 2u + v$

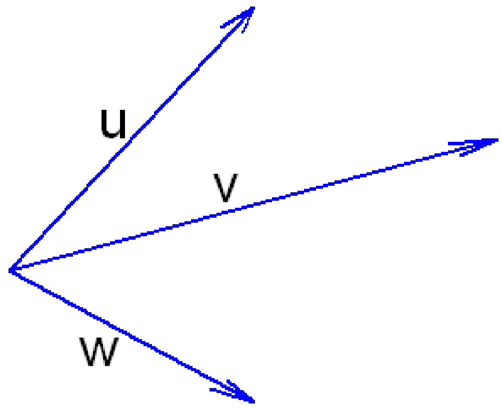
(b) $w = u + v$

(c) $w = -u + v$

(d) $w = u - v$

(e) w cannot be written as a linear combination of u and v.

52. Write the vector w as a linear combination of u and v.



(a) $w = 2u + v$

(b) $w = u + v$

(c) $w = -u + v$

(d) $w = u - v$

(e) w cannot be written as a linear combination of u and v.

53. Write $z = \begin{bmatrix} 2 \\ -2 \\ 6 \end{bmatrix}$ as a linear combination of $x = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$ and $y = \begin{bmatrix} 4 \\ -5 \\ 7 \end{bmatrix}$.

(a) $z = x + y$

(b) $z = -x + y$

(c) $z = 3x + 2y$

(d) $z = -3x + y$

(e) z cannot be written as a linear combination of x and y.

(f) None of the above

54. Suppose we have the vectors $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$. Which of the following is *not* a linear combination of these?

(a) $\begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix}$

(b) $\begin{bmatrix} 8 \\ 0 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 8 \\ 1 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$

(e) $\begin{bmatrix} 40 \\ 5 \\ 15 \end{bmatrix}$

(f) More than one of the above is not a linear combination of the given vectors.

55. Suppose we have the vectors $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$. Which of the following is true?

(a) Every vector in $\Re^3$ can be written as a linear combination of these vectors.

(b) Some, but not all, vectors in $\Re^3$ can be written as a linear combination of these vectors.

(c) Every vector in $\Re^2$ can be written as a linear combination of these vectors.

(d) More than one of the above is true.

(e) None of the above are true.

56. Which of the following vectors can be written as a linear combination of the vectors (1, 0) and (0, 1)?

(a) $(2, 0)$

(b) $(-3, 1)$

(c) $(0.4, 3.7)$

(d) All of the above

57. How do you describe the set of all linear combinations of the vectors (1, 0) and (0, 1)?

(a) A point

(b) A line segment

(c) A line

(d) $\Re^2$
(e) $\Re^3$

58. Which of the following vectors can be written as a linear combination of the vectors (1, 0, 0), (0, 1, 0), and (0, 0, 1)?
    (a) $(0, 2, 0)$
    (b) $(-3, 0, 1)$
    (c) $(0.4, 3.7, -1.5)$
    (d) All of the above

59. How do you describe the set of all linear combinations of the vectors (1, 0, 0), (0, 1, 0), (0, 0, 1)?
    (a) A point
    (b) A line segment
    (c) A line
    (d) $\Re^2$
    (e) $\Re^3$

60. How do you describe the set of all linear combinations of the vectors (1, 2, 0) and (-1, 1, 0)?
    (a) A point
    (b) A line
    (c) A plane
    (d) $\Re^2$
    (e) $\Re^3$

61. Let z be any vector from $\Re^3$. If we have a set V of unknown vectors from $\Re^3$, how many vectors must be in V to guarantee that z can be written as a linear combination of the vectors in V?
    (a) 2
    (b) 3
    (c) 4
    (d) It is not possible to make such a guarantee.

62. Suppose y and z are both solutions to $Ax = b$. **True or False** All linear combinations of y and z also solve $Ax = b$. (You should be prepared to support your answer with either a proof or a counterexample.)

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

63. Suppose y and z are both solutions to $Ax = 0$. **True or False** All linear combinations of y and z also solve $Ax = 0$. (You should be prepared to support your answer with either a proof or a counterexample.)

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

64. Lucinda owns two ice cream parlors. The first ice cream shop sells 5 gallons of vanilla ice cream and 8 gallons of chocolate ice cream each day. The daily sales at the second store are 6 gallons of vanilla ice cream and 10 gallons of chocolate ice cream. The daily sales at stores one and two can be represented by the vectors $s_1 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ and $s_2 = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$, respectively. In this context, what interpretation can be given to the vector $15s_1$?

(a) $15s_1$ shows the number of people that can be served with 15 gallons of vanilla ice cream.

(b) $15s_1$ shows the gallons of vanilla and chocolate ice cream sold by store 1 in 15 days.

(c) $15s_1$ gives the total revenue from selling 15 gallons of ice cream at store 1.

(d) $15s_1$ represents the number of days it will take to sell 15 gallons of ice cream at store 1.

65. Lucinda owns two ice cream parlors. The first ice cream shop sells 5 gallons of vanilla ice cream and 8 gallons of chocolate ice cream each day. The daily sales at the second store are 6 gallons of vanilla ice cream and 10 gallons of chocolate ice cream. The daily sales at stores one and two can be represented by the vectors $s_1 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ and

$s_2 = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$, respectively. The stores are run by different managers, and they are not always able to be open the same number of days in a month. If store 1 is open for c_1 days in March, and store 2 is open for c_2 days in March, which of the following represents the total sales of each flavor of ice cream between the two stores?

(a) $c_1 s_1 + c_2 s_2$

(b) $\begin{bmatrix} 5 & 6 \\ 8 & 10 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

(c) $\begin{bmatrix} 5c_1 \\ 8c_1 \end{bmatrix} + \begin{bmatrix} 6c_2 \\ 10c_2 \end{bmatrix}$

(d) All of the above

(e) None of the above

66. Lucinda owns two ice cream parlors. The first ice cream shop sells 5 gallons of vanilla ice cream and 8 gallons of chocolate ice cream each day. The daily sales at the second store are 6 gallons of vanilla ice cream and 10 gallons of chocolate ice cream. The daily sales at stores one and two can be represented by the vectors $s_1 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ and $s_2 = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$, respectively. Lucinda is getting ready to close her ice cream parlors for the winter. She has a total of 39 gallons of vanilla ice cream in her warehouse, and 64 gallons of chocolate ice cream. She would like to distribute the ice cream to the two stores so that it is used up before the stores close for the winter. How much ice cream should she take to each store? The stores may stay open for different number of days, but no store may run out of ice cream before the end of the day on which it closes.

(a) Lucinda should take 3 gallons of each kind of ice cream to store 1 and 4 gallons of each kind to store 2.

(b) Lucinda should take 3 gallons of vanilla to each store and 4 gallons of chocolate to each store.

(c) Lucinda should take 15 gallons of vanilla and 24 gallons of chocolate to store 1, and she should take 24 gallons of vanilla and 40 gallons of chocolate to store 2.

(d) Lucinda should take 15 gallons of vanilla and 32 gallons of chocolate to store 1, and she should take 18 gallons of vanilla and 40 gallons of chocolate to store 2.

(e) This cannot be done unless ice cream is thrown out or a store runs out of ice cream before the end of the day.

Solution Sets of Linear Systems

67. Which of the following are solutions to the system of equations?

$$\begin{aligned} 2x + y + 2z &= 0 \\ -x + 2y - 6z &= 0 \end{aligned}$$

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ 6 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \\ -2 \end{bmatrix}$

(e) None of the above.

(f) More than one of the above.

68. What is the solution to the following system of equations?

$$\begin{aligned} x + 2y + z &= 0 \\ x + 3y - 2z &= 0 \end{aligned}$$

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \\ 1 \end{bmatrix} s$

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 3 \\ 0 \end{bmatrix}$

(e) None of the above.

(f) More than one of the above.

69. What is the solution to the following system of equations?

$$\begin{aligned} x + 2y + z &= 3 \\ x + 3y - 2z &= 4 \end{aligned}$$

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} s$

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

(d) None of the above.

(e) More than one of the above.

70. What is the solution to the following system of equations?

$$\begin{aligned} x + 2y + z &= -2 \\ x + 3y - 2z &= 1 \end{aligned}$$

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

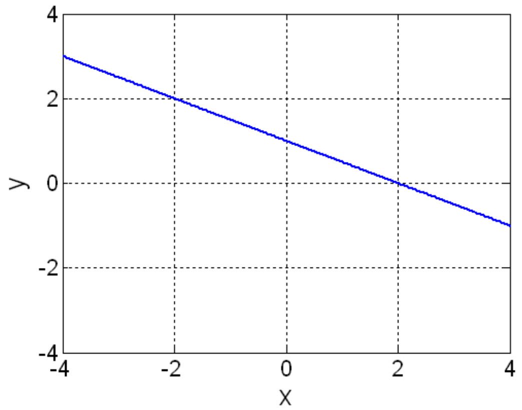
(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -3 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 7 \\ -3 \\ -1 \end{bmatrix} s$

(d) None of the above.

(e) More than one of the above.

71. The set of solutions to a system of linear equations is plotted below. Which of the following expressions represents this solution set?



(a) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1/2 \end{bmatrix} s$

(b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -1/2 \\ 2 \end{bmatrix} s$

(c) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1/2 \\ 2 \end{bmatrix} s$

(d) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} s$

(e) None of the above.

(f) More than one of the above.

72. The set of solutions to a linear system are represented by the expression below. How can we geometrically represent this solution set?

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} t$$

(a) As a line in $\Re^2$

(b) As a line in $\Re^3$

(c) As a plane in $\Re^3$

(d) As a volume in $\Re^3$

(e) None of the above

73. Let $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$. If R is the reduced row echelon form of the augmented matrix for the system $Ax = b$, what are the solutions to that system?

(a) $x_1 = 1, x_2 = 1,$ and $x_3 = 2$

(b) $x_1 = 1, x_2 = 1, x_3 = 2,$ and $x_4 = 0$

(c) $x_1 = -t, x_2 = -t, x_3 = -2t,$ and $x_4 = t$

(d) There are no solutions to this system.

74. Let $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$. If R is the reduced row echelon form of the coefficient matrix for the system $Ax = 0$, what are the solutions to that system?

(a) $x_1 = 1, x_2 = 1,$ and $x_3 = 2$

(b) $x_1 = 1, x_2 = 1, x_3 = 2,$ and $x_4 = 0$

(c) $x_1 = -t, x_2 = -t, x_3 = -2t,$ and $x_4 = t$

(d) There are no solutions to this system.

75. Let matrix R be the reduced row echelon form of matrix A. **True or False** The solutions to $Rx = 0$ are the same as the solutions to $Ax = 0$.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

76. Let matrix R be the reduced row echelon form of matrix A. **True or False** The solutions to $Rx = b$ are the same as the solutions to $Ax = b$.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

77. Consider a homogeneous linear system with n unknowns. Suppose the reduced row echelon form of its augmented matrix has $r \leq n$ nonzero rows. We can conclude that:

(a) $x_1 = 0, x_2 = 0, \ldots, x_n = 0$ is a solution to the system.

(b) The system has $n - r$ free variables (parameters).

(c) The system has infinitely many solutions.

(d) None of the above.

(e) More than one of the above.

Linear Independence

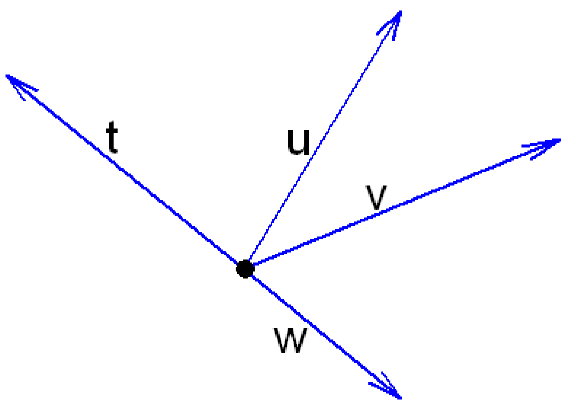
78. **True or False** The following vectors are linearly independent: (1,0,0), (0,0,2), (3,0,4)

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

79. Which set of vectors is linearly independent?

    (a) $(2,3),(8,12)$
    (b) $(1,2,3),(4,5,6),(7,8,9)$
    (c) $(-3,1,0),(4,5,2),(1,6,2)$
    (d) None of these sets are linearly independent.
    (e) Exactly two of these sets are linearly independent.
    (f) All of these sets are linearly independent.

80. Which subsets of the set of the vectors shown below are linearly dependent?



    (a) u,w
    (b) t,w
    (c) t,v
    (d) t,u,v
    (e) None of these sets are linearly dependent.
    (f) More than one of these sets is linearly dependent.

81. Suppose you wish to determine whether a set of vectors is linearly independent. You form a matrix with those vectors as the columns, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. What do you decide?

(a) These vectors are linearly independent.
(b) These vectors are not linearly independent.

82. Suppose you wish to determine whether a set of vectors $\{v_1, v_2, v_3, v_4\}$ is linearly independent. You form the matrix $A = [v_1 v_2 v_3 v_4]$, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. You now decide to write v_4 as a linear combination of v_1, v_2, and v_3. Which is a correct linear combination?

(a) $v_4 = v_1 + v_2$
(b) $v_4 = -v_1 - 2v_3$
(c) v_4 cannot be written as a linear combination of v_1, v_2, and v_3.
(d) We cannot determine the linear combination from this information.

83. Suppose you wish to determine whether a set of vectors $\{v_1, v_2, v_3, v_4\}$ is linearly independent. You form the matrix $A = [v_1 v_2 v_3 v_4]$, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. You now decide to write v_3 as a linear combination of v_1, v_2, and v_4. Which is a correct linear combination?

(a) $v_3 = (1/2)v_1 - (1/2)v_4$
(b) $v_3 = (1/2)v_1 + (1/3)v_2$
(c) $v_3 = 2v_1 + 3v_2$
(d) $v_3 = -2v_1 - 3v_2$
(e) v_3 cannot be written as a linear combination of v_1, v_2, and v_4.
(f) We cannot determine the linear combination from this information.

84. Suppose you wish to determine whether a set of vectors $\{v_1, v_2, v_3, v_4\}$ is linearly independent. You form the matrix $A = [v_1 v_2 v_3 v_4]$, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. You now decide to write v_2 as a linear combination of v_1, v_3, and v_4. Which is a correct linear combination?

(a) $v_2 = 3v_3 + v_4$

(b) $v_2 = -3v_3 - v_4$

(c) $v_2 = v_4 - 3v_3$

(d) $v_2 = -v_1 + v_4$

(e) v_2 cannot be written as a linear combination of v_1, v_3, and v_4.

(f) We cannot determine the linear combination from this information.

85. Are the vectors $\left\{ \begin{bmatrix} 1 \\ 4 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -14 \\ 13 \\ 7 \\ -19 \end{bmatrix} \right\}$ linearly independent?

(a) Yes, they are linearly independent.

(b) No, they are not linearly independent.

86. To determine whether a set of n vectors from $\Re^n$ is independent, we can form a matrix A whose columns are the vectors in the set and then put that matrix in reduced row echelon form. If the vectors are linearly independent, what will we see in the reduced row echelon form?

(a) A row of all zeros.

(b) A row that has all zeros except in the last position.

(c) A column of all zeros.

(d) An identity matrix.

87. To determine whether a set of fewer than n vectors from $\Re^n$ is independent, we can form a matrix A whose columns are the vectors in the set and then put that matrix in reduced row echelon form. If the vectors are linearly independent, what will we see in the reduced row echelon form?

(a) An identity submatrix with zeros below it.

(b) A row that has all zeros except in the last position.

(c) A column that is not an identity matrix column.

(d) A column of all zeros.

88. If the columns of A are not linearly independent, how many solutions are there to the system $Ax = 0$?

(a) 0
(b) 1
(c) infinite
(d) Not enough information is given.

89. **True or False** A set of 4 vectors from $\Re^3$ could be linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

90. **True or False** A set of 2 vectors from $\Re^3$ must be linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

91. **True or False** A set of 3 vectors from $\Re^3$ could be linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

92. **True or False** A set of 5 vectors from $\Re^4$ could be linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

93. Which statement is equivalent to saying that v_1, v_2, and v_3 are linearly independent vectors?

(a) The only solution to $c_1v_1 + c_2v_2 + c_3v_3 = 0$ is $c_1 = c_2 = c_3 = 0$.

(b) v_3 cannot be written as a linear combination of v_1 and v_2.

(c) No vector is a multiple of any other.

(d) Exactly two of (a), (b), and (c) are true.

(e) All three statements are true.

Linear Transformations and Projections

94. Define $T(v) = Av$, where $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Then $T(v)$

(a) reflects v about the x_2-axis.

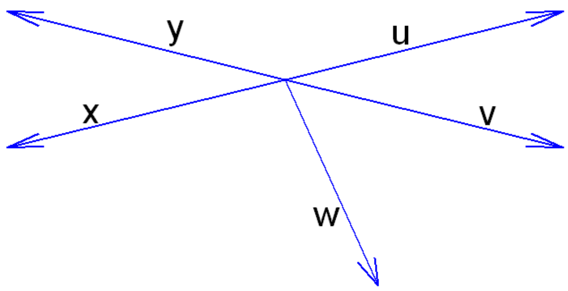
(b) reflects v about the x_1-axis.

(c) rotates v clockwise $\pi/2$ radians about the origin.

(d) rotates v counterclockwise $\pi/2$ radians about the origin.

(e) None of the above

95. Define $T(u) = Au$, where $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$. Using the vectors from $\Re^2$ plotted below, this means that



(a) $T(u) = v$.

(b) $T(u) = w$.

(c) $T(u) = x$.

(d) $T(u) = y$.

(e) None of the above

96. If the linear transformation $T(v) = Av$ rotates the vectors $v_1 = (-1, 0)$ and $v_2 = (0, 1)$ clockwise $\pi/2$ radians, the resulting vectors are

(a) $T(v_1) = \left(-\sqrt{2}/2, \sqrt{2}/2\right)$ and $T(v_2) = \left(\sqrt{2}/2, \sqrt{2}/2\right)$
(b) $T(v_1) = \left(-\sqrt{2}/2, -\sqrt{2}/2\right)$ and $T(v_2) = \left(-\sqrt{2}/2, \sqrt{2}/2\right)$
(c) $T(v_1) = (0, -1)$ and $T(v_2) = (-1, 0)$
(d) $T(v_1) = (0, 1)$ and $T(v_2) = (1, 0)$
(e) None of the above

97. If the linear transformation $T(v) = Av$ rotates the vectors $(-1, 0)$ and $(0, 1)$ clockwise $\pi/2$ radians then

(a) $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(d) $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(e) None of the above

98. If the linear transformation $T(v) = Av$ rotates the vectors $v_1 = (-1, 0)$ and $v_2 = (0, 1)$ clockwise π radians, the resulting vectors are

(a) $T(v_1) = (1, 0)$ and $T(v_2) = (0, -1)$
(b) $T(v_1) = (-1, 0)$ and $T(v_2) = (0, 1)$
(c) $T(v_1) = (0, 1)$ and $T(v_2) = (1, 0)$
(d) $T(v_1) = (0, -1)$ and $T(v_2) = (-1, 0)$
(e) None of the above

99. If the linear transformation $T(v) = Av$ rotates the vectors $(-1, 0)$ and $(0, 1)$ π radians clockwise then

(a) $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $A = \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

(d) $A = \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$

(e) None of the above

100. If the linear transformation $T(v) = Av$ rotates the vector v θ radians clockwise, then

(a) $A = \begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix}$

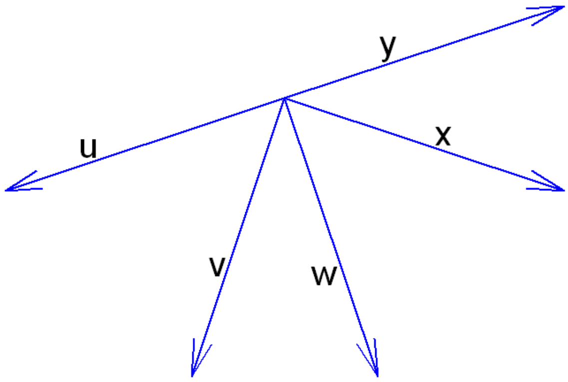
(b) $A = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$

(c) $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

(d) $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

(e) None of the above

101. The linear transformation $T(v) = Av$ produces $T(u) = w$, $T(v) = x$ and $T(w) = y$, as shown below. Which of the following could be the matrix A?



(a) $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(d) $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(e) None of the above

102. The linear transformation $T(x, y) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$, can be written as

(a) $T(x,y) = (x,y)$
(b) $T(x,y) = (y,x)$
(c) $T(x,y) = (-x,y)$
(d) $T(x,y) = (-y,x)$
(e) None of the above

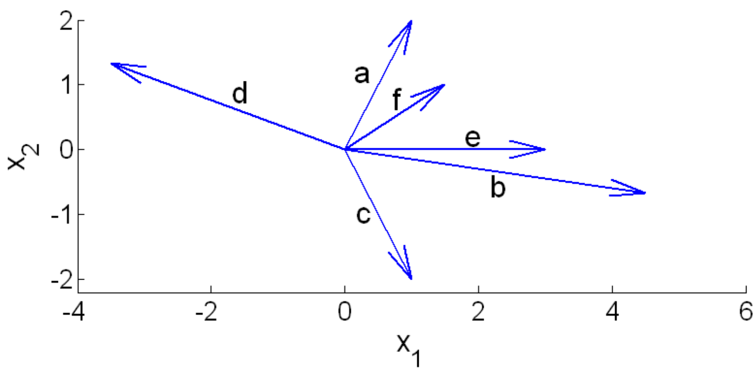
103. The linear transformation $T(x,y) = (x+2y, x-2y)$, can be written as a matrix transformation $T(x,y) = A \begin{bmatrix} x \\ y \end{bmatrix}$ where

(a) $A = \begin{bmatrix} x & 2y \\ x & -2y \end{bmatrix}$
(b) $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$
(c) $A = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}$
(d) It can't be written in matrix form

104. Which of the following is not a linear transformation?

(a) $T(x,y) = (x, y+1)$
(b) $T(x,y) = (x-2y, x)$
(c) $T(x,y) = (4y, x-2y)$
(d) $T(x,y) = (x,0)$
(e) All are linear transformations
(f) More than one of these are not linear transforms

105. **True or False** If a transformation produces $T(a) = b$, $T(c) = d$, and $T(e) = f$, for the vectors plotted below, then this transformation must be nonlinear.

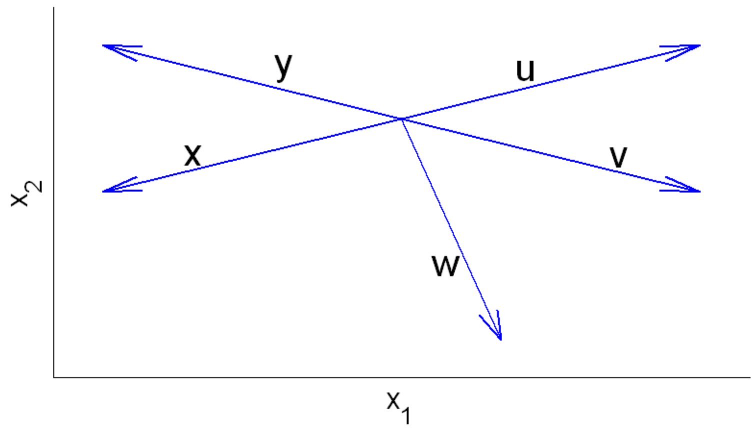


(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

106. Is the transformation $T(x, y, z) = (x, y, 0)$ linear?

(a) No, it is not linear because all z components map to 0.
(b) No, it is not linear because it does not satisfy the scalar multiplication property.
(c) No, it is not linear because it does not satisfy the vector addition property.
(d) No, it is not linear for a reason not listed here.
(e) Yes, it is linear.

107. **True or False** If a transformation produces $T(x) = y$, $T(y) = u$, $T(u) = v$, and $T(v) = w$ for the vectors plotted below, then this transformation must be nonlinear.



(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

108. If f is a function, is the transformation $T(f) = f'$ linear?

(a) No, it is not linear because it does not satisfy the scalar multiplication property.
(b) No, it is not linear because it does not satisfy the vector addition property.
(c) No, it is not linear for a reason not listed here.
(d) Yes, it is linear.

109. What is the range of $T(v) = Av$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 0 \end{bmatrix}$?

(a) All of $\Re^3$
(b) All of $\Re^2$
(c) A line in $\Re^2$
(d) A plane in $\Re^3$
(e) A line in $\Re^3$

110. When we map w to Aw and w is an eigenvector of A, what is the geometric effect?

(a) Aw is a rotation of w.
(b) Aw is a reflection of w in the x-axis.
(c) Aw is a reflection of w in the y-axis.
(d) Aw is parallel to w but may have a different length.

Chapter 2: Matrix Algebra

Matrix Operations

111. What size is this matrix?

$$\begin{bmatrix} 6 & 11 & -2 \\ 23 & 31 & 5 \end{bmatrix}$$

(a) 2x3
(b) 3x2
(c) 6

112. Let $A = \begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$
What is $A + B$?

(a) 71
(b)

$$\begin{bmatrix} 6 & 9 \\ 7 & 11 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 6 & 11 \\ 23 & 31 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 26 & 62 \\ 112 & 268 \end{bmatrix}$$

(e)

$$\begin{bmatrix} 4 & 6 & 2 & 5 \\ 20 & 24 & 3 & 7 \end{bmatrix}$$

113. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$ what is A^T?

(a) $A^T = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$

(b) $A^T = \begin{bmatrix} 2 & 0 & -2 \\ 3 & -1 & 0 \\ 1 & 3 & 4 \end{bmatrix}$

(c) $A^T = \begin{bmatrix} -2 & 0 & 4 \\ 0 & -1 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

(d) $A^T = \begin{bmatrix} 1 & 3 & 4 \\ 3 & -1 & 0 \\ 2 & 0 & -2 \end{bmatrix}$

114. If $A = \begin{bmatrix} 4 & 6 \\ 20 & 7 \end{bmatrix}$ what is $5A$?

(a) $5A = \begin{bmatrix} 9 & 6 \\ 20 & 7 \end{bmatrix}$

(b) $5A = \begin{bmatrix} 9 & 11 \\ 25 & 12 \end{bmatrix}$

(c) $5A = \begin{bmatrix} 20 & 6 \\ 20 & 7 \end{bmatrix}$

(d) $5A = \begin{bmatrix} 20 & 30 \\ 100 & 35 \end{bmatrix}$

115. If A is a matrix and c a scalar such that $cA = 0$ (here 0 represents a matrix with all entries equal to zero), then

(a) A is the identity matrix.

(b) $A = 0$

(c) $c = 0$

(d) Both $A = 0$ and $c = 0$

(e) Either $A = 0$ or $c = 0$

(f) We can't deduce anything.

116. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ then calculate the product AB.

(a) $AB = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

(b) $AB = \begin{bmatrix} 10 & 7 \end{bmatrix}$

(c) $AB = \begin{bmatrix} 8 & 4 \\ -3 & -2 \end{bmatrix}$

(d) $AB = \begin{bmatrix} 7 \\ 10 \end{bmatrix}$

(e) None of the above.

(f) This matrix multiplication is impossible.

117. Calculate $\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix}$.

(a) $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -2 \\ 2 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 \\ -6 & 2 \end{bmatrix}$

(d) None of the above.

(e) This matrix multiplication is impossible.

118. Calculate $\begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$.

(a) $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -2 \\ 2 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 \\ -6 & 2 \end{bmatrix}$

(d) None of the above.

(e) This matrix multiplication is impossible.

119. **True or False** If A and B are square matrices with the same dimensions, then $(A + B) \times (A + B) = A^2 + 2AB + B^2$.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

120. If A and B are both 2x3 matrices, then which of the following is not defined?

(a) $A + B$

(b) $A^T B$

(c) BA

(d) AB^T

(e) More than one of the above

(f) All of these are defined.

121. If A is a 2x3 matrix and B is a 3x6 matrix, what size is AB?

(a) 2x6

(b) 6x2

(c) 3x3

(d) 2x3

(e) 3x6

(f) This matrix multiplication is impossible.

122. In order to compute the matrix product AB, what must be true about the sizes of A and B?

(a) A and B must have the same number of rows.

(b) A and B must have the same number of columns.

(c) A must have as many rows as B has columns.

(d) A must have as many columns as B has rows.

123. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 & 2 \\ 1 & 2 & -1 \\ 3 & 1 & 0 \end{bmatrix}$ what is the (3,2)-entry of AB? (You should be able to determine this without computing the entire matrix product.)

(a) 1

(b) 3

(c) 4

(d) 8

124. You have a business that sells tables and chairs. You have brown tables and white tables, and corresponding chairs. Your May sales are 4 brown tables, 6 white tables, 20 brown chairs, and 24 white chairs, which is represented by the matrix $M = \begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$. where the first row is tables, the second row is chairs, the first column is brown items, and the second column is white items. If your October sales are 50% more than your May sales, which of the following would represent your October sales?

(a) $M + 50$

(b) $0.5M$

(c) $1.5M$

(d) $M^{.5}$

125. You have a business that sells tables and chairs. You have brown tables and white tables, and corresponding chairs. Your May sales are 4 brown tables, 6 white tables, 20 brown chairs, and 24 white chairs, which is represented by the matrix $M = \begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$. where the first row is tables, the second row is chairs, the first column is brown items, and the second column is white items. Your June sales are given by the analogous matrix J, where $J = \begin{bmatrix} 6 & 8 \\ 22 & 32 \end{bmatrix}$. Which of the following matrix operations would make sense in this scenario? Be prepared to explain what the result tells you.

(a) $M + J$

(b) $M - J$

(c) $1.2J$

(d) MJ

(e) All of the above make sense.

(f) More than one, but not all, of the above make sense.

126. You have a business that sells tables and chairs. You have brown tables and white tables, and corresponding chairs. Your May sales are 4 brown tables, 6 white tables, 20 brown chairs, and 24 white chairs, which is represented by the matrix $M = \begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$. where the first row is tables, the second row is chairs, the first column is brown items, and the second column is white items. All tables cost \$350 and all chairs cost \$125, which we represent with the cost vector $C = \begin{bmatrix} 350 \\ 125 \end{bmatrix}$. Which of the following matrix operations could be useful in this scenario? Be prepared to explain what the result tells you.

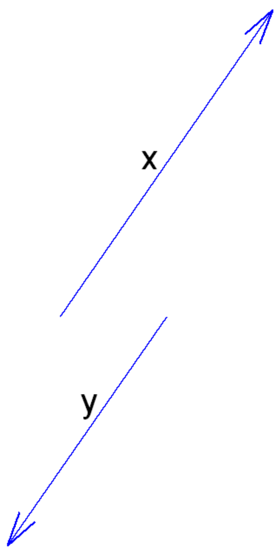
(a) MC

(b) CM

(c) C^TM

(d) MC^T

127. **True or False** Given the vectors x and y plotted below and some matrix A, if we know that $Ax = 0$, this means that $Ay = 0$ as well.



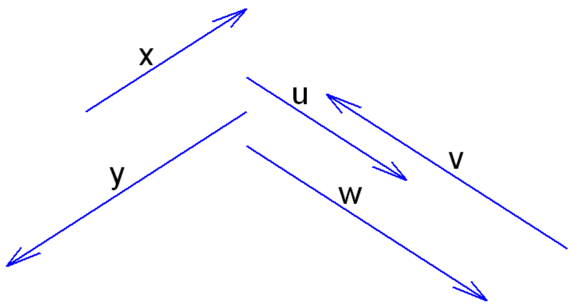
(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

128. Given the vectors x, y, u, v, and w plotted below and some matrix A, if we know that $Ax = u$, what does this tell us about the product Ay?



(a) $Ay = u$
(b) $Ay = v$
(c) $Ay = w$
(d) We cannot say anything about Ay without knowing more about A.

129. Let A, B, C be 3 matrices such that the product ABC is defined. What is $(ABC)^T$?

(a) $(ABC)^T = A^T B^T C^T$
(b) $(ABC)^T = B^T C^T A^T$
(c) $(ABC)^T = C^T A^T B^T$
(d) $(ABC)^T = C^T B^T A^T$

Matrix Inverses

130. Which of the following matrices does not have an inverse?

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$

(e) More than one of the above do not have inverses.
(f) All have inverses.

131. When we put a matrix A into reduced row echelon form, we get the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$. This means that

(a) Matrix A has no inverse.
(b) The matrix we have found is the inverse of matrix A.
(c) Matrix A has an inverse, but this isn't it.
(d) This tells us nothing about whether A has an inverse.

132. Let $A = \begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$. What is A^{-1}?

(a) $\begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$.
(b) $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$.
(c) $\begin{bmatrix} 0 & 1/4 \\ 1/2 & 0 \end{bmatrix}$.
(d) $\begin{bmatrix} 0 & 1/2 \\ 1/4 & 0 \end{bmatrix}$.

133. We find that for a square coefficient matrix A, the homogeneous matrix equation $AX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, has only the trivial solution $X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. This means that

(a) Matrix A has no inverse.
(b) Matrix A has an inverse.
(c) This tells us nothing about whether A has an inverse.

134. **True or False** If A, B, and C are square matrices and we know that $AB = AC$, this means that matrix B is equal to matrix C.

(a) True, and I am very confident
(b) True, but I am not very confident

(c) False, but I am not very confident
(d) False, and I am very confident

135. **True or False** Suppose that A, B, and C are square matrices, and $CA = B$, and A is invertible. This means that $C = A^{-1}B$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

136. We know that $(5A)^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. What is matrix A?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
(b) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$
(c) $\begin{bmatrix} 1/5 & 0 \\ 0 & 1/5 \end{bmatrix}$
(d) $\begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$
(e) There is no matrix A which solves this equation.

137. A and B are invertible matrices. If $AB = C$, then what is the inverse of C?

(a) $C^{-1} = A^{-1}B^{-1}$
(b) $C^{-1} = B^{-1}A^{-1}$
(c) $C^{-1} = AB^{-1}$
(d) $C^{-1} = BA^{-1}$
(e) More than one of the above is true.
(f) Just because A and B have inverses, this doesn't mean that C has an inverse.

138. Let A be a 2×2 matrix. The inverse of $3A$ is

(a) $\frac{1}{9}A^{-1}$
(b) $\frac{1}{3}A^{-1}$

(c) A^{-1}

(d) $3A^{-1}$

(e) Not enough information is given.

139. If A is an invertible matrix, what else must be true?

(a) If $AB = C$ then $B = A^{-1}C$.

(b) A^2 is invertible.

(c) A^T is invertible.

(d) $5A$ is invertible.

(e) The reduced row echelon form of A is I.

(f) All of the above must be true.

Fundamental Vector Subspaces

140. How many linearly independent columns are there in the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(a) 2

(b) 1

(c) 0

141. The *column space* of a matrix A is the set of vectors that can be created by taking all linear combinations of the columns of A. Is the vector $b = \begin{bmatrix} -4 \\ 12 \end{bmatrix}$ in the column space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(a) Yes, since we can find a vector x so that $Ax = b$.

(b) Yes, since $-2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} - 1 \begin{bmatrix} 2 \\ 6 \end{bmatrix} = \begin{bmatrix} -4 \\ 12 \end{bmatrix}$.

(c) No, because there is no vector x so that $Ax = b$.

(d) No, because we can't find c_1 and c_2 such that $c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 6 \end{bmatrix} = \begin{bmatrix} -4 \\ 12 \end{bmatrix}$.

(e) More than one of the above

(f) None of the above

142. The column space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ is

(a) the set of all linear combinations of the columns of A.

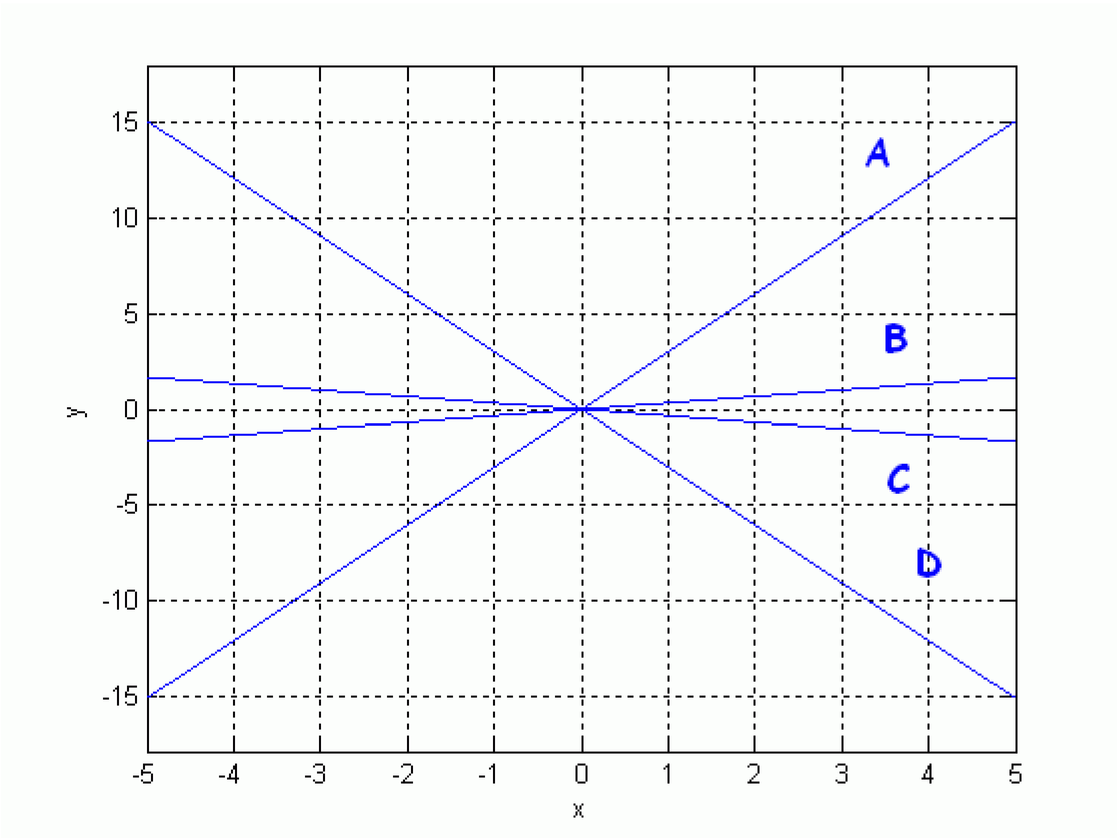
(b) a line in $\Re^2$.

(c) the set of all multiples of the vector $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

(d) All of the above

(e) None of the above

143. Which line in the graph below represents the column space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?



(a) line A

(b) line B

(c) line C

(d) line D

(e) None of the above

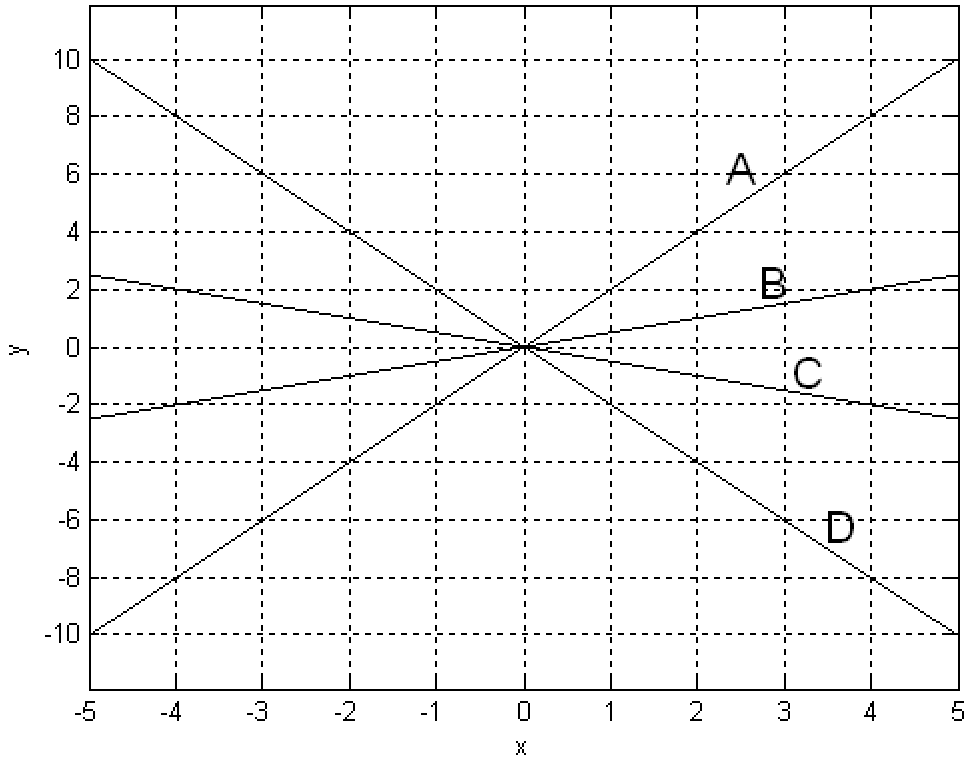
144. How many solutions x are there to $Ax = 0$ where $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(a) 0 solutions
(b) 1 solution
(c) 2 solutions
(d) Infinite number of solutions

145. The *null space* of a matrix A is the set of all vectors x that are solutions of $Ax = 0$. Which of the following vectors is in the null space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(a) $x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$
(b) $x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
(c) $x = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$
(d) All of the above
(e) None of the above

146. Which line in the graph below represents the null space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?



(a) line A
(b) line B
(c) line C
(d) line D
(e) None of the above

147. The *row space* of a matrix A is the set of vectors that can be created by taking all linear combinations of the rows of A. Which of the following vectors is in the row space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(a) $x = \begin{bmatrix} -2 & 4 \end{bmatrix}$
(b) $x = \begin{bmatrix} 4 & 8 \end{bmatrix}$
(c) $x = \begin{bmatrix} 0 & 0 \end{bmatrix}$
(d) $x = \begin{bmatrix} 8 & 4 \end{bmatrix}$
(e) More than one of the above
(f) None of the above

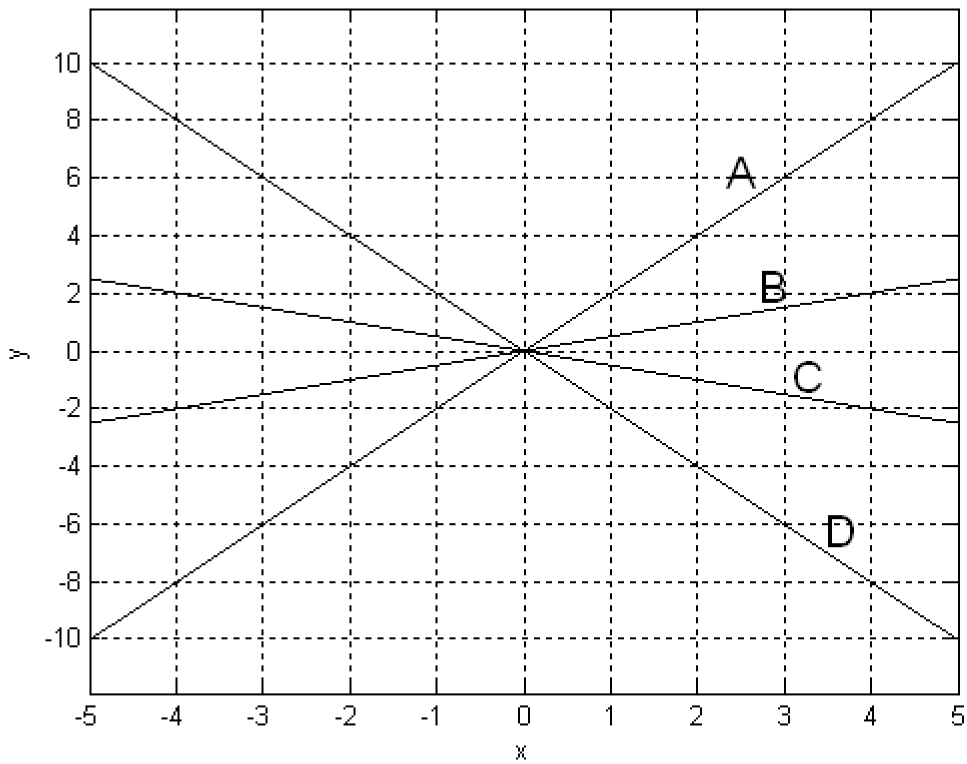
148. **True or False:** The row space of a matrix A is the same as the column space of A^T.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

149. The row space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ consists of

(a) All linear combinations of the columns of A^T.
(b) All multiples of the vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.
(c) All linear combinations of the rows of A.
(d) All of the above
(e) None of the above

150. Which line in the graph below represents the row space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?



(a) line A

(b) line B

(c) line C

(d) line D

(e) None of the above

151. The *left null space* of a matrix A is the set of vectors x that solve $xA = 0$. Which of the following vectors is in the left null space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(a) $x = \begin{bmatrix} -2 & 1 \end{bmatrix}$

(b) $x = \begin{bmatrix} -3 & 1 \end{bmatrix}$

(c) $x = \begin{bmatrix} 1 & -3 \end{bmatrix}$

(d) $x = \begin{bmatrix} 1 & -2 \end{bmatrix}$

(e) More than one of the above

(f) None of the above

152. **True or False:** Since $xA = 0$ can be rewritten as $A^T x^T = 0$, we can think of the left null space as the null space of A^T.

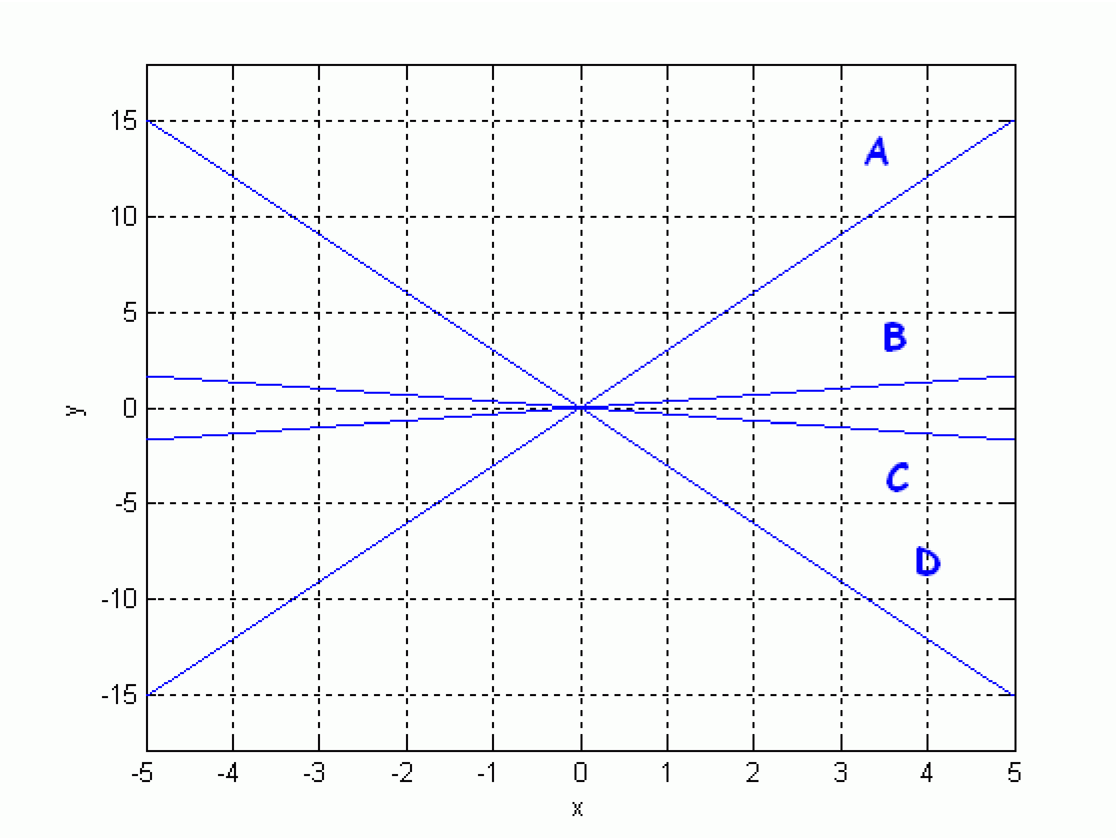
(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

153. Which line in the graph below represents the left null space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?



(a) line A

(b) line B

(c) line C

(d) line D

(e) None of the above

154. Let $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 2 & -1 & 1 & 1 \end{bmatrix}$. Which of the following vectors are in the nullspace of A?

(a) $\begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 3 \\ -1 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 3 \\ -1 \\ 3 \\ 2 \end{bmatrix}$

155. Let $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 2 & -1 & 1 & 1 \end{bmatrix}$. How many vectors are in the nullspace of A?

(a) Only one
(b) Probably more than one, but it's hard to say how many
(c) An infinite number

156. If A is an $m \times n$ matrix, then the column space of A is

(a) A subset of $\Re^m$ that may not include the origin.
(b) A subset of $\Re^m$ that includes the origin.
(c) A subset of $\Re^n$ that may not include the origin.
(d) A subset of $\Re^n$ that includes the origin.
(e) None of the above

157. If A is an $m \times n$ matrix, then the row space of A is

(a) A subset of $\Re^m$ that may not include the origin.
(b) A subset of $\Re^m$ that includes the origin.
(c) A subset of $\Re^n$ that may not include the origin.
(d) A subset of $\Re^n$ that includes the origin.
(e) None of the above

158. If A is an $m \times n$ matrix, then the null space of A is

(a) A subset of $\Re^m$ that may not include the origin.
(b) A subset of $\Re^m$ that includes the origin.
(c) A subset of $\Re^n$ that may not include the origin.
(d) A subset of $\Re^n$ that includes the origin.
(e) None of the above

159. If A is an $m \times n$ matrix, then the left null space of A is

(a) A subset of $\Re^m$ that may not include the origin.
(b) A subset of $\Re^m$ that includes the origin.
(c) A subset of $\Re^n$ that may not include the origin.
(d) A subset of $\Re^n$ that includes the origin.
(e) None of the above

160. Two vector spaces, V and W are *orthogonal complements* if and only if V is the set of all vectors which are orthogonal to every vector in W. Recall that for the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ the null space consists of all multiples of the vector $(-2, 1)$ and the left null space consists of all multiples of the vector $(-3, 1)$. Which of the following are true?

(a) The column space and null space are orthogonal complements.
(b) The column space and row space are orthogonal complements.
(c) The column space and left null space are orthogonal complements.
(d) None of the above

161. Recall that for the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ the null space consists of all multiples of the vector $(-2, 1)$ and the left null space consists of all multiples of the vector $(-3, 1)$. Which of the following vector subspaces are orthogonal complements?

(a) The row space and null space are orthogonal complements.
(b) The row space and column space are orthogonal complements.
(c) The row space and left null space are orthogonal complements.
(d) None of the above

Dimension and Rank

162. Let $A = \begin{bmatrix} 5 & 4 & -8 & 1 \\ 1 & 3 & 4 & 8 \\ 0 & 2 & 1 & 3 \\ -1 & -2 & 4 & 1 \end{bmatrix}$. The reduced row echelon form of A is $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. What is the rank of A?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

163. Suppose a 4×4 matrix A has rank 3. Are the columns of A linearly independent?

(a) Yes, they are linearly independent.
(b) No, they are not linearly independent.
(c) We do not have enough information to decide.

164. Suppose a 4×4 matrix A has rank 4. How many solutions does the system $Ax = b$ have?

(a) 0
(b) 1
(c) Infinite
(d) Not enough information is given.

165. Suppose a 4×4 matrix A has rank 3. How many solutions does the system $Ax = b$ have?

(a) 0
(b) 1
(c) Infinite
(d) Not enough information is given.

166. Suppose a 4×4 matrix A has rank 3. If it is known that (4, 5, 0, 1) is a solution to the system $Ax = b$, then how many solutions does $Ax = b$ have?

(a) 1
(b) Infinite
(c) Not enough information is given.

167. Suppose a 5×5 matrix A has rank 3. If it is known that (-1, 4, 2, 0, 3) is a solution to the system $Ax = b$, then how many parameters does the solution set have?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4
(f) Not enough information is given.

168. **True or False** If $AX = BX$ for all matrices X where the products are defined, then A and B have to be the same matrix.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

169. **True or False** If $Ax = Bx$ for all vectors x where the products are defined, then A and B have to be the same matrix.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

Chapter 3: Determinants

Determinants

170. What is the determinant of $\begin{bmatrix} 5 & 4 \\ 1 & 3 \end{bmatrix}$?

(a) 4
(b) 11
(c) 15

(d) 19

171. What is the determinant of $\begin{bmatrix} 5 & 1 & 0 \\ 1 & 3 & 2 \\ 0 & -1 & 1 \end{bmatrix}$?

(a) 0
(b) 15
(c) 24
(d) 26

172. What is the determinant of $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$?

(a) 0
(b) 9
(c) 15

173. What is the determinant of $\begin{bmatrix} 5 & 2 & -1 \\ 0 & 3 & 4 \\ 0 & 0 & 1 \end{bmatrix}$?

(a) 0
(b) 6
(c) 15
(d) 22

174. Which of the following matrices are not invertible?

(a) $\begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix}$
(b) $\begin{bmatrix} -2 & 3 \\ 2 & 3 \end{bmatrix}$
(c) $\begin{bmatrix} 3 & -3 \\ -2 & 2 \end{bmatrix}$
(d) $\begin{bmatrix} 3 & -3 & 3 \\ -2 & 2 & -2 \\ 0 & 0 & 0 \end{bmatrix}$

(e) More than one of the above

(f) All of the above have inverses

175. **True or False** $\det(A+B) = \det A + \det B$. Be prepared to support your answer either with a proof (at least for the 2×2 case) or a counterexample.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

176. **True or False** $\det(AB) = \det A \det B$. Be prepared to support your answer either with a proof (at least for the 2×2 case) or a counterexample.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

177. Suppose the determinant of a 2×2 matrix A is equal to 3. What is the determinant of A^{-1}?

(a) 1/3

(b) 3

(c) 9

(d) Not enough information is given.

178. Suppose the determinant of a 2×2 matrix A is equal to 3. What is the determinant of $5A$?

(a) 3

(b) 9

(c) 15

(d) 75

(e) Not enough information is given.

179. If A is a 2×2 matrix, then $\det(kA)$ is

(a) $k \det(A)$
(b) $2k \det(A)$
(c) $k^2 \det(A)$
(d) Not enough information is given.

180. Which of the following statements is true?

(a) If a square matrix has two identical rows then its determinant is zero.
(b) If the determinant of a matrix is zero, then the matrix has two identical rows.
(c) Both are true.
(d) Neither is true.

181. Suppose the determinant of matrix A is zero. How many solutions does the system $Ax = b$ have?

(a) 0
(b) 1
(c) Infinite
(d) Not enough information is given.

182. Suppose the determinant of matrix A is zero. How many solutions does the system $Ax = 0$ have?

(a) 0
(b) 1
(c) Infinite
(d) Not enough information is given.

Chapter 4: Vector Spaces

Vector Spaces and Subspaces

183. Which property of vector spaces is not true for the following set?

$$\left\{ \begin{bmatrix} -2 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$$

(a) Closure under vector addition

(b) Existence of an additive identity

(c) Existence of an additive inverse for each vector

(d) None of the above

184. A vector subspace does *not* have to satisfy which of the following properties?

(a) Associativity under vector addition

(b) Existence of an additive identity

(c) Commutativity under vector addition

(d) A vector subspace must satisfy all of the above properties.

(e) A vector subspace need not satisfy any of the above properties.

185. A vector space does *not* have to satisfy which of the following properties?

(a) Closure under vector addition

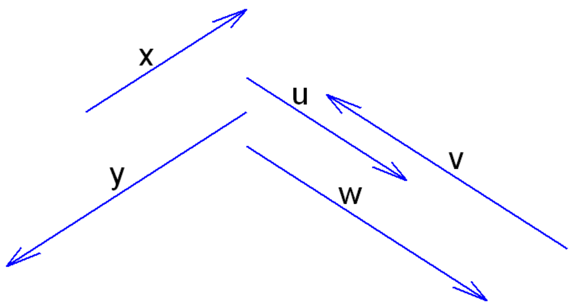
(b) Closure under scalar multiplication

(c) Closure under vector multiplication

(d) A vector subspace must satisfy all of the above properities.

(e) A vector subspace need not satisfy any of the above properties.

186. Which of the following sets of vectors are contained within a proper subspace of $\Re^2$?



i. x, y

ii. u, v, w

iii. x, v

iv. y, u, w

(a) i, ii, iii, and iv

(b) ii, iii, and iv only

(c) i and ii only

(d) ii and iv only

(e) iii and iv only

(f) ii only

187. The set of all 2×2 matrices with determinant equal to zero is not a vector subspace. Why?

(a) 2×2 matrices are not vectors.

(b) With matrices, AB need not equal BA.

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$ is not in the set.

(d) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is not in the set.

(e) None of the above

188. Which of the following sets of vectors is a basis for $\Re^3$?

(a) $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

(b) $\{(1, 0, 1), (1, 1, 0), (1, 1, 1)\}$

(c) $\{(2, 0, 0), (0, 5, 0), (0, 0, 8)\}$

(d) All are bases for $\Re^3$.

189. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$. Which of the following sets has the same span as the set of all three vectors $\{v_1, v_2, v_3\}$?

(a) $\{v_1, v_2\}$

(b) $\{v_2, v_3\}$

(c) $\{v_1, v_3\}$

(d) None of the above

(e) More than one of the above

190. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$. Which of the following vectors is *not* in the subspace of $\Re^3$ spanned by $\{v_1, v_2, v_3\}$?

(a) $(1, 0, 0)$

(b) $(4, 1, 1)$
(c) $(3, 3, 6)$
(d) All of these are in the subspace of $\Re^3$ spanned by $\{v_1, v_2, v_3\}$.

191. Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$. Geometrically, what is the subspace spanned by the set $\{v_1, v_2, v_3\}$?

(a) a point
(b) a line
(c) a plane
(d) a volume
(e) all of $\mathcal{R}^3$

192. Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, w = \begin{bmatrix} k \\ 2 \\ -3 \end{bmatrix}$. For how many values of k will the vector w be in the subspace spanned by $\{v_1, v_2, v_3\}$?

(a) No values of k - vector w will never be in this subspace
(b) Exactly one value of k will work.
(c) Any value of k will work.

193. Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, w = \begin{bmatrix} k \\ 8 \\ 11 \end{bmatrix}$. For how many values of k will the vector w be in the subspace spanned by $\{v_1, v_2, v_3\}$?

(a) No values of k - vector w will never be in this subspace
(b) Exactly one value of k will work.
(c) Any value of k will work.

194. Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, w = \begin{bmatrix} 1 \\ k \\ k \end{bmatrix}$. For how many values of k will the vector w be in the subspace spanned by $\{v_1, v_2, v_3\}$?

(a) No values of k - vector w will never be in this subspace
(b) Exactly one value of k will work.
(c) Any value of k will work.

Linear Independence

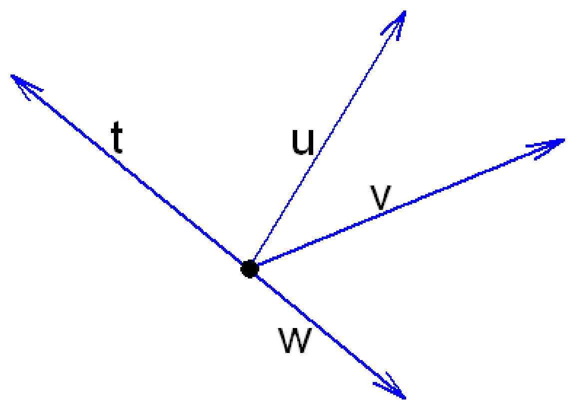
195. **True or False** The following vectors are linearly independent: (1,0,0), (0,0,2), (3,0,4)

    (a) True, and I am very confident
    (b) True, but I am not very confident
    (c) False, but I am not very confident
    (d) False, and I am very confident

196. Which set of vectors is linearly independent?

    (a) $(2, 3), (8, 12)$
    (b) $(1, 2, 3), (4, 5, 6), (7, 8, 9)$
    (c) $(-3, 1, 0), (4, 5, 2), (1, 6, 2)$
    (d) None of these sets are linearly independent.
    (e) Exactly two of these sets are linearly independent.
    (f) All of these sets are linearly independent.

197. Which subsets of the set of the vectors shown below are linearly dependent?



    (a) u, w
    (b) t, w
    (c) t, v
    (d) t, u, v
    (e) None of these sets are linearly dependent.
    (f) More than one of these sets is linearly dependent.

198. Suppose you wish to determine whether a set of vectors is linearly independent. You form a matrix with those vectors as the columns, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. What do you decide?

(a) These vectors are linearly independent.

(b) These vectors are not linearly independent.

199. Suppose you wish to determine whether a set of vectors $\{v_1, v_2, v_3, v_4\}$ is linearly independent. You form the matrix $A = [v_1 v_2 v_3 v_4]$, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. You now decide to write v_4 as a linear combination of v_1, v_2, and v_3. Which is a correct linear combination?

(a) $v_4 = v_1 + v_2$

(b) $v_4 = -v_1 - 2v_3$

(c) v_4 cannot be written as a linear combination of v_1, v_2, and v_3.

(d) We cannot determine the linear combination from this information.

200. Suppose you wish to determine whether a set of vectors $\{v_1, v_2, v_3, v_4\}$ is linearly independent. You form the matrix $A = [v_1 v_2 v_3 v_4]$, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. You now decide to write v_3 as a linear combination of v_1, v_2, and v_4. Which is a correct linear combination?

(a) $v_3 = (1/2)v_1 - (1/2)v_4$

(b) $v_3 = (1/2)v_1 + (1/3)v_2$

(c) $v_3 = 2v_1 + 3v_2$

(d) $v_3 = -2v_1 - 3v_2$

(e) v_3 cannot be written as a linear combination of v_1, v_2, and v_4.

(f) We cannot determine the linear combination from this information.

201. Suppose you wish to determine whether a set of vectors $\{v_1, v_2, v_3, v_4\}$ is linearly independent. You form the matrix $A = [v_1 v_2 v_3 v_4]$, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. You now decide to write v_2 as a linear combination of v_1, v_3, and v_4. Which is a correct linear combination?

(a) $v_2 = 3v_3 + v_4$
(b) $v_2 = -3v_3 - v_4$
(c) $v_2 = v_4 - 3v_3$
(d) $v_2 = -v_1 + v_4$
(e) v_2 cannot be written as a linear combination of v_1, v_3, and v_4.
(f) We cannot determine the linear combination from this information.

202. Are the vectors $\left\{ \begin{bmatrix} 1 \\ 4 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -14 \\ 13 \\ 7 \\ -19 \end{bmatrix} \right\}$ linearly independent?

(a) Yes, they are linearly independent.
(b) No, they are not linearly independent.

203. To determine whether a set of n vectors from $\Re^n$ is independent, we can form a matrix A whose columns are the vectors in the set and then put that matrix in reduced row echelon form. If the vectors are linearly independent, what will we see in the reduced row echelon form?

(a) A row of all zeros.
(b) A row that has all zeros except in the last position.
(c) A column of all zeros.
(d) An identity matrix.

204. To determine whether a set of fewer than n vectors from $\Re^n$ is independent, we can form a matrix A whose columns are the vectors in the set and then put that matrix in reduced row echelon form. If the vectors are linearly independent, what will we see in the reduced row echelon form?

(a) An identity submatrix with zeros below it.
(b) A row that has all zeros except in the last position.
(c) A column that is not an identity matrix column.
(d) A column of all zeros.

205. If the columns of A are not linearly independent, how many solutions are there to the system $Ax = 0$?

(a) 0
(b) 1
(c) infinite
(d) Not enough information is given.

206. **True or False** A set of 4 vectors from $\Re^3$ could be linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

207. **True or False** A set of 2 vectors from $\Re^3$ must be linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

208. **True or False** A set of 3 vectors from $\Re^3$ could be linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

209. **True or False** A set of 5 vectors from $\Re^4$ could be linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

210. Which statement is equivalent to saying that v_1, v_2, and v_3 are linearly independent vectors?

(a) The only solution to $c_1v_1 + c_2v_2 + c_3v_3 = 0$ is $c_1 = c_2 = c_3 = 0$.

(b) v_3 cannot be written as a linear combination of v_1 and v_2.

(c) No vector is a multiple of any other.

(d) Exactly two of (a), (b), and (c) are true.

(e) All three statements are true.

Linearly Independent Sets

211. To determine whether a set S of vectors is linearly independent, you form a matrix which has those vectors as columns, and you calculate its reduced row echelon form. Suppose the resulting form is $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. How many linearly independent vectors are in S?

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

212. To determine whether a set S of vectors is linearly independent, you form a matrix which has those vectors as columns, and you calculate its reduced row echelon form. Suppose the resulting form is $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Which of the following subsets of S are linearly independent?

(a) The first, second, and third vectors

(b) The first, second, and fourth vectors

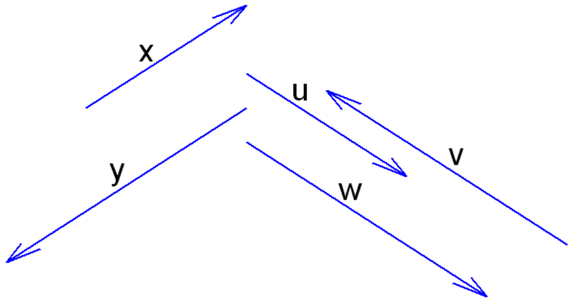
(c) The first, third, and fourth vectors

(d) The second, third, and fourth vectors

(e) All of the above

(f) More than one, but not all, of the above

213. Consider the vectors x, y, u, v, and w in $\Re^2$ plotted below and form a matrix M which has these vectors as columns. What is the rank of this matrix?



(a) $\text{rank}(M) = 1$

(b) $\text{rank}(M) = 2$

(c) $\text{rank}(M) = 3$

(d) $\text{rank}(M) = 4$

(e) $\text{rank}(M) = 5$

214. To determine whether a set of vectors is linearly independent, you form a matrix which has those vectors as columns. If the matrix is square and its determinant is zero, what do you conclude?

(a) The vectors are linearly independent.

(b) The vectors are not linearly independent.

(c) This test is inconclusive, and further work must be done.

215. Which of the following expressions is a linear combination of the functions $f(t)$ and $g(t)$?

(a) $2f(t) + 3g(t) + 4$

(b) $f(t) - 2g(t) + t$

(c) $2f(t)g(t) - 3f(t)$

(d) $f(t) - g(t)$

(e) All of the above

(f) None of the above

(g) Some of the above

216. **True or False** The function $h(t) = 4 + 3t$ is a linear combination of the functions $f(t) = (1 + t)^2$ and $g(t) = 2 - t - 2t^2$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

217. **True or False** The function $h(t) = \sin(t + 2)$ is a linear combination of the functions $f(t) = \sin t$ and $g(t) = \cos t$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

218. **True or False** $h(t) = t^2$ is a linear combination of $f(t) = (1 - t)^2$ and $g(t) = (1 + t)^2$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

219. Let $y_1(t) = \sin(2t)$. For which of the following functions $y_2(t)$ will $\{y_1(t), y_2(t)\}$ be a linearly independent set?

(a) $y_2(t) = \sin(t)\cos(t)$
(b) $y_2(t) = 2\sin(2t)$
(c) $y_2(t) = \cos(2t - \pi/2)$
(d) $y_2(t) = \sin(-2t)$
(e) All of the above
(f) None of the above

220. Let $y_1(t) = e^{2t}$. For which of the following functions $y_2(t)$ will $\{y_1(t), y_2(t)\}$ be a linearly independent set?

(a) $y_2(t) = e^{-2t}$
(b) $y_2(t) = te^{2t}$
(c) $y_2(t) = 1$

(d) $y_2(t) = e^{3t}$
(e) All of the above
(f) None of the above

221. The functions $y_1(t)$ and $y_2(t)$ are linearly independent on the interval $a < t < b$ if

(a) for some constant k, $y_1(t) = ky_2(t)$ for $a < t < b$.
(b) there exists some $t_0 \in (a, b)$ and some constants c_1 and c_2 such that $c_1y_1(t_0) + c_2y_2(t_0) \neq 0$.
(c) the equation $c_1y_1(t) + c_2y_2(t) = 0$ holds for all $t \in (a, b)$ only if $c_1 = c_2 = 0$.
(d) the ratio $y_1(t)/y_2(t)$ is a constant function.
(e) All of the above
(f) None of the above

222. The functions $y_1(t)$ and $y_2(t)$ are linearly dependent on the interval $a < t < b$ if

(a) there exist two constants c_1 and c_2 such that $c_1y_1(t)+c_2y_2(t) = 0$ for all $a < t < b$.
(b) there exist two constants c_1 and c_2, not both 0, such that $c_1y_1(t)+c_2y_2(t) = 0$ for all $a < t < b$.
(c) for each t in (a, b), there exists constants c_1 and c_2 such that $c_1y_1(t)+c_2y_2(t) = 0$.
(d) for some $a < t_0 < b$, the equation $c_1y_1(t_0) + c_2y_2(t_0) = 0$ can only be true if $c_1 = c_2 = 0$.
(e) All of the above
(f) None of the above

Spanning Sets, Bases, and Dimension

223. Write $d = (3, -5, 10)$ as a linear combination of the vectors $a = (-1, 0, 3), b = (0, 1, 5)$, and $c = (4, -2, 0)$.

(a) $d = -3a - 5b + c$
(b) $d = 5a - b + 2c$
(c) $d = (10/3)a + (5/2)c$
(d) d cannot be written as a linear combination of a, b, and c.

224. Which of the following sets of vectors spans $\Re^3$?

i. $\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}$

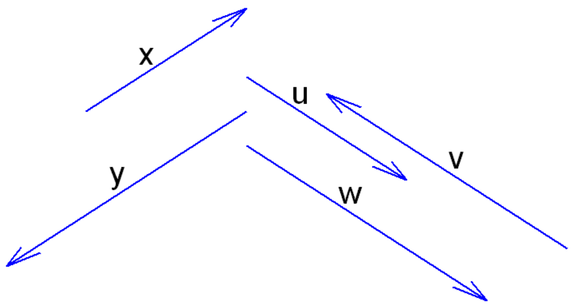
ii. $\begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

iii. $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ -2 \end{bmatrix}$

iv. $\begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 12 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$

(a) i, ii, iii, and iv
(b) ii, iii, and iv only
(c) ii and iii only
(d) i, ii and iii only
(e) iii and iv only
(f) ii only

225. Which of the following sets of vectors spans $\Re^2$?



i. x, y

ii. u, v, w

iii. x, v

iv. y, u, w

(a) i, ii, iii, and iv
(b) ii, iii, and iv only
(c) ii and iii only
(d) ii and iv only
(e) iii and iv only
(f) ii only

226. Which of the following sets of vectors forms a basis for $\Re^3$?

i. $\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}$

ii. $\begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

iii. $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ -2 \end{bmatrix}$ iv. $\begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 12 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$

(a) i, ii, iii, and iv
(b) ii, iii, and iv only
(c) ii and iii only
(d) i, ii and iii only
(e) iii and iv only
(f) ii only

227. Which of the following describes the subspace of $\Re^3$ spanned by the vectors $\begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 12 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$?

(a) A line
(b) A plane
(c) $\Re^2$
(d) All of $\Re^3$
(e) Both (b) and (c)

228. Which of the following describes a basis for a subspace V?

(a) A basis is a linearly independent spanning set for V.
(b) A basis is a minimal spanning set for V.
(c) A basis is a largest possible set of linearly independent vectors in V.
(d) All of the above
(e) Some of the above
(f) None of the above

229. Let $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 3 & -1 & 0 & 2 \end{bmatrix}$. The reduced row echelon form of A is $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.
What is the dimension of the column space of A?

(a) 1

(b) 2

(c) 3

(d) 4

230. Let $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 3 & -1 & 0 & 2 \end{bmatrix}$. The reduced row echelon form of A is $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Which columns would form a basis for the column space of A?

(a) All four

(b) The first three

(c) Any three

(d) Any two

231. Let $B = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$. Which of the following describes the column space of B?

(a) The column space of B is all of $\Re^3$.

(b) The column space of B is a proper subset of $\Re^3$.

(c) The column space of B is $\Re^4$.

(d) The column space of B is a proper subset of $\Re^4$.

(e) None of the above

232. Let $B = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$. What is the dimension of the column space of B?

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

(f) Infinite

233. Let $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 2 & -1 & 1 & 1 \end{bmatrix}$. What is the dimension of the nullspace of A?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4
(f) Infinite

234. Let A be an $n \times n$ matrix. If A is an invertible matrix, what else must be true?

(a) The columns of A form a basis of $\Re^n$.
(b) The rank of A is n.
(c) The dimension of the column space of A is n.
(d) The dimension of the null space of A is 0.
(e) All of the above must be true.
(f) More than one, but not all, of the above have to be true.

235. Howard's store sells three blends of flour: standard, extra wheat, and extra soy. Each is a blend of whole wheat flour and soy flour, and the table below shows how many pounds of each type of flour is needed to make one pound of each blend.

Standard Blend	Extra Wheat	Extra Soy	
0.5	0.8	0.3	whole wheat flour
0.5	0.2	0.7	soy flour

Do the column vectors in this table span $\Re^2$? Do they form a basis for $\Re^2$?

(a) Yes, they span $\Re^2$, and they form a basis.
(b) They do span $\Re^2$, but they do not form a basis.
(c) They do not span $\Re^2$, but they do form a basis for $\Re^2$.
(d) They do not span $\Re^2$, nor do they form a basis.

236. Howard's store sells three blends of flour: standard, extra wheat, and extra soy. Each is a blend of whole wheat flour and soy flour, and the table below shows how many pounds of each type of flour is needed to make one pound of each blend.

Standard Blend	Extra Wheat	Extra Soy	
0.5	0.8	0.3	whole wheat flour
0.5	0.2	0.7	soy flour

To save rent money, the store will be moving to a smaller space and will need to cut back on inventory. If possible, the manager would like to only stock two of these blends, and make the third from those as necessary. Which blends can be made from the others?

(a) Standard Blend can be made from Extra Wheat Blend and Extra Soy Blend.

(b) Extra Wheat Blend can be made from Standard Blend and Extra Soy Blend.

(c) Extra Soy Blend can be made from Standard Blend and Extra Wheat Blend.

(d) Any one blend can be made from the other two.

237. Howard's store sells three blends of flour: standard, extra wheat, and extra soy. Each is a blend of whole wheat flour and soy flour, and the table below shows how many pounds of each type of flour is needed to make one pound of each blend.

Standard Blend	Extra Wheat	Extra Soy	
0.5	0.8	0.3	whole wheat flour
0.5	0.2	0.7	soy flour

If the store continues to stock all three of these blends, which special-request blends could be made from these three?

(a) Any special request could be accomodated by mixing the right combination of these three blends.

(b) It would be possible to make any blend that is between 30% and 80% whole wheat.

(c) It would be possible to make a broader range of blends than what is described in answer (b), but there are still some blends that would not be possible.

(d) It would be possible to satisfy some special requests, but not all of the ones described in answer (b).

Markov Chains

238. A vector with nonnegative entries that add up to one is called a probability vector. Which of the following vectors is a probability vector?

(a) $\begin{bmatrix} 0.2 \\ 0.5 \\ 0.1 \end{bmatrix}$

(b) $\begin{bmatrix} -2.1 \\ 2.8 \\ 0.3 \end{bmatrix}$

(c) $\begin{bmatrix} 0.1 \\ 0 \\ 0.9 \end{bmatrix}$

(d) More than one of the given vectors are probability vectors.

(e) None of the given vectors are probability vectors.

239. A stochastic matrix is a square matrix whose columns are probability vectors. Which of the following matrices is a stochastic matrix?

(a) $\begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}$

(b) $\begin{bmatrix} 0.2 & 0.6 \\ 0.8 & 0.4 \end{bmatrix}$

(c) $\begin{bmatrix} 0.2 & 0.8 \\ 0.3 & 0.7 \end{bmatrix}$

(d) Both (a) and (b) are stochastic matrices.

(e) Both (a) and (c) are stocahstic matrices.

(f) All three are stochastic matrices.

240. A small, isolated town has two grocery stores, Mike's Market and Sharon's Shoppe. While some customers are completely loyal to one store or another, there is another group of customers who change their shopping habits each month. Of the shoppers who favor Mike's Market one month, only 70% will still shop there the following month, while Sharon's Shoppe retains 78% of its customer base each month. Everyone in the town shops at one of the two stores, and no one from out of town ever shops at either store. If Mike's Market currently has 2500 customers and Sharon's Shoppe has 1900 customers, how many customers will Mike's Market have next month?

(a) 418

(b) 1750

(c) 2168

(d) 3080

241. Referring to the scenario in the previous question, what will the product

$$\begin{bmatrix} 0.70 & 0.22 \\ 0.30 & 0.78 \end{bmatrix} \begin{bmatrix} 2500 \\ 1900 \end{bmatrix}$$

tell us?

(a) This product will tell us the percentage of customers that will switch from one store to the other store next month.

(b) This product will tell us the number of customers who will shop at each store next month.

(c) This product will tell us the total number of customers who switched stores this month.

(d) This product doesn't have any meaning.

242. Continuing the scenario from the previous questions, what does the (2, 1)-entry of the matrix $\begin{bmatrix} 0.70 & 0.22 \\ 0.30 & 0.78 \end{bmatrix}^3$ represent?

(a) This represents the probability that a customer will switch from Mike's Market to Sharon's Shoppe between months 3 and 4.

(b) This represents the probability that a customer will switch from Sharon's Shoppe to Mike's Market between months 3 and 4.

(c) This represents the probability that a customer who currently shops at Mike's Market will be shopping at Sharon's Shoppe three months from now.

(d) This represents the probability that a customer who currently shops at Sharon's Shoppe will be shopping at Mike's Market three months from now.

243. A steady-state (or equilibrium) vector for a stochastic matrix P is a probability vector x such that $Px = x$. Which of the following is a steady-state vector for $\begin{bmatrix} 0.70 & 0.22 \\ 0.30 & 0.78 \end{bmatrix}$?

(a) $\begin{bmatrix} 22 \\ 30 \end{bmatrix}$

(b) $\begin{bmatrix} 11/26 \\ 15/26 \end{bmatrix}$

(c) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(d) All of the above

244. What does the steady-state vector mean in the context of Mike's Market and Sharon's Shoppe?

(a) In the long-run, the probability of staying at Mike's Market will be 11/26 and the probability of switching to Sharon's Shoppe will be 15/26.
(b) In the long-run, the probability of staying at Mike's Market will be 11/26 and the probability of staying at Sharon's Shoppe will be 15/26.
(c) In the long-run, Mike's Market will approach 11/26 of the market share, while Sharon's Shoppe will approach 15/26 of the market share.

245. **True or False** A stochastic matrix will always have a steady-state vector.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

Chapter 5: Eigenvalues and Eigenvectors

Eigenvalues and Eigenvectors

246. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(a) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$
(b) $\begin{bmatrix} 3 \\ 3 \end{bmatrix}$
(c) $\begin{bmatrix} 3 & 3 \end{bmatrix}$
(d) $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$
(e) None of the above

247. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^2 \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(a) $\begin{bmatrix} 3 \\ 3 \end{bmatrix}$
(b) $\begin{bmatrix} 6 \\ 6 \end{bmatrix}$

(c) $\begin{bmatrix} 9 \\ 9 \end{bmatrix}$

(d) $\begin{bmatrix} 12 \\ 12 \end{bmatrix}$

(e) None of the above

(f) This matrix multiplication is impossible.

248. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^4 \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(a) $\begin{bmatrix} 27 \\ 27 \end{bmatrix}$

(b) $\begin{bmatrix} 81 \\ 81 \end{bmatrix}$

(c) $\begin{bmatrix} 243 \\ 243 \end{bmatrix}$

(d) $\begin{bmatrix} 729 \\ 729 \end{bmatrix}$

(e) None of the above

249. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(a) $\begin{bmatrix} 3n \\ 3n \end{bmatrix}$

(b) $3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(c) $n^3 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(d) $3^n \begin{bmatrix} n \\ n \end{bmatrix}$

(e) $\begin{bmatrix} 3 \\ 3 \end{bmatrix}^n$

(f) More than one of the above

250. Suppose A is an $n \times n$ matrix, c is a scalar, and x is an $n \times 1$ vector. If $Ax = cx$, what is A^2x?

(a) $2cx$

(b) c^2x

(c) cx

(d) None of the above

251. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(a) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 \\ -3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) $\begin{bmatrix} -3 \\ 3 \end{bmatrix}$

(e) None of the above

252. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(a) $(-1)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(b) $(-1)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(c) $(-3)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) $\begin{bmatrix} (-1)^n \\ (-1)^{n+1} \end{bmatrix}$

(e) None of the above

(f) More than one of the above

253. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

(a) $3^n \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

(b) $2^n \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

(c) $6^n \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(d) $3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

254. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} -5 \\ 5 \end{bmatrix}$

(a) $3^n \begin{bmatrix} -5 \\ 5 \end{bmatrix}$

(b) $(-1)^n \begin{bmatrix} -5 \\ 5 \end{bmatrix}$

(c) $(-5)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) $5 \begin{bmatrix} (-1)^n \\ (-1)^n \end{bmatrix}$

(e) None of the above

(f) More than one of the above

255. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(a) $\begin{bmatrix} 3 \\ 15 \end{bmatrix}$

(b) $\begin{bmatrix} -1 \\ -5 \end{bmatrix}$

(c) $\begin{bmatrix} 11 \\ 7 \end{bmatrix}$

(d) $\begin{bmatrix} 7 \\ 11 \end{bmatrix}$

(e) None of the above

256. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(a) $11^n \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(b) $7^n \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(c) $\begin{bmatrix} 11^n \\ 7^n \end{bmatrix}$

(d) $\begin{bmatrix} 25 \\ 29 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

257. Write the vector $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 5 \end{bmatrix} = \frac{1}{2}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$

(c) $\begin{bmatrix} 1 \\ 5 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 1 \end{bmatrix} - 2\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) None of the above

(e) More than one of the above

258. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \times \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

(a) $-1 \times 3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 3 \times (-2)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(b) $3 \times (-1)^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + (-2) \times 3^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) $3 \times 3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + (-2) \times (-1)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) $3 \times 3^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + (-1) \times (-2)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

259. Which of the following is an eigenvector of the matrix $\begin{bmatrix} 2 & -1 \\ -4 & -1 \end{bmatrix}$?

(a) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ -4 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

260. Which of the following is an eigenvector of the matrix $\begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$?

(a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} -3/2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} -3 \\ -3 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ -2/3 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

261. Suppose the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has an eigenvalue 1 with associated eigenvector $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. What is $A^{50}x$?

(a) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(b) $\begin{bmatrix} a^{50} & b^{50} \\ c^{50} & d^{50} \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} 2^{50} \\ 3^{50} \end{bmatrix}$

(e) Way too hard to compute.

262. Vector x is an eigenvector of matrix A. If $x = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $Ax = \begin{bmatrix} 4 \\ 12 \end{bmatrix}$, then what is the associated eigenvalue?

(a) 1

(b) 3

(c) 4

(d) Not enough information is given.

263. Which of the following is an eigenvector of $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$? (You should be able to answer this by checking the vectors given, rather than by finding the eigenvectors of A from scratch.)

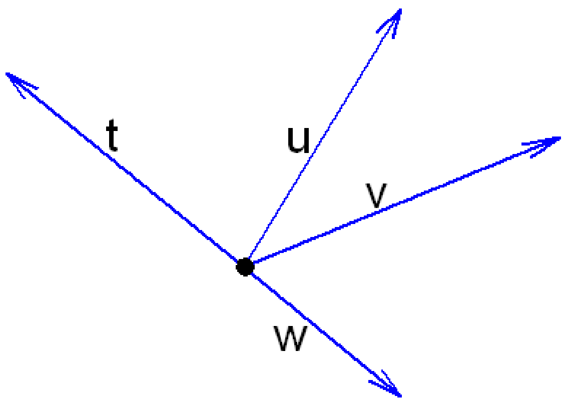
(a) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) None of the above

264. The vector t is an eigenvector of the matrix A. What could be the result of the product At?



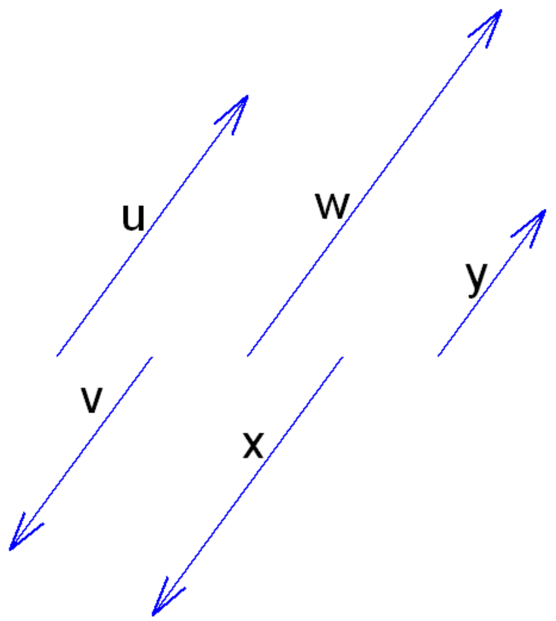
(a) $At = u$

(b) $At = v$

(c) $At = w$

(d) None of the above

265. The vector u is an eigenvector of the matrix A and $Au = v$, where the vectors u and v are shown below. What could be the result of the product Av?



(a) $Av = u$

(b) $Av = v$

(c) $Av = w$

(d) $Av = x$

(e) $Av = y$

266. $\begin{bmatrix} 4/3 \\ 1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$. What is the associated eigenvalue? (Think! Don't solve for all the eigenvalues and eigenvectors.)

(a) 4/3

(b) 5

(c) -2

267. The matrix $A = \begin{bmatrix} -1 & 4 \\ 3 & 0 \end{bmatrix}$ has an eigenvalue 3 with associated eigenvector $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Let $y = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. Which of the following statements is true?

(a) $Ax = 3x$

(b) $Ay = 3y$

(c) For any scalars c and d, $A(cx + dy) = 3(cx + dy)$

(d) All of the above are true.

(e) Only (a) and (b) are true.

268. The matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ has an eigenvalue 2 with associated eigenvectors $x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. Which of the following statements is true?

(a) $Ax = 2x$

(b) $Ay = 2y$

(c) For any scalars c and d, $A(cx + dy) = 2(cx + dy)$.

(d) For any nonzero scalars c and d, $cx + dy$ is an eigenvector of A corresponding to the eigenvalue 2.

(e) All of the above are true.

(f) Only (a) and (b) are true.

269. **True or False** Any nonzero linear combination of two eigenvectors of a matrix A is an eigenvector of A.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

270. If w is an eigenvector of A, how does the vector Aw compare geometrically to the vector w?

(a) Aw is a rotation of w.

(b) Aw is a reflection of w in the x-axis.

(c) Aw is a reflection of w in the y-axis.

(d) Aw is parallel to w but may have a different length.

271. What does it mean if 0 is an eigenvalue of a matrix A?

(a) The determinant of A is zero.

(b) The columns of A are linearly dependent.
(c) There are an infinite number of solutions to the system $Ax = 0$.
(d) All of the above
(e) None of the above

272. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 6 \\ 0 & 4 & 2 \end{bmatrix}$ and note that all of the rows sum to six. Which of the following is true?

(a) $w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector of A.
(b) 6 is an eigenvalue of A.
(c) Both statements are true.
(d) Neither statement is true.

Eigenspaces

273. If a vector x is in the eigenspace of A corresponding to λ, and $\lambda \neq 0$, then x is

(a) in the nullspace of the matrix A.
(b) in the nullspace of the matrix $A - \lambda I$.
(c) not the zero vector.
(d) More than one of the above correctly completes the sentence.

274. Which of the following statements is correct?

(a) The set of eigenvectors of a matrix A forms the eigenspace of A.
(b) The set of eigenvectors of a matrix A spans the eigenspace of A.
(c) Since any multiple of an eigenvector is also an eigenvector, the eigenspace always has infinite dimension.
(d) More than one of the above statements are correct.
(e) None of the above statements are correct.

275. Which of the following statements is correct?

(a) The set of eigenvectors of a matrix A corresponding to a particular eigenvalue λ_1, together with the zero vector, forms the eigenspace of A corresponding to λ_1.

(b) An eigenspace corresponding to a non-repeated eigenvalue has dimension one.

(c) An eigenvalue of multiplicity two has a corresponding eigenspace of dimension two.

(d) All of the above statements are correct.

(e) Exactly two of the above statements are correct.

Diagonalization

276. What are the eigenvalues of $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$?

(a) 2 and 3

(b) 0 and 2

(c) 0 and 3

(d) 5 and 6

277. If $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, what is D^5?

(a) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 10 & 0 \\ 0 & 15 \end{bmatrix}$

(c) $\begin{bmatrix} 2^5 & 0 \\ 0 & 3^5 \end{bmatrix}$

(d) Too hard to compute by hand.

278. Why might we be interested in diagonalizing a matrix?

(a) Because it is easy to find the eigenvalues of a diagonal matrix.

(b) Because it is easy to compute powers of a diagonal matrix.

(c) Both of these reasons.

279. Which of the following statements are true?

(a) An $n \times n$ matrix with n linearly independent eigenvectors is diagonalizable.
(b) Any diagonalizable $n \times n$ matrix has n linearly independent eigenvectors.
(c) Both are true.
(d) Neither is true.

280. Which of the following statements are true?

(a) An $n \times n$ matrix with n distinct eigenvalues is diagonalizable.
(b) Any diagonalizable $n \times n$ matrix has n distinct eigenvalues.
(c) Both are true.
(d) Neither is true.

281. Which of the following statements are true?

(a) If A is a diagonalizable matrix, then A does not have any zero eigenvalues.
(b) If A does not have any zero eigenvalues, then A is diagonalizable.
(c) Both are true.
(d) Neither is true.

282. **True or False** Invertible matrices are diagonalizable.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

Complex Eigenvalues

283. **True or False** Real matrices have only real eigenvalues.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

284. Which of the following could *not* be the set of distinct eigenvalues for a 3×3 real matrix?

(a) 2, 5
(b) 1, 3, 5
(c) 2, 3, $4 + 7i$
(d) 3, $2 + i$, $2 - i$

285. **True or False** Real eigenvalues of a real matrix correspond to real eigenvectors only.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

Solving Homogeneous Systems of Difference Equations

286. If we are told that the general solution to a system of difference equations is

$$A_n = \begin{bmatrix} a_n \\ b_n \end{bmatrix} = c_1 \cdot (0.9)^n \begin{bmatrix} 1 \\ \frac{7}{8} \end{bmatrix} + c_2(-0.5)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix},$$

then which is an equivalent form of the solution?

(a) $a_n = c_1(0.9)^n + \frac{7}{8}c_1(0.9)^n$ and $b_n = -c_2(-0.5)^n + c_2(-0.5)^n$
(b) $a_n = c_1(0.9)^n - c_2(-0.5)^n$ and $b_n = \frac{7}{8}c_1(0.9)^n + c_2(-0.5)^n$
(c) $a_n = c_1(0.9)^n - c_1(-0.5)^n$ and $b_n = \frac{7}{8}c_2(0.9)^n + c_2(-0.5)^n$
(d) All of the above
(e) None of the above

287. The solution to a system of difference equations is

$$A_n = \begin{bmatrix} a_n \\ b_n \end{bmatrix} = c_1 \cdot (0.9)^n \begin{bmatrix} 1 \\ \frac{7}{8} \end{bmatrix} + c_2(-0.5)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

Which of the following is a true statement?

(a) This system has an unstable equilibrium.
(b) In the long-run, b will hold 7/8 of the population.

(c) The equilibrium value of this system is $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

(d) All of the above

(e) None of the above

288. If we wish to solve this system,

$$\begin{aligned} a_{n+1} &= a_n - 0.2a_n + 0.3b_n \\ b_{n+1} &= b_n - 0.3b_n \end{aligned}$$

which matrix do we need to find eigenvalues and eigenvectors for?

(a)

$$\begin{bmatrix} 1 & -0.2 & 0.3 \\ 1 & -0.3 & 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & -0.2 & 0.3 \\ 0 & 1 & -0.3 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 0.8 & 0.3 \\ 0.7 & 0 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 0.8 & 0.3 \\ 0 & 0.7 \end{bmatrix}$$

(e) None of the above

289. In solving the system

$$\begin{aligned} a_{n+1} &= a_n - 0.2a_n + 0.3b_n \\ b_{n+1} &= b_n - 0.3b_n \end{aligned}$$

we find that the eigenvalues of the coefficient matrix are 0.8 and 0.7 with corresponding eigenvectors of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$. What is the solution to this system?

(a) $A_n = c_1(0.8)^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2(0.7)^n \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

(b) $A_n = c_1(0.8)^n \begin{bmatrix} -3 \\ 1 \end{bmatrix} + c_2(0.7)^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(c) $A_n = c_1(0.8)\begin{bmatrix} 1 \\ 0 \end{bmatrix}^n + c_2(0.7)\begin{bmatrix} -3 \\ 1 \end{bmatrix}^n$

(d) $A_n = c_1(0.8)\begin{bmatrix} -3 \\ 1 \end{bmatrix}^n + c_2(0.7)\begin{bmatrix} 1 \\ 0 \end{bmatrix}^n$

(e) None of the above

290. The solution to a system of difference equations is $A_n = c_1(0.8)^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2(0.7)^n \begin{bmatrix} -3 \\ 1 \end{bmatrix}$. If $A_0 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, what are c_1 and c_2?

(a) $c_1 = 2$ and $c_2 = 3$

(b) $c_1 = 55/4$ and $c_2 = 110/8$

(c) $c_1 = 11$ and $c_2 = 3$

(d) $c_1 = -7$ and $c_2 = 3$

(e) None of the above

291. The following system of difference equations allows us to predict how the populations of two towns, A and B, change each year.

$$\begin{aligned} a_{n+1} &= a_n - 0.2a_n + 0.3b_n \\ b_{n+1} &= b_n - 0.3b_n \end{aligned}$$

The solution to this system is

$$A_n = c_1(0.8)^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2(0.7)^n \begin{bmatrix} -3 \\ 1 \end{bmatrix}.$$

Which of the following is a true statement?

(a) This system has a stable equilibrium.

(b) In the long-run, both of these towns will be ghost towns.

(c) If there are initially 10,000 people in town B, then $b_{10} = 282$ people.

(d) All of the above

(e) None of the above

292. If $A_n = (2)^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \left(\frac{1}{3}\right)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is a solution to the system of difference equations $A_{n+1} = RA_n$, which of the following is also a solution?

(a) $(2^n)\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(b) $3 \cdot (2)^n \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 4 \cdot \left(\frac{1}{3}\right)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(c) $8 \cdot \left(\frac{1}{3}\right)^n \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(d) All of the above

(e) None of the above

293. **True or False** If either column of the coefficient matrix of a system of homogeneous difference equations sums to a value greater than one, then the system has an unstable equilibrium.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

294. **True or False** When solving a system of two homogeneous difference equations, if one eigenvalue is greater than one and one is between 0 and 1, then one population will grow without bound while the other declines.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

Solutions to Linear Systems

295. Consider the linear system given by

$$\frac{d\vec{Y}}{dt} = \begin{bmatrix} 2 & 3 \\ 0 & -4 \end{bmatrix} \vec{Y}.$$

True or False: $\vec{Y}_1(t) = \begin{pmatrix} e^{2t} \\ 0 \end{pmatrix}$ is a solution.

(a) True, and I am very confident

(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

296. Consider the linear system $\dfrac{d\vec{Y}}{dt} = \begin{bmatrix} 2 & 3 \\ 0 & -4 \end{bmatrix} \vec{Y}$ with solution $\vec{Y}_1(t) = \begin{pmatrix} e^{2t} \\ 0 \end{pmatrix}$.

True or False: The function $k \cdot \vec{Y}_1(t)$ formed by multiplying $\vec{Y}_1(t)$ by a constant k is also a solution to the linear system.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

297. Consider the linear system $\dfrac{d\vec{Y}}{dt} = \begin{bmatrix} 2 & 3 \\ 0 & -4 \end{bmatrix} \vec{Y}$. The functions $\vec{Y}_1(t)$ and $\vec{Y}_2(t)$ are solutions to the linear system.

True or False: The function $\vec{Y}_1(t) + \vec{Y}_2(t)$ formed by adding the two solutions $\vec{Y}_1(t)$ and $\vec{Y}_2(t)$ is also a solution to the linear system.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

298. **True or False:** The functions $\vec{Y}_1(t) = \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix}$ and $\vec{Y}_2(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$ are linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

299. **True or False:** The functions $\vec{Y}_1(t) = \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix}$ and $\vec{Y}_2(t) = \begin{pmatrix} -2\sin(t) \\ -2\cos(t) \end{pmatrix}$ are linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

300. If we are told that the general solution to the linear homogeneous system $Y' = AY$ is $Y = c_1 e^{-4t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, then an equivalent form of the solution is

(a) $y_1 = -2c_1 e^{-4t} + c_2 e^{-4t}$ and $y_2 = 2c_1 e^{3t} + 3c_2 e^{3t}$
(b) $y_1 = -2c_1 e^{-4t} + 2c_2 e^{3t}$ and $y_2 = c_1 e^{-4t} + 3c_2 e^{3t}$
(c) $y_1 = -2c_1 e^{-4t} + c_1 e^{-4t}$ and $y_2 = 2c_2 e^{3t} + 3c_2 e^{3t}$
(d) $y_1 = -2c_1 e^{-4t} + 2c_1 e^{3t}$ and $y_2 = c_2 e^{-4t} + 3c_2 e^{3t}$
(e) All of the above
(f) None of the above

301. If $Y = e^{-2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + e^{-4t} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is a solution to the linear homogeneous system $Y' = AY$, which of the following is also a solution?

(a) $Y = 2e^{-2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
(b) $Y = 3e^{-2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} - 4e^{-4t} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$
(c) $Y = 1/4e^{-4t} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$
(d) All of the above
(e) None of the above

302. The eigenvalues and eigenvectors for the coefficient matrix A in the linear homogeneous system $Y' = AY$ are $\lambda_1 = 4$ with $v_1 =< 1, 2 >$ and $\lambda_2 = -3$ with $v_2 =< -2, 1 >$. What is a form of the solution?

(a) $Y = c_1 e^{-4t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$
(b) $Y = c_1 e^{4t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{-3t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(c) $Y = c_1 e^{4t}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{-3t}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

(d) $Y = c_1 e^{-4t}\begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{3t}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

303. You have a linear, homogeneous, system of differential equations with constant coefficients whose coefficient matrix A has the eigensystem: eigenvalues -5 and -2 and eigenvectors $< -1, 2 >$ and $< -4, 5 >$, respectively. Then a general solution to $\frac{d\vec{Y}}{dt} = A\vec{Y}$ is given by:

(a) $Y = \begin{bmatrix} -k_1 e^{-5t} + 2k_2 e^{-2t} \\ -4k_1 e^{-5t} + 5k_2 e^{-2t} \end{bmatrix}$

(b) $Y = \begin{bmatrix} -k_1 e^{-2t} - 4k_2 e^{-5t} \\ 2k_1 e^{-2t} + 5k_2 e^{-5t} \end{bmatrix}$

(c) $Y = \begin{bmatrix} -k_1 e^{-5t} - 4k_2 e^{-2t} \\ 2k_1 e^{-5t} + 5k_2 e^{-2t} \end{bmatrix}$

(d) $Y = \begin{bmatrix} -k_1 e^{-2t} + 2k_2 e^{-5t} \\ -4k_1 e^{-2t} + 5k_2 e^{-5t} \end{bmatrix}$

304. The eigenvalues and eigenvectors for the coefficient matrix A in the linear homogeneous system $Y' = AY$ are $\lambda_1 = 4$ with $v_1 =< 1, 2 >$ and $\lambda_2 = -3$ with $v_2 =< -2, 1 >$. In the long term, phase trajectories:

(a) become parallel to the vector $v_2 =< -2, 1 >$.

(b) tend towards positive infinity.

(c) become parallel to the vector $v_1 =< 1, 2 >$.

(d) tend towards 0.

(e) None of the above

305. If the eigenvalues and eigenvectors for the coefficient matrix A in the linear homogeneous system $Y' = AY$ are $\lambda_1 = -4$ with $v_1 =< 1, 2 >$ and $\lambda_2 = 3$ with $v_2 =< 2, 3 >$, is $y_a = e^{-4t}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ a solution?

(a) Yes, it is a solution.

(b) No, it is not a solution because it does not contain λ_2.

(c) No, it is not a solution because it is a vector.

(d) No, it is not a solution because of a different reason.

306. The eigenvalues and eigenvectors for the coefficient matrix A in the linear homogeneous system $Y' = AY$ are $\lambda_1 = 4$ with $v_1 =< 1, 0 >$ and $\lambda_2 = 4$ with $v_2 =< 0, 1 >$. What is a form of the solution?

(a) $Y = c_1 e^{4t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{4t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(b) $Y = c_1 e^{-4t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{-4t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(c) $Y = c_1 e^{4t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 t e^{4t} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(d) $Y = c_1 e^{-4t} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + c_2 e^{-4t} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

307. The system of differential equations $Y' = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} Y$ has eigenvalue $\lambda = 2$ with multiplicity 2, and all eigenvectors are multiples of $v =< 1, -1 >$. Testing $Y = e^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, we find that

(a) Y is a solution.

(b) Y is not a solution.

308. The system of differential equations $Y' = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} Y$ has eigenvalue $\lambda = 2$ with multiplicity 2, and all eigenvectors are multiples of $v =< 1, -1 >$. One solution to this equation is $Y = e^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Testing $Y = te^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, we find that

(a) $Y = te^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is also a solution.

(b) $Y = te^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is not a solution.

309. The eigenvalues and eigenvectors for the coefficient matrix A in the linear homogeneous system $Y' = AY$ are $\lambda = -4$ with multiplicity 2, and all eigenvectors are multiples of $v =< 1, -2 >$. What is the form of the general solution?

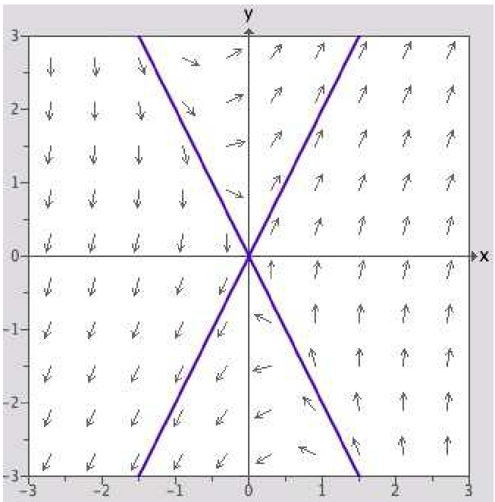
(a) $Y = c_1 e^{-4t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 t e^{-4t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(b) $Y = c_1 e^{-4t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 e^{-4t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(c) $Y = c_1 e^{-4t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 \left(te^{-4t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + e^{-4t} \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right)$

Geometry of Systems

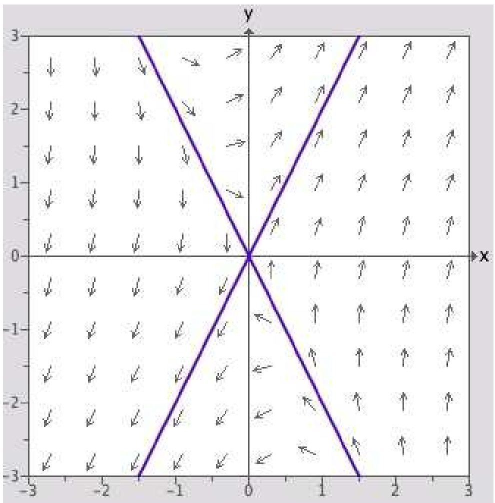
310. The differential equation $\frac{d\vec{Y}}{dt} = A\vec{Y}$ has two straight line solutions corresponding to eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ that are shown on the direction field below. We denote the associated eigenvalues by λ_1 and λ_2.



We can deduce that λ_1 is

(a) positive real
(b) negative real
(c) zero
(d) complex
(e) There is not enough information

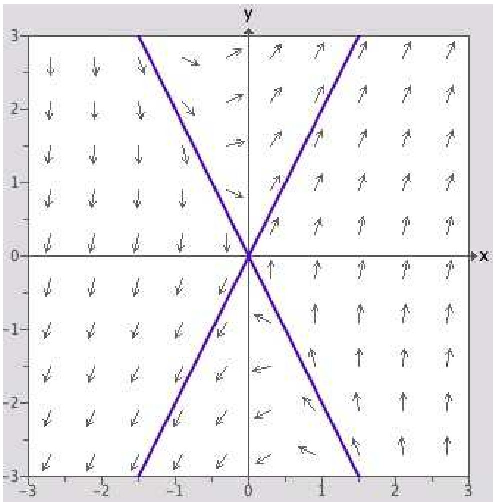
311. The differential equation $\frac{d\vec{Y}}{dt} = A\vec{Y}$ has two straight line solutions corresponding to eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ that are shown on the direction field below. We denote the associated eigenvalues by λ_1 and λ_2.



We can deduce that λ_2 is

(a) positive real
(b) negative real
(c) zero
(d) complex
(e) There is not enough information

312. The differential equation $\dfrac{d\vec{Y}}{dt} = A\vec{Y}$ has two straight line solutions corresponding to eigenvectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ that are shown on the direction field below.



Suppose we have a solution $\vec{Y}(t)$ to this system of differential equations which satisfies initial condition $\vec{Y}(t) = (x_0, y_0)$ where the point (x_0, y_0) is not on the line through the point $(1, -2)$. Which statement best describes the behavior of the solution as $t \to \infty$?

(a) The solution tends towards the origin.

(b) The solution moves away from the origin and asymptotically approaches the line through $< 1, 2 >$.

(c) The solution moves away from the origin and asymptotically approaches the line through $< 1, -2 >$.

(d) The solution spirals and returns to the point (x_0, y_0).

(e) There is not enough information.

313. Suppose you have a linear, homogeneous system of differential equations with constant coefficients whose coefficient matrix has the following eigensystem: eigenvalues -4 and -1 and eigenvectors $< 1, 1 >$ and $< -2, 1 >$ respectively. The function $\vec{Y}(t)$ is a solution to this system of differential equations which satisfies initial value $\vec{Y}(0) = (-15, 20)$. Which statement best describes the behavior of the solution as $t \to \infty$?

(a) The solution tends towards the origin.

(b) The solution moves away from the origin and asymptotically approaches the line through $< 1, 1 >$.

(c) The solution moves away from the origin and asymptotically approaches the line through $< -2, 1 >$.

(d) The solution spirals and returns to the point $(-15, 20)$.

(e) There is not enough information.

314. Suppose we have a linear, homogeneous system of differential equations with constant coefficients who coefficient matrix has the following eigensystem: eigenvalues -4 and -1 and eigenvectors $< 1, 1 >$ and $< -2, 1 >$ respectively. Suppose we have a solution $\vec{Y}(t)$ which satisfies $\vec{Y}(0) = (x_0, y_0)$ where the point (x_0, y_0) is not on the line through the point $(1, 1)$. How can we best describe the manner in which the solution $\vec{Y}(t)$ approaches the origin?

(a) The solution will approach the origin in the same manner as the line which goes through the point $(1, 1)$.

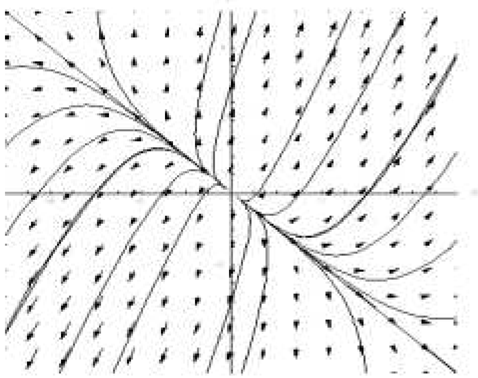
(b) The solution will approach the origin in the same manner as the line which goes through the point $(-2, 1)$.

(c) The solution will directly approach the origin in a straight line from the point (x_0, y_0).

(d) The answer can vary greatly depending on what the point (x_0, y_0) is.

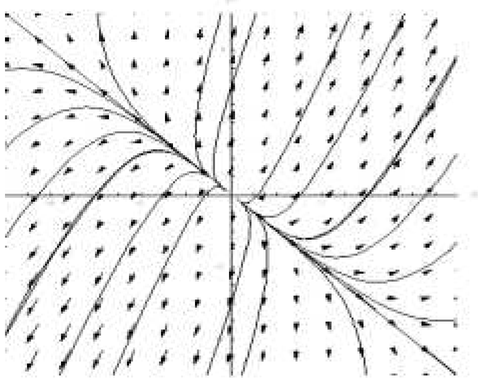
(e) The solution doesn't approach the origin.

315. Using the phase portrait below for the system $Y' = AY$, we can deduce that the eigenvalues of the coefficient matrix A are:



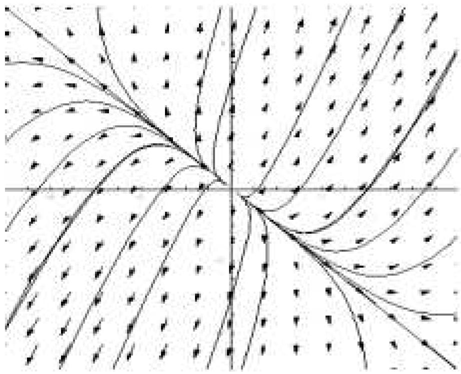
(a) both real
(b) both complex
(c) one real, one complex
(d) Not enough information is given

316. Using the phase portrait below for the system $Y' = AY$, we can deduce that the eigenvalues are:



(a) of mixed sign
(b) both negative
(c) both positive
(d) Not enough information is given

317. Using the phase portrait below for the system $Y' = AY$, we can deduce that the dominant eigenvector is:



(a) $< -1, 1 >$

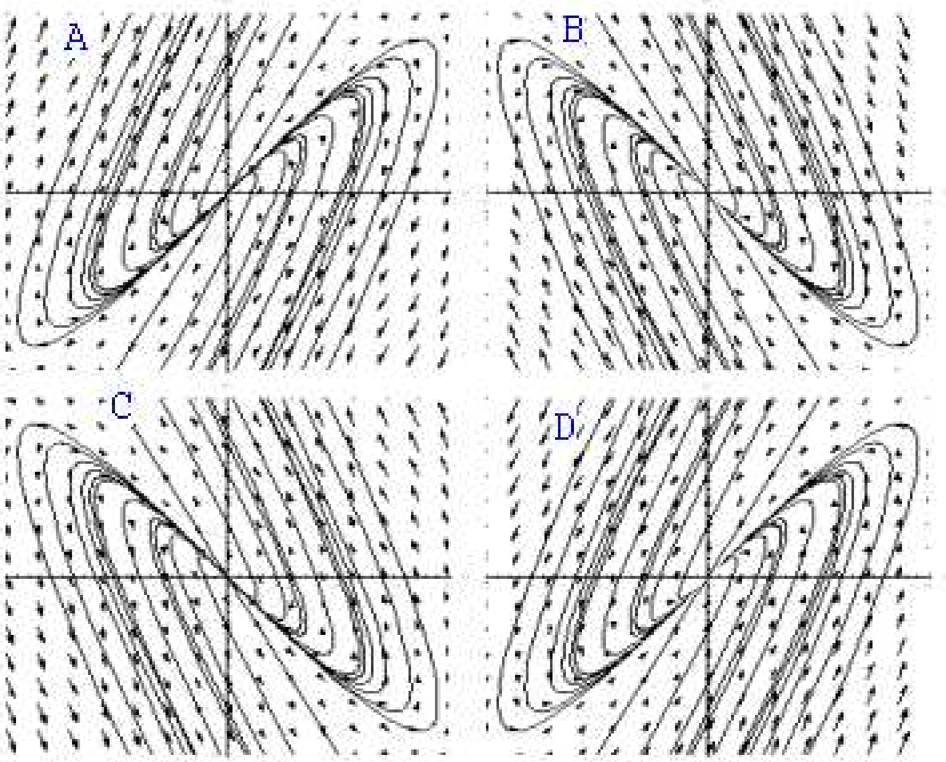
(b) $< 1, 3 >$

(c) $< 1, -2 >$

(d) There is no dominant eigenvector because there is no vector that is being approached by all of the solution curves.

(e) Not enough information is given

318. Which phase portrait below corresponds to the linear, homogeneous, system of differential equations with constant coefficients whose coefficient matrix has the following eigensystem: eigenvalues -5 and -2 and eigenvectors $< -1, 2 >$ and $< -4, 5 >$, respectively?



319. Classify the equilibrium point at the origin for the system

$$\frac{d\vec{Y}}{dt} = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \vec{Y}.$$

(a) Sink
(b) Source
(c) Saddle
(d) None of the above

Chapter 6: Orthogonality and Least Squares

Dot Products

320. What is the dot product of $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix}$?

(a) $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$
(b) 5
(c) 0
(d) The dot product cannot be computed for these vectors.

321. What is the dot product of $\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$?

(a) $\begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}$
(b) -4
(c) 0
(d) The dot product cannot be computed for these vectors.

322. The magnitude of a vector v is defined to be its dot product with itself $v \cdot v$. What is the magnitude of the vector $(2, -1, -1)$?

(a) 0
(b) 2
(c) 4
(d) 6

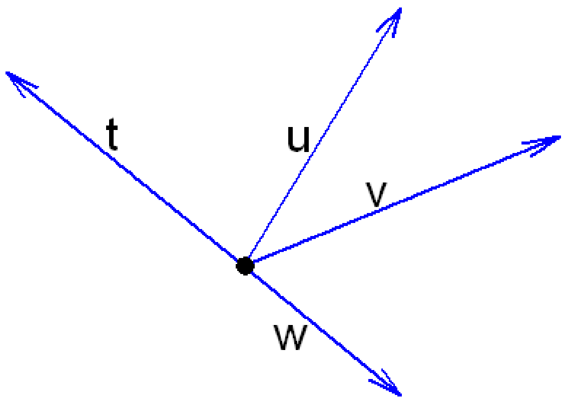
323. It is possible for a vector to have a negative magnitude?

(a) Yes
(b) No
(c) Not enough information is given

324. What can we say about two vectors whose dot product is negative?

(a) The vectors are orthogonal.
(b) The angle between the two vectors is less than 90°.
(c) The angle between the two vectors is greater than 90°.

325. Rank the dot products $u \cdot v$, $u \cdot t$ and $u \cdot w$.



(a) $u \cdot v > u \cdot w > u \cdot t$
(b) $u \cdot v > u \cdot t > u \cdot w$
(c) $u \cdot w > u \cdot t > u \cdot v$
(d) $u \cdot w > u \cdot v > u \cdot t$

326. **True or False** If x and y are $n \times 1$ vectors, then $x^T y = y^T x$.

(a) True, and I am very confident

(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

327. **True or False** If x and y are $n \times 1$ vectors, then xy^T is an $n \times n$ matrix.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

328. **True or False** If x and y are $n \times 1$ vectors, then $xy^T = yx^T$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

329. **True or False** If x and y are $n \times 1$ nonzero vectors, then xy^T is an $n \times n$ matrix with rank 1.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

330. If A and B are matrices which can be multiplied, then the (i, j)-entry of AB is

(a) $\left[\text{the } i^{\text{th}} \text{ row of } A\right] \cdot \left[\text{the } j^{\text{th}} \text{ column of } B\right]$
(b) $\left[\text{the } i^{\text{th}} \text{ column of } A\right] \cdot \left[\text{the } j^{\text{th}} \text{ row of } B\right]$
(c) None of the above

331. When we are in the vector space of real valued functions, it is often useful to have the equivalent of a dot product, which we call an inner product: We define the inner product of two functions $f(x)$ and $g(x)$ as $\langle f, g\rangle \equiv \int_a^b f(x)g(x)dx$. Consider the functions $f(x) = \sin 2\pi x$ on the interval $(a, b) = (0, 1)$. What is the inner product of this function with itself $\langle f, f\rangle$?

(a) 0

(b) $\frac{1}{2}$

(c) 1

(d) 2

(e) This is not a meaningful statement.

332. $\langle \sin(2\pi x), \sin(4\pi x) \rangle \equiv \int_0^1 \sin(2\pi x)\sin(4\pi x)dx = 0$. What does this mean?

(a) These are parallel functions.

(b) These are orthogonal functions.

(c) These are acute functions.

(d) These are obtuse functions.

Orthogonal Sets

333. Which of the following sets of vectors is *not* an orthogonal set?

(a) $(1, 1, 1), (1, 0, -1)$

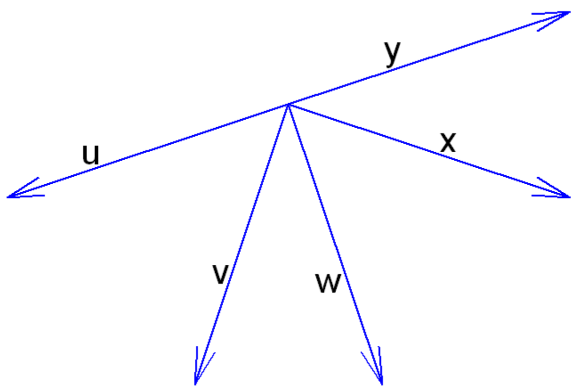
(b) $(2, 3), (-6, 4)$

(c) $(3, 0, 0, 2), (0, 1, 0, 1)$

(d) $(0, 2, 0), (-1, 0, 3)$

(e) $(\cos\theta, \sin\theta), (\sin\theta, -\cos\theta)$

334. Which of the following sets of vectors is *not* an orthogonal set?



(a) u, w

(b) x, v

(c) v, y

(d) u, w, y

(e) More than one of the above

(f) None of the above

335. Let A be a square matrix whose columns are mutually orthogonal, nonzero vectors. Which of the following are true?

(a) The dot product of any two different column vectors is zero.

(b) The set of column vectors is linearly independent.

(c) $\det(A) \neq 0$.

(d) For any b, there is a unique solution to $Ax = b$.

(e) All of the above.

336. **True or False** If two vectors are linearly independent, they must be orthogonal.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

337. **True or False** Any orthogonal set of nonzero vectors that spans a vector space must be a basis for that space.

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

338. Let A be any matrix. Which of the following are true?

(a) The row space of A and the nullspace of A are orthogonal to each other.

(b) The column space of A and the row space of A are orthogonal to each other.

(c) The column space of A and the nullspace of A are orthogonal to each other.

(d) Exactly two of (a), (b), and (c) are true.

(e) All of (a), (b), and (c) are true.

339. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$. Which of the following vectors is orthogonal to the row space of A?

(a) $(1, 1, -1)$
(b) (1, 4, 2)
(c) (0, 0, 5)
(d) $(-1, 0, 1)$

340. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$. Which of the following vectors is orthogonal to the column space of A?

(a) $(1, 1, -1)$
(b) (1, 4, 2)
(c) $(0, 1, -2)$
(d) $(2, 0, 2)$

341. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$. Which of the following vectors is orthogonal to the nullspace of A?

(a) $(1, 1, -1)$
(b) (1, 4, 2)
(c) $(0, 1, -2)$
(d) $(2, 0, 2)$

342. Which of the following sets of vectors is an orthonormal set?

(a) $(1, 1, 1), (1, 0, -1)$
(b) $(2, 3), (-6, 4)$
(c) $(0, 2, 0), (-1, 0, 3)$
(d) $(\cos\theta, \sin\theta), (\sin\theta, -\cos\theta)$

343. Let A be a matrix whose columns are mutually orthogonal. Which of the following must be true? Try several examples of matrices with mutually orthogonal columns to build your intuition, then try to provide a proof.

(a) A is symmetric.
(b) $A^{-1} = A^T$.
(c) $A^T A$ is diagonal.
(d) $\det(A) \neq 0$.
(e) All of the above must be true.
(f) More than one, but not all, of the above must be true.

344. Let M be any matrix. **True or False** The columns of M are orthonormal if and only if $M^T M$ is an identity matrix.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

345. Let Q be a square matrix with orthonormal columns. **True or False** $Q^{-1} = Q^T$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

346. **True or False** Any set of nonzero orthogonal vectors must also be linearly independent.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

347. **True or False** The only orthonormal basis for $\Re^2$ is $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

Orthogonal Projections

348. If $b = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ and $y = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, then the orthogonal projection of b onto y is

(a) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix}$

(c) $\begin{bmatrix} 10 \\ 5 \end{bmatrix}$

(d) $\begin{bmatrix} 1/10 \\ 3/10 \end{bmatrix}$

349. If $b = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ and l is the line $y = \frac{1}{2}x$, then the orthogonal projection of b onto l is

(a) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix}$

(c) $\begin{bmatrix} 10 \\ 5 \end{bmatrix}$

(d) $\begin{bmatrix} 1/10 \\ 3/10 \end{bmatrix}$

350. If l is the line $y = 3x$, $b \in \Re^2$, and z is the orthogonal projection of b on l, then which of the following are true?

(a) $b - z$ is perpendicular to l.

(b) $b - z$ is a point on l.

(c) z is of the form $(c, 3c)$

(d) Exactly two of the statements are true.

(e) None of the above are true.

351. Let A be an $n \times p$ matrix. Let W be the column space of A, so W is a subspace of $\Re^n$. Let $b \in \Re^n$ and let z be an orthogonal projection of b on W. Then which of the following is *not* true?

(a) $A^T(b-z)=0$.

(b) z is orthogonal to W.

(c) $b-z$ is orthogonal to W.

(d) z is the vector in W closest to b.

352. Let $v_1 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$, and $v = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$. Let z be the orthogonal projection of v on the span of $\{v_1, v_2\}$, and let $A = \begin{bmatrix} v_1 & v_2 \end{bmatrix}$. Which of the following are true?

(a) $z = Ax$ for some x.

(b) z is a linear combination of v_1 and v_2.

(c) $z = -\frac{1}{5}\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + \frac{7}{30}\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$

(d) All of the above statements are true.

(e) Exactly two of the above statements are true.

(f) None of the above statements are true.

Gram-Schmidt Orthogonalization

353. Let A be an $m \times n$ matrix with linearly independent columns $x_1, x_2, \ldots, x_n$. Applying the Gram-Schmidt process to $x_1, x_2, \ldots, x_n$ will produce

(a) an orthogonal basis for A.

(b) an orthogonal basis for the column space of A.

(c) an orthogonal basis for the row space of A.

(d) an orthogonal basis for the null space of A.

354. Let $v_1 = (2, -1, 0)$ and $v_2 = (1, 1, 1)$. The Gram-Schmidt process, when applied to these vectors, produces $\{v_1', v_2'\}$ where

(a) $v_1' = (2, -1, 0)$ and $v_2' = (-1, 2, 1)$.

(b) $v_1' = (2, -1, 0)$ and $v_2' = (3/5, 6/5, 1)$.

(c) $v_1' = (2, -1, 0)$ and $v_2' = (2/5, -1/5, 0)$.

(d) $v_1' = (2, -1, 0)$ and $v_2' = (7/5, 6/5, 1)$.

(e) $v_1' = (2, -1, 0)$ and $v_2' = (3/2, 3, 1)$.

355. **True or False** If $W = span\left\{\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right\}$, then $S = \left\{\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 5 \end{bmatrix}\right\}$ is an orthogonal basis for W.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

Chapter 7: Symmetric Matrices and Quadratic Forms

Symmetric Matrices

356. **True or False** If A is a symmetric, invertible matrix, then $A^{-1} = A^T$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

357. If A is an $n \times n$ real symmetric matrix, then which of the following is true?

(a) Each eigenvalue of A is real.
(b) If A is invertible, then its inverse is also symmetric.
(c) If $Ax = 2x$ and $Ay = 3y$ then $x \cdot y = 0$.
(d) If λ_1 and λ_2 are two different eigenvalues of A and W_1 and W_2 are the corresponding eigenspaces, then W_1 and W_2 are orthogonal sets.
(e) All of the above are true.
(f) More than one, but not all, of the above are true.

Chapter 8: The Geometry of Vector Spaces

Affine and Convex Combinations

358. How do you describe the set of all affine combinations of the vectors (1, 0) and (0, 1)?

    (a) A point
    (b) A line segment
    (c) A line
    (d) $\Re^2$
    (e) $\Re^3$

359. How do you describe the set of all convex combinations of the vectors (1, 0) and (0, 1)?

    (a) A point
    (b) A line segment
    (c) A line
    (d) $\Re^2$
    (e) $\Re^3$

360. How do you describe the set of all affine combinations of the vectors (1, 0) and (0, 1) and (1, 1)?

    (a) Three lines (the lines through each pair of vectors)
    (b) The boundary of the triangle formed by these three vectors
    (c) The boundary and interior of the triangle formed by these three vectors
    (d) $\Re^2$

361. How do you describe the set of all convex combinations of the vectors (1, 0) and (0, 1) and (1, 1)?

    (a) Three lines (the lines through each pair of vectors)
    (b) The boundary of the triangle formed by these three vectors
    (c) The boundary and interior of the triangle formed by these three vectors
    (d) $\Re^2$

362. Suppose x and y both solve $Ax = b$. **True or False** All linear combinations of x and y also solve $Ax = b$. (You should be prepared to support your answer with either a proof or a counterexample.)

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

363. Suppose x and y both solve $Ax = b$. **True or False** All affine combinations of x and y also solve $Ax = b$. (You should be prepared to support your answer with either a proof or a counterexample.)

(a) True, and I am very confident

(b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

364. What is the maximum number of affinely independent vectors in $\Re^n$?

(a) $n - 1$

(b) n

(c) $n + 1$

365. Which of the following statements is correct?

(a) A set of vectors that is linearly independent must be affinely independent.

(b) A set of vectors that is affinely independent must be linearly independent.

(c) Both statements are true.