Classroom Voting Questions: Multivariable Calculus

14.3 Local Linearity and the Differential

1. Let $f(2,3) = 7$, $f_x(2,3) = -1$, and $f_y(2,3) = 4$. Then the tangent plane to the surface $z = f(x,y)$ at the point $(2,3)$ is

   (a) $z = 7 - x + 4y$

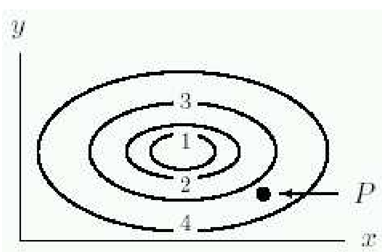
   (b) $x - 4y + z + 3 = 0$

   (c) $-x + 4y + z = 7$

   (d) $-x + 4y + z + 3 = 0$

   (e) $z = 17 + x - 4y$

2. The figure below shows level curves of the function $f(x,y)$. The tangent plane approximation to $f(x,y)$ at the point $P = (x_0, y_0)$ is $f(x,y) \approx c + m(x - x_0) + n(y - y_0)$. What are the signs of c, m, and n?

   (a) $c > 0$, $m > 0$, $n > 0$

   (b) $c < 0$, $m > 0$, $n < 0$

   (c) $c > 0$, $m < 0$, $n > 0$

   (d) $c < 0$, $m < 0$, $n < 0$

   (e) $c > 0$, $m > 0$, $n < 0$

3. Suppose that $f(x,y) = 2x^2y$. What is the tangent plane to this function at $x = 2$, $y = 3$?

   (a) $z = 4xy(x-2) + 2x^2(y-3) + 24$

   (b) $z = 4x(x-2) + 2(y-3) + 24$

   (c) $z = 8(x-2) + 2(y-3) + 24$

(d) $z = 24(x-2) + 8(y-3) + 24$

(e) $z = 24x + 8y + 24$

4. The differential of a function $f(x, y)$ at the point (a, b) is given by the formula $df = f_x(a, b)dx + f_y(a, b)dy$. Doubling dx and dy doubles df.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

5. The differential of a function $f(x, y)$ at the point (a, b) is given by the formula $df = f_x(a, b)dx + f_y(a, b)dy$. Moving to a different point (a, b) may change the formula for df.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

6. The differential of a function $f(x, y)$ at the point (a, b) is given by the formula $df = f_x(a, b)dx + f_y(a, b)dy$. If dx and dy represent small changes in x and y in moving away from the point (a, b), then df approximates the change in f.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

7. The differential of a function $f(x, y)$ at the point (a, b) is given by the formula $df = f_x(a, b)dx + f_y(a, b)dy$. The equation of the tangent plane to $z = f(x, y)$ at the point (a, b) can be used to calculate values of df from dx and dy.

   (a) True, and I am very confident
   (b) True, but I am not very confident
   (c) False, but I am not very confident
   (d) False, and I am very confident

8. A small business has \$300,000 worth of equipment and 100 workers. The total monthly production, P (in thousands of dollars), is a function of the total value of the equipment, V (in thousands of dollars), and the total number of workers, N. The differential of P is given by $dP = 4.9dN + 0.5dV$. If the business decides to lay off 3 workers and buy additional equipment worth \$20,000, then

   (a) Monthly production increases.

   (b) Monthly production decreases.

   (c) Monthly production stays the same.

9. Which of the following could be the equation of the tangent plane to the surface $z = x^2 + y^2$ at a point (a, b) in the first quadrant?

   (a) $z = -3x + 4y + 7$

   (b) $z = 2x - 4y + 5$

   (c) $z = 6x + 6y - 18$

   (d) $z = -4x - 4y + 24$

10. Suppose $f_x(3, 4) = 5$, $f_y(3, 4) = -2$, and $f(3, 4) = 6$. Assuming the function is differentiable, what is the best estimate for $f(3.1, 3.9)$ using this information?

    (a) 6.3

    (b) 9

    (c) 6

    (d) 6.7

11. We need to figure out the area of the floor of a large rectangular room, however our measurements aren't very precise. We find that the room is 52.3 ft by 44.1 ft so we get an area of 2,306.43 square feet, but our measurements are only good to within a couple of inches, roughly an error of 0.2 feet in both directions, so our estimate of the area is probably off by a bit. Use differentials to determine the likely error in our estimate of the floor's area.

    (a) 0.04 square feet

    (b) 19.24 square feet

    (c) 19.28 square feet

    (d) 19.32 square feet

    (e) 19.56 square feet

12. A giant stone cylinder suddenly appears on the campus lawn outside, and we of course ask ourselves: What could its volume be? We send out a student with a meter stick to measure the cylinder, who reports that the cylinder has a height of 3.34 meters (with a measurement error of 2 centimeters) and has a radius of 2.77 meters (give or take 3 centimeters). We know that the volume of a cylinder is $V = \pi r^2 h$, so we estimate its volume as 80.5 m^3. What is our measurement error on this volume?

    (a) 5 cm
    (b) 1.00 m^3
    (c) 1.89 m^3
    (d) 2.23 m^3