14.5 Gradients and Directional Derivatives in Space

1. Suppose the temperature at a point \((x, y, z)\) in a room is given by \(T(x, y, z)\). Suppose heat is being radiated out from a hot spot at the origin. Which of the following could be \(\nabla T(a, b, c)\) where \(a, b, c\) are all positive?

(a) \(2\hat{i} + 2\hat{j} - 4\hat{k}\)
(b) \(-3\hat{i} - 3\hat{j} - 5\hat{k}\)
(c) \(-2\hat{i} + 2\hat{j} + 5\hat{k}\)
(d) \(3\hat{i} + 3\hat{j} + 5\hat{k}\)

2. Let \(f(x, y, z) = x^2 + y^2 + z^2\). Which statement best describes the vector \(\nabla f(x, y, z)\)? It is always perpendicular to:

(a) vertical cylinder passing through \((x, y, z)\).
(b) a horizontal plane passing through \((x, y, z)\).
(c) a sphere centered on the origin passing through \((x, y, z)\).
(d) None of the above

3. For \(f(x, y, z)\), suppose \(\nabla f(a, b, c) \cdot \hat{i} > \nabla f(a, b, c) \cdot \hat{j} > \nabla f(a, b, c) \cdot \hat{k} > 0\). The tangent plane to the surface \(f(x, y, z) = 0\) through the point \((a, b, c)\) is given by \(z = p + mx + ny\). Which of the following is correct?

(a) \(m > n > 0\)
(b) \(n > m > 0\)
(c) \(m < n < 0\)
(d) \(n < m < 0\)

4. The function \(f(x, y)\) has gradient \(\nabla f\) at the point \((a, b)\). The vector \(\nabla f\) is perpendicular to the level curve \(f(x, y) = f(a, b)\).

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

5. The function $f(x, y)$ has gradient $\nabla f$ at the point $(a, b)$. The vector $\nabla f$ is perpendicular to the surface $z = f(x, y)$ at the point $(a, b, f(a, b))$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

6. The function $f(x, y)$ has gradient $\nabla f$ at the point $(a, b)$. The vector $f_x(a, b)i + f_y(a, b)j + k$ is perpendicular to the surface $z = f(x, y)$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident