Classroom Voting Questions: Multivariable Calculus

14.5 Gradients and Directional Derivatives in Space

1. Suppose the temperature at a point (x, y, z) in a room is given by $T(x, y, z)$. Suppose heat is being radiated out from a hot spot at the origin. Which of the following could be $\nabla T(a, b, c)$ where a, b, c are all positive?

   (a) $2\hat{i} + 2\hat{j} - 4\hat{k}$

   (b) $-3\hat{i} - 3\hat{j} - 5\hat{k}$

   (c) $-2\hat{i} + 2\hat{j} + 5\hat{k}$

   (d) $3\hat{i} + 3\hat{j} + 5\hat{k}$

2. Let $f(x, y, z) = x^2 + y^2 + z^2$. Which statement best describes the vector $\nabla f(x, y, z)$? It is always perpendicular to:

   (a) vertical cylinder passing through (x, y, z).

   (b) a horizontal plane passing through (x, y, z).

   (c) a sphere centered on the origin passing through (x, y, z).

   (d) None of the above

3. For $f(x, y, z)$, suppose $\nabla f(a, b, c) \cdot \hat{i} > \nabla f(a, b, c) \cdot \hat{j} > \nabla f(a, b, c) \cdot \hat{k} > 0$. The tangent plane to the surface $f(x, y, z) = 0$ through the point (a, b, c) is given by $z = p + mx + ny$. Which of the following is correct?

   (a) $m > n > 0$

   (b) $n > m > 0$

   (c) $m < n < 0$

   (d) $n < m < 0$

4. The function $f(x, y)$ has gradient ∇f at the point (a, b). The vector ∇f is perpendicular to the level curve $f(x, y) = f(a, b)$.

   (a) True, and I am very confident

   (b) True, but I am not very confident

(c) False, but I am not very confident

(d) False, and I am very confident

5. The function $f(x, y)$ has gradient ∇f at the point (a, b). The vector ∇f is perpendicular to the surface $z = f(x, y)$ at the point $(a, b, f(a, b))$.

   (a) True, and I am very confident

   (b) True, but I am not very confident

   (c) False, but I am not very confident

   (d) False, and I am very confident

6. The function $f(x, y)$ has gradient ∇f at the point (a, b). The vector $f_x(a, b)\hat{i} + f_y(a, b)\hat{j} + \hat{k}$ is perpendicular to the surface $z = f(x, y)$.

   (a) True, and I am very confident

   (b) True, but I am not very confident

   (c) False, but I am not very confident

   (d) False, and I am very confident