Classroom Voting Questions:
Multivariable Calculus

16.1 The Definite Integral of a Function of Two Variables

1. Suppose the contour plot shown shows the height of a pile of dirt in feet. Which of the following is clearly a lower bound for the volume of dirt?

   (a) $0 \times 1 + 0 \times 1 + 6 \times 1 + 6 \times 1$
   (b) $9 \times 1 + 9 \times 1 + 9 \times 1 + 9 \times 1$
   (c) $9$
   (d) $6 \times 1 + 6 \times 1 + 9 \times 1 + 9 \times 1$

2. Let $R$ be the region $10 \leq x \leq 14$; $20 \leq y \leq 30$. The table below gives values of $f(x, y)$. Using upper and lower Riemann sums, what are the best possible upper and lower estimates for the integral

$$I = \int_{R} f(x, y) dx dy$$
(a) $23 < I < 990$
(b) $92 < I < 300$
(c) $160 < I < 396$
(d) $160 < I < 300$
(e) $92 < I < 396$

3. Let $R$ be the square defined by $-1 \leq x \leq 1$, $-1 \leq y \leq 1$. The sign of the definite integral of $x^4$ over $R$ is:

(a) positive
(b) negative
(c) zero
(d) cannot be determined

4. The value of $(1/\pi)$ times the integral of $1 + x$ over the unit circle $R$ is:

(a) 0
(b) 1
(c) $\pi$
(d) $\pi/2$

5. The integral $\int_R x\,dA$ over the region where $R$ is the rectangle $-1 \leq x \leq 1$, $-1 \leq y \leq 1$ is

(a) positive
(b) negative
(c) zero

6. The integral $\int_T y\,dA$ over the region where $T$ is the rectangle $-1 \leq x \leq 1$, $0 \leq y \leq 1$ is

(a) positive
7. The integral \( \int_R (x - x^2) \, dA \) over the region where \( R \) is the rectangle \(-1 \leq x \leq 1, -1 \leq y \leq 1\) is

(a) positive  
(b) negative  
(c) zero

8. The integral \( \int_T (y - y^2) \, dA \) over the region where \( T \) is the rectangle \(-1 \leq x \leq 1, 0 \leq y \leq 1\) is

(a) positive  
(b) negative  
(c) zero

9. The integral \( \int_L (x^2 - x) \, dA \) over the region where \( L \) is the rectangle \(-1 \leq x \leq 0, -1 \leq y \leq 1\) is

(a) positive  
(b) negative  
(c) zero

10. The integral \( \int_L (y + y^2) \, dA \) over the region where \( L \) is the rectangle \(-1 \leq x \leq 0, -1 \leq y \leq 1\) is

(a) positive  
(b) negative  
(c) zero

11. The integral \( \int_R (2x + 3y) \, dA \) over the region where \( R \) is the rectangle \(-1 \leq x \leq 1, -1 \leq y \leq 1\) is

(a) positive  
(b) negative  
(c) zero