16.5 Integrals in Cylindrical and Spherical Coordinates

1. What are the Cartesian coordinates of the point with cylindrical coordinates \((r, \theta, z) = (4, \pi, 6)\)?
   
   (a) \((x, y, z) = (0, -4, 4)\)
   (b) \((x, y, z) = (0, 4, 6)\)
   (c) \((x, y, z) = (-4, 4, 4)\)
   (d) \((x, y, z) = (4, 0, 4)\)
   (e) \((x, y, z) = (-4, 0, 6)\)

2. What are the cylindrical coordinates of the point with Cartesian coordinates \((x, y, z) = (3, 3, 7)\)?
   
   (a) \((r, \theta, z) = (3, \pi, 7)\)
   (b) \((r, \theta, z) = (3, \pi/4, 3)\)
   (c) \((r, \theta, z) = (3\sqrt{2}, \pi/4, 7)\)
   (d) \((r, \theta, z) = (3\sqrt{2}, \pi, 7)\)
   (e) \((r, \theta, z) = (3\sqrt{2}, \pi, 3)\)

3. What are the Cartesian coordinates of the point with spherical coordinates \((\rho, \phi, \theta) = (4, \pi, 0)\)?
   
   (a) \((x, y, z) = (0, 0, -4)\)
   (b) \((x, y, z) = (0, 0, 4)\)
   (c) \((x, y, z) = (4, 0, 0)\)
   (d) \((x, y, z) = (-4, 0, 0)\)
   (e) \((x, y, z) = (0, 4, 0)\)

4. What are the spherical coordinates of the point with Cartesian coordinates \((x, y, z) = (0, -3, 0)\)?
(a) \((\rho, \phi, \theta) = (3, \pi, \frac{\pi}{2})\)
(b) \((\rho, \phi, \theta) = (3, \pi, -\frac{\pi}{2})\)
(c) \((\rho, \phi, \theta) = (3, \frac{\pi}{2}, \frac{\pi}{2})\)
(d) \((\rho, \phi, \theta) = (3, \frac{\pi}{2}, -\frac{\pi}{2})\)
(e) \((\rho, \phi, \theta) = (3, \frac{\pi}{2}, \pi)\)

5. Which of the following regions represents the portion of a cylinder of height 4 and radius 3 above the 3rd quadrant of the \(xy\) plane?

(a) \(1 \leq r \leq 3, 0 \leq z \leq 4, 0 \leq \theta \leq \frac{\pi}{2}\)
(b) \(0 \leq r \leq 3, 0 \leq z \leq 4, \pi \leq \theta \leq 3\pi/2\)
(c) \(0 \leq r \leq 4, 0 \leq z \leq 3, \pi \leq \theta \leq 3\pi/2\)
(d) \(0 \leq r \leq 3, 0 \leq z \leq 4, 0 \leq \theta \leq \pi/2\)

6. Which of the following is equivalent to

\[
\int_{-5}^{5} \int_{0}^{3} \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} x \, dy \, dz \, dx
\]

(a) \(\int_{0}^{5} \int_{0}^{3} \int_{0}^{\frac{\pi}{2}} r^2 \cos \theta \, d\theta \, dz \, dr\)
(b) \(\int_{0}^{5} \int_{0}^{3} \int_{0}^{\frac{\pi}{2}} r^2 \cos \theta \, d\theta \, dz \, dr\)
(c) \(\int_{0}^{3} \int_{0}^{5} \int_{0}^{2\pi} r \cos \theta \, d\theta \, dz \, dr\)
(d) \(\int_{0}^{5} \int_{0}^{3} \int_{0}^{2\pi} r^2 \cos \theta \, d\theta \, dz \, dr\)

7. Which of the following describes the bottom half of a sphere of radius 4 centered on the origin?

(a) \(0 \leq \rho \leq 4, \pi/2 \leq \phi \leq \pi, 0 \leq \theta \leq 2\pi\)
(b) \(0 \leq \rho \leq 4, 0 \leq \phi \leq \pi/2, 0 \leq \theta \leq 2\pi\)
(c) \(0 \leq \rho \leq 4, 0 \leq \phi \leq \pi, 0 \leq \theta \leq \pi\)
(d) \(0 \leq \rho \leq 4, 0 \leq \phi \leq \pi, 0 \leq \theta \leq 2\pi\)

8. Which of the following describes the surface of the cylinder of radius 3 centered on the \(z\)-axis?

(a) \(0 \leq \rho < \infty, \theta = \pi, 0 \leq \phi \leq \pi\)
(b) \( r = 3, \theta = \frac{\pi}{2}, -\infty < z < \infty \)
(c) \( 1 \leq r \leq 4, 0 \leq \theta \leq 2\pi, -5 \leq z \leq 2 \)
(d) \( r = 3, 0 \leq \theta \leq 2\pi, -\infty < z < \infty \)

9. Which of the following describes the solid cylinder of radius 4, centered on the \( z \)-axis, with the central cylindrical core removed?

(a) \( 0 \leq \rho < \infty, \theta = \pi, 0 \leq \phi \leq \pi \)
(b) \( r = 3, \theta = \frac{\pi}{2}, -\infty < z < \infty \)
(c) \( 1 \leq r \leq 4, 0 \leq \theta \leq 2\pi, -5 \leq z \leq 2 \)
(d) \( r = 3, 0 \leq \theta \leq 2\pi, -\infty < z < \infty \)

10. Which of the following integrals give the volume of the unit sphere?

(a) \( \int_0^{2\pi} \int_0^2 \int_0^1 d\rho d\theta d\phi \)
(b) \( \int_0^{2\pi} \int_0^\pi \int_0^1 d\rho d\theta d\phi \)
(c) \( \int_0^{2\pi} \int_0^\pi \int_0^1 \rho^2 \sin \phi d\rho d\theta d\phi \)
(d) \( \int_0^{2\pi} \int_0^1 \int_0^1 \rho^2 \sin \phi d\rho d\phi d\theta \)
(e) \( \int_0^{2\pi} \int_0^\pi \int_0^1 \rho d\rho d\phi d\theta \)