17.1 Parameterized Curves

1. Which of the following is an equation of a line in three dimensions \((x, y, z)\)?
   
   (a) \(x = 4\)
   (b) \(y = 2x + 3\)
   (c) \(z = 3x + 2y + 7\)
   (d) All of the above
   (e) None of the above

2. Which of the following best describes the path of a particle defined by the parametric equations \(x(t) = \cos(t^2), y(t) = \sin(t^2)\)?
   
   (a) a circle around which the particle moves faster and faster
   (b) a parabola on which the particle travels at constant speed
   (c) a parabola on which the particle travels faster and faster
   (d) a circle on which the particle moves slower and slower

3. Which of the following is not a parameterization of the entire curve \(y = x^3\)?
   
   (a) \(x(t) = t; y(t) = t^3\)
   (b) \(x(t) = t^2; y(t) = t^6\)
   (c) \(x(t) = t^3; y(t) = t^9\)
   (d) \(x(t) = 2t; y(t) = 8t^3\)

4. What does the path of the particle described by \(x(t) = \cos(t), y(t) = \sin(t), z(t) = -t\) look like?
   
   (a) a circle in the \(xz\) plane
   (b) a helix on which the particle is traveling up
   (c) a helix on which the particle is traveling down
   (d) a sine wave in the \(xz\) plane
5. Which of the following parameterizations does not describe the quarter circle in the figure below?

\begin{center}
\begin{tikzpicture}
  \fill[fill=white] (0,0) circle (1.5ex);
  \fill[fill=white] (1,0) circle (1.5ex);
  \fill[fill=white] (0,1) circle (1.5ex);
  \draw[->] (-1.5,0) -- (1.5,0) node[right] {$x$};
  \draw[->] (0,-1.5) -- (0,1.5) node[above] {$y$};
\end{tikzpicture}
\end{center}

(a) \((\cos t, \sin t), 0 \leq t \leq \pi/2\)
(b) \((\sin t, \cos t), 0 \leq t \leq \pi/2\)
(c) \((- \cos t, \sin t), \pi/2 \leq t \leq \pi\)
(d) \((\cos t, -\sin t), 3\pi/2 \leq t \leq 2\pi\)

6. Let \((\cos at, \sin at)\) be the position at time \(t\) seconds of a particle moving around a circle, where \(a > 0\). If \(a\) is increased,

(a) The radius of the circle increases.
(b) The speed of the particle increases.
(c) The center of the circle changes.
(d) The path ceases to be a circle.

7. Let \((a \cos t, a \sin t)\) be the position at time \(t\) seconds of a particle moving around a circle, where \(a > 0\). If \(a\) is increased,

(a) The radius of the circle increases.
(b) The speed of the particle increases.
(c) The center of the circle changes.
(d) The path ceases to be a circle.

8. Which of the following parametric curves does not trace out the unit circle?

(a) \((\cos t, \sin t), 0 \leq t \leq 2\pi\)
(b) \((\sin^2 t, \cos^2 t), 0 \leq t \leq 2\pi\)
(c) \((\sin(t^2), \cos(t^2)), 0 \leq t \leq 2\pi\)
(d) \((\sin(2t), \cos(2t)), 0 \leq t \leq 2\pi\)

9. Which of the following parametric paths describe particles that are not traveling along a straight line in 3-space?

(a) \((1 - t, 2 + 2t, 3 - t)\)
(b) \((1 - t^2, 2 + 2t^2, 3 - t^2)\)
(c) \((1, 2, 1 - t)\)
(d) \((1, t, 1 - t^2)\)
(e) More than one of the above

10. The value of \(c\) for which the lines \(l(t) = (c+4t, 2-t, 3+t)\) and \(m(t) = (4t, 1-8t, 4+4t)\) intersect is

(a) 4
(b) 0
(c) -4
(d) There is no such \(c\).