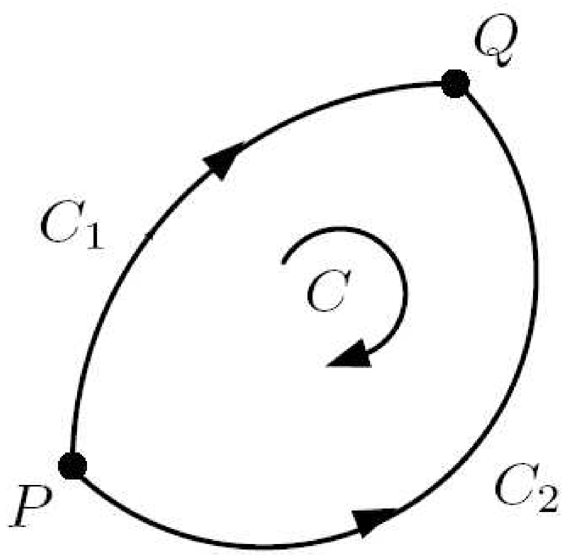
Classroom Voting Questions: Multivariable Calculus

18.4 Path-Dependent Vector Fields and Green's Theorem

1. What will guarantee that $\vec{F}(x,y) = y\hat{i} + g(x,y)\hat{j}$ is not a gradient vector field?

   (a) $g(x,y)$ is a function of y only
   (b) $g(x,y)$ is a function of x only
   (c) $g(x,y)$ is always larger than 1
   (d) $g(x,y)$ is a linear function

2. The figure shows a curve C broken into two pieces C_1 and C_2. Which of the following statements is true for any smooth vector field $\vec{F}$?



   (a) $\int_C \vec{F} \cdot d\vec{r} = \int_{C_1} \vec{F} \cdot d\vec{r} + \int_{C_2} \vec{F} \cdot d\vec{r}$
   (b) $\int_C \vec{F} \cdot d\vec{r} = \int_{C_1} \vec{F} \cdot d\vec{r} - \int_{C_2} \vec{F} \cdot d\vec{r}$
   (c) $\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$
   (d) $\int_C \vec{F} \cdot d\vec{r} = 0$
   (e) More than one of the above is true.

3. A smooth two dimensional vector field $\vec{F} = F_1\hat{\imath} + F_2\hat{\jmath}$, with $\vec{F} \neq \vec{0}$ satisfies $\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$ at every point in the plane. Which of the following statments is not true?

   (a) $\int_C \vec{F} \cdot d\vec{r} = 0$ for all smooth paths C.

   (b) $\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$ for any two smooth paths C_1 and C_2 with the same starting and ending points.

   (c) $\vec{F} = \nabla f$ for some function f.

   (d) If C_1 is the straight line from -1 to 1 on the y-axis and if C_2 is the right half of the unit circle, traversed counter-clockwise, then $\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$.

   (e) More than one of the above is not true.