Classroom Voting Questions: Multivariable Calculus

19.1 The Idea of a Flux Integral

1. A river is flowing downstream at a constant rate of 5 ft/s. We take a rectangular net that is 6 ft wide and 3 ft deep and place it in the river so that a vector perpendicular to the net (a normal vector) is parallel to the velocity of the water. What is the rate at which water flows through the net?

   (a) 0 ft^3/s
   (b) 15 ft^2/s
   (c) 30 ft^2/s
   (d) 90 ft^3/s
   (e) None of the above

2. A river is flowing downstream at a constant rate of 5 ft/s. We take a rectangular net that is 6 ft wide and 3 ft deep and place it in the river so that there is a 30 degree angle between a vector perpendicular to the net (a normal vector) and the velocity of the water. What is the rate at which water flows through the net?

   (a) 0 ft^3/s
   (b) 90 ft^3/s
   (c) 45 ft^3/s
   (d) ≈ 78 ft^3/s
   (e) None of the above

3. Through which surface is the flux of $\vec{F}(x, y, z) = 2\hat{i}$ negative?

   (a) A square of side length 2 in the yz plane, oriented in the negative x direction
   (b) A square of side length 2 in the xz plane, oriented in the positive y direction
   (c) A square of side length 2 in the yz plane, oriented in the positive x direction
   (d) A square of side length 2 in the xz plane, oriented up.

4. Through which surface is the flux of $\vec{F}(x, y, z) = x\hat{i}$ the most positive?

(a) A square of side length 2 in the yz plane, oriented in the positive x direction

(b) A square of side length 2 in the plane $x = 4$, oriented in the positive x direction

(c) A square of side length 4 in the plane $x = 2$, oriented in the positive x direction

(d) A square of side length 1 in the plane $x = 8$, oriented in the positive x direction

5. Consider the flux of $\vec{F} = x\hat{i}$ through a disk of radius 1 oriented as described below. In which case is the flux positive?

   (a) In the yz-plane, centered at the origin and oriented in the direction of increasing x.

   (b) In the plane $x = 2$, centered on the x-axis and oriented away from the origin.

   (c) In the plane $y = 2$, centered on the y-axis and oriented away from the origin.

   (d) In the plane $x + y = 2$, centered on the x-axis and oriented away from the origin.

   (e) More than one of the above has positive flux.

   (f) None of the above.

6. Consider the flux of $\vec{F} = y\hat{i}$ through a disk of radius 1 oriented as described below. In which case is the flux positive?

   (a) In the yz-plane, centered at the origin and oriented in the direction of increasing x.

   (b) In the plane $x = 2$, centered on the x-axis and oriented away from the origin.

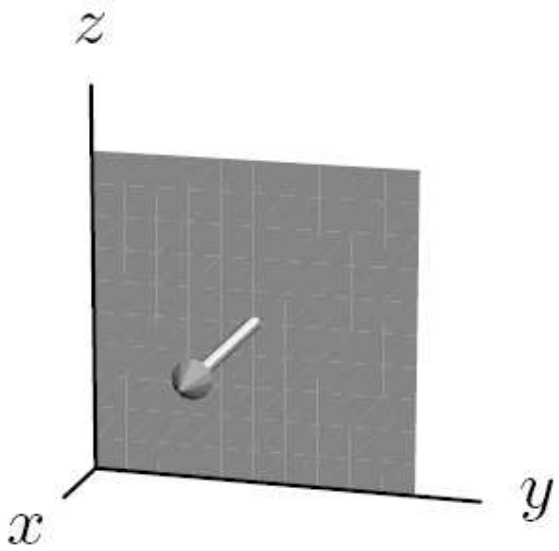
   (c) In the plane $y = 2$, centered on the y-axis and oriented away from the origin.

   (d) In the plane $x + y = 2$, centered on the x-axis and oriented away from the origin.

   (e) More than one of the above has positive flux.

   (f) None of the above.

7. Which vector field has a positive flux through the surface below?



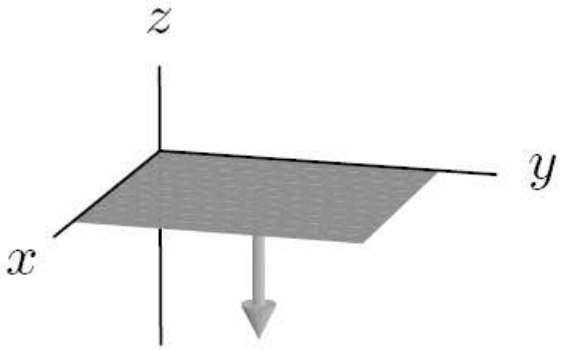
(a) $\vec{F} = x\hat{j}$

(b) $\vec{F} = y\hat{j}$

(c) $\vec{F} = -z\hat{i}$

(d) $\vec{F} = (z + x)\hat{i}$

8. Which vector field has a positive flux through the surface below?



(a) $\vec{F} = -y\hat{k}$

(b) $\vec{F} = y\hat{j}$

(c) $\vec{F} = -z\hat{i}$

(d) $\vec{F} = x\hat{k}$

9. Let $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$. Which of the surfaces below has positive flux?

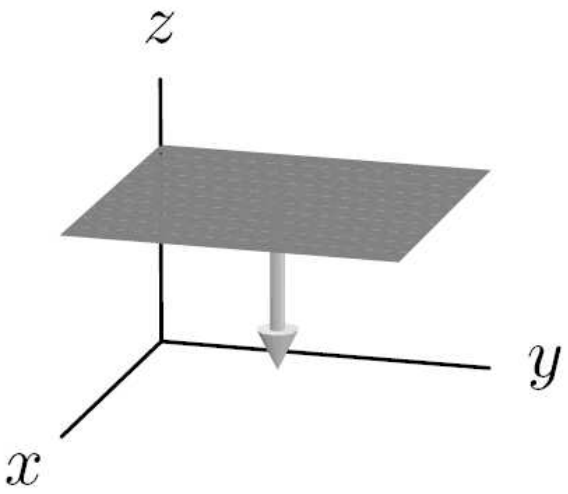
(a) Sphere of radius 1 centered at the origin, oriented outward.

(b) Unit disk in the xy-plane, oriented upward.

(c) Unit disk in the plane $x = 2$, oriented toward the origin.

(d) None of the above.

10. Choose the vector field with the largest flux through the surface below.



(a) $\vec{F}_1 = 2\hat{i} - 3\hat{j} - 4\hat{k}$

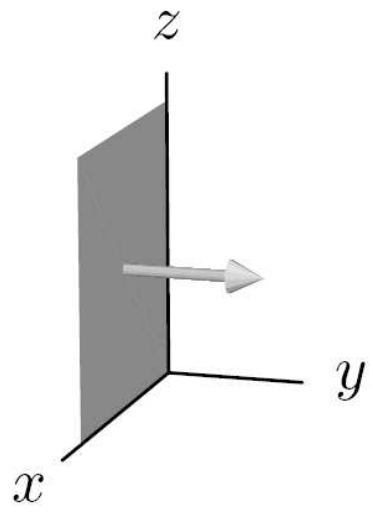
(b) $\vec{F}_2 = \hat{i} - 2\hat{j} + 7\hat{k}$

(c) $\vec{F}_3 = -7\hat{i} + 5\hat{j} + 6\hat{k}$

(d) $\vec{F}_4 = -11\hat{i} + 4\hat{j} - 5\hat{k}$

(e) $\vec{F}_5 = -5\hat{i} + 3\hat{j} + 5\hat{k}$

11. Choose the vector field with the largest flux through the surface below.



(a) $\vec{F}_1 = 2\hat{i} - 3\hat{j} - 4\hat{k}$

(b) $\vec{F}_2 = \hat{i} - 2\hat{j} + 7\hat{k}$

(c) $\vec{F}_3 = -7\hat{i} + 5\hat{j} + 6\hat{k}$

(d) $\vec{F}_4 = -11\hat{i} + 4\hat{j} - 5\hat{k}$

(e) $\vec{F}_5 = -5\hat{i} + 3\hat{j} + 5\hat{k}$

12. Which of the following vector fields has the largest flux through the surface of a sphere of radius 2 centered at the origin?

(a) $\vec{F}_1 = \frac{\vec{\rho}}{||\vec{\rho}||}$

(b) $\vec{F}_2 = \frac{\vec{\rho}}{||\vec{\rho}||^2}$

(c) $\vec{F}_3 = x\hat{j}$

(d) $\vec{F}_4 = \vec{\rho}||\vec{\rho}||$