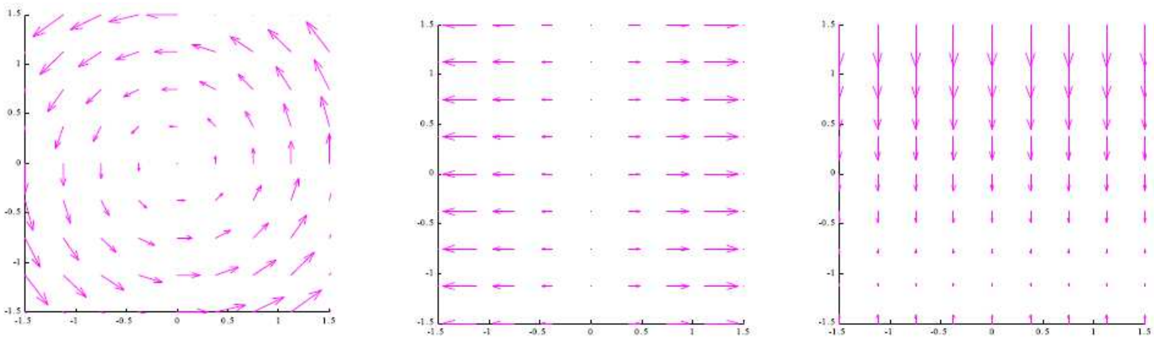
Classroom Voting Questions: Multivariable Calculus

20.1 The Divergence of a Vector Field

1. Moving from the picture on the left to the picture on the right, what are the signs of $\nabla \cdot \vec{F}$?



   (a) positive, positive, negative
   (b) zero, positive, negative
   (c) positive, negative, zero
   (d) zero, negative, positive

2. If $\vec{F}(x, y, z)$ is a vector field and $f(x, y, z)$ is a scalar function, which of the following is not defined?

   (a) ∇f
   (b) $\nabla \cdot \vec{F} + f$
   (c) $\vec{F} + \nabla f$
   (d) $\nabla \cdot \vec{F} + \nabla f$
   (e) More than one of the above
   (f) None of the above

3. If $\vec{F}(x, y, z)$ is a vector field and $f(x, y, z)$ is a scalar function, which of the following quantities is a vector?

   (a) $\nabla \cdot \vec{F}$
   (b) $\nabla f \cdot \vec{u}$

(c) $\nabla \cdot \nabla f$

(d) $(\nabla \cdot \vec{F})\vec{F}$

4. True or False? If all the flow lines of a vector field $\vec{F}$ are parallel straight lines, then $\nabla \cdot \vec{F} = 0$.

    (a) True, and I am very confident

    (b) True, but I am not very confident

    (c) False, but I am not very confident

    (d) False, and I am very confident

5. True or False? If all the flow lines of a vector field $\vec{F}$ radiate outward along straight lines from the origin, then $\nabla \cdot \vec{F} > 0$.

    (a) True, and I am very confident

    (b) True, but I am not very confident

    (c) False, but I am not very confident

    (d) False, and I am very confident

6. In Cartesian coordinates given the vector field $\vec{F} = F_1\hat{\imath} + F_2\hat{\jmath} + F_3\hat{k}$,

$$\vec{\nabla} \cdot \vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

.

Which of the following vector fields has zero divergence, so that it could represent the flow of a liquid which does not expand or contract?

    (a) $\vec{F} = 2\sin(3z^2)\hat{\imath} + 5xyz\hat{\jmath} + 3e^{7x}\hat{k}$

    (b) $\vec{F} = 3\ln(yz)\hat{\imath} + 2x^3z^7\hat{\jmath} + 4\cos(2x)\hat{k}$

    (c) $\vec{F} = 6e^{2y}\hat{\jmath} + 3\sin(4z)\hat{k}$

    (d) None of the above

7. In cylindrical coordinates given the vector field $\vec{F} = F_1\hat{r} + F_2\hat{\theta} + F_3\hat{z}$,

$$\vec{\nabla} \cdot \vec{F} = \frac{1}{r}\frac{\partial (rF_1)}{\partial r} + \frac{1}{r}\frac{\partial F_2}{\partial \theta} + \frac{\partial F_3}{\partial z}$$

.

What is the divergence of the vector field $\vec{F} = 2\theta\hat{r} + 3z\hat{\theta} + 4r\hat{z}$?

(a) $\vec{\nabla} \cdot \vec{F} = 2\theta + 3z + 4r$

(b) $\vec{\nabla} \cdot \vec{F} = 9$

(c) $\vec{\nabla} \cdot \vec{F} = \frac{2\theta}{r}$

(d) $\vec{\nabla} \cdot \vec{F} = 0$

(e) None of the above

8. In spherical coordinates given the vector field $\vec{F} = F_1\hat{\rho} + F_2\hat{\theta} + F_3\hat{\phi}$,

$$\vec{\nabla} \cdot \vec{F} = \frac{1}{\rho^2}\frac{\partial(\rho^2 F_1)}{\partial\rho} + \frac{1}{\rho\sin\phi}\frac{\partial F_2}{\partial\theta} + \frac{1}{\rho\sin\phi}\frac{\partial(F_3\sin\phi)}{\partial\phi}$$

.

What is the divergence of the vector field $\vec{F} = \frac{3}{\rho^2}\hat{\rho} + 2r\hat{\theta}$?

(a) $\vec{\nabla} \cdot \vec{F} = \frac{2r}{\rho\sin\phi}$

(b) $\vec{\nabla} \cdot \vec{F} = \frac{3}{\rho^2}$

(c) $\vec{\nabla} \cdot \vec{F} = \frac{\cos\phi}{\rho\sin\phi}$

(d) $\vec{\nabla} \cdot \vec{F} = 0$

(e) None of the above