Classroom Voting Questions: Multivariable Calculus

20.1 The Divergence of a Vector Field

1. Moving from the picture on the left to the picture on the right, what are the signs of $\nabla \cdot \vec{F}$?

(a) positive, positive, negative
(b) zero, positive, negative
(c) positive, negative, zero
(d) zero, negative, positive

2. If $\vec{F}(x, y, z)$ is a vector field and $f(x, y, z)$ is a scalar function, which of the following is not defined?

(a) $\nabla f$
(b) $\nabla \cdot \vec{F} + f$
(c) $\vec{F} + \nabla f$
(d) $\nabla \cdot \vec{F} + \nabla f$
(e) More than one of the above
(f) None of the above

3. If $\vec{F}(x, y, z)$ is a vector field and $f(x, y, z)$ is a scalar function, which of the following quantities is a vector?

(a) $\nabla \cdot \vec{F}$
(b) $\nabla f \cdot \vec{u}$
(c) $\nabla \cdot \nabla f$
(d) $(\nabla \cdot \vec{F})\vec{F}$

4. True or False? If all the flow lines of a vector field $\vec{F}$ are parallel straight lines, then $\nabla \cdot \vec{F} = 0$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

5. True or False? If all the flow lines of a vector field $\vec{F}$ radiate outward along straight lines from the origin, then $\nabla \cdot \vec{F} > 0$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

6. In Cartesian coordinates given the vector field $\vec{F} = F_1 \hat{i} + F_2 \hat{j} + F_3 \hat{k}$,

$$\nabla \cdot \vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

Which of the following vector fields has zero divergence, so that it could represent the flow of a liquid which does not expand or contract?

(a) $\vec{F} = 2 \sin(3z^2) \hat{i} + 5xyz \hat{j} + 3e^{7x} \hat{k}$
(b) $\vec{F} = 3 \ln(yz) \hat{i} + 2x^3z^7 \hat{j} + 4 \cos(2x) \hat{k}$
(c) $\vec{F} = 6e^{2y} \hat{j} + 3 \sin(4z) \hat{k}$
(d) None of the above

7. In cylindrical coordinates given the vector field $\vec{F} = F_1 \hat{r} + F_2 \hat{\theta} + F_3 \hat{z}$,

$$\nabla \cdot \vec{F} = \frac{1}{r} \frac{\partial (rF_1)}{\partial r} + \frac{1}{r} \frac{\partial F_2}{\partial \theta} + \frac{\partial F_3}{\partial z}$$

What is the divergence of the vector field $\vec{F} = 2\theta \hat{r} + 3z \hat{\theta} + 4r \hat{z}$?
(a) $\nabla \cdot \vec{F} = 2\theta + 3z + 4r$
(b) $\nabla \cdot \vec{F} = 9$
(c) $\nabla \cdot \vec{F} = \frac{2\theta}{r}$
(d) $\nabla \cdot \vec{F} = 0$
(e) None of the above

8. In spherical coordinates given the vector field $\vec{F} = F_1\hat{\rho} + F_2\hat{\theta} + F_3\hat{\phi}$,

$$\nabla \cdot \vec{F} = \frac{1}{\rho^2} \frac{\partial (\rho^2 F_1)}{\partial \rho} + \frac{1}{\rho \sin \phi} \frac{\partial F_2}{\partial \theta} + \frac{1}{\rho \sin \phi} \frac{\partial (F_3 \sin \phi)}{\partial \phi}$$

What is the divergence of the vector field $\vec{F} = \frac{3}{\rho^2}\hat{\rho} + 2r\hat{\theta}$?

(a) $\nabla \cdot \vec{F} = \frac{2r}{\rho \sin \phi}$
(b) $\nabla \cdot \vec{F} = \frac{3}{\rho^2}$
(c) $\nabla \cdot \vec{F} = \frac{\cos \phi}{\rho \sin \phi}$
(d) $\nabla \cdot \vec{F} = 0$
(e) None of the above