20.2 The Divergence Theorem

1. Given a small cube resting on the $xy$ plane with corners at $(0,0,0)$, $(a,0,0)$, $(a,a,0)$, and $(0,a,0)$, which vector field will produce positive flux through that cube?

(a) $\mathbf{F} = 3\mathbf{i}$
(b) $\mathbf{F} = x\mathbf{i} - y\mathbf{j}$
(c) $\mathbf{F} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$
(d) $\mathbf{F} = z\mathbf{k}$

2. Let $\mathbf{F} = (5x + 7y)\mathbf{i} + (7y + 9z)\mathbf{j} + (9z + 11x)\mathbf{k}$. This vector field produces the largest flux through which of the following surfaces?

(a) $S_1$, a sphere of radius 2 centered at the origin.
(b) $S_2$, a cube of side 2 centered at the origin, with sides parallel to the axes.
(c) $S_3$, a sphere of radius 1 centered at the origin.
(d) $S_4$, a pyramid contained inside $S_3$.

3. True or False? The vector field $\mathbf{F}$ is defined everywhere in a region $W$ bounded by a surface $S$. If $\nabla \cdot \mathbf{F} > 0$ at all points of $W$, then the vector field $\mathbf{F}$ points outward at all points of $S$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

4. True or False? The vector field $\mathbf{F}$ is defined everywhere in a region $W$ bounded by a surface $S$. If $\nabla \cdot \mathbf{F} > 0$ at all points of $W$, then the vector field $\mathbf{F}$ points outward at some points of $S$.

(a) True, and I am very confident
(b) True, but I am not very confident
5. True or False? The vector field $\vec{F}$ is defined everywhere in a region $W$ bounded by a surface $S$. If $\int_S \vec{F} \cdot d\vec{A} > 0$, then $\nabla \cdot \vec{F} > 0$ at some points of $W$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

6. Which of the following vector fields produces the largest flux out of the unit sphere centered at the origin?

(a) $\vec{F}_1 = (e^z + x^3)\hat{i} + e^{x^2}\hat{j} + y^3\hat{k}$
(b) $\vec{F}_2 = (z^2 + \cos y)\hat{i} - y^3\hat{j} + x^3y^3\hat{k}$
(c) $\vec{F}_3 = z^2\hat{i} - (x^2 + z^2)\hat{j} + (z^3 + zy^2)\hat{k}$
(d) $\vec{F}_4 = (x^4 - y^4)\hat{i} - (z^4 - 2x^3y)\hat{j} + (y^4 - 2x^3z)\hat{k}$