20.3 The Curl of a Vector Field

1. The pictures below show top views of three vector fields, all of which have no \( z \) component. Which one has the curl pointing in the positive \( \hat{k} \) direction at the origin?

   (a) the one on the left
   (b) the one in the middle
   (c) the one on the right
   (d) none of them

2. Let \( \vec{F}(x, y, z) \) be a vector field and let \( f(x, y, z) \) be a scalar function. If \( \vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \), which of the following is not defined?

   (a) \( \nabla \times f \)
   (b) \( \nabla \times \vec{F} + \nabla f \)
   (c) \( \nabla \times (\vec{r} \times \nabla f) \)
   (d) \( f + \nabla \cdot \vec{F} \)
   (e) More than one of the above

3. Which one of the following vector fields has a curl which points purely in the \( \hat{j} \)?

   (a) \( y\hat{i} - x\hat{j} + z\hat{k} \)
   (b) \( y\hat{i} + z\hat{j} + x\hat{k} \)
   (c) \( -z\hat{i} + y\hat{j} + x\hat{k} \)
(d) $x\hat{i} + z\hat{j} - y\hat{k}$

4. True or False? If all the flow lines of a vector field $\vec{F}$ are straight lines, then $\nabla \times \vec{F} = 0$.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident

5. True or False? If all the flow lines of a vector field $\vec{F}$ lie in planes parallel to the $xy$-plane, then the curl of $\vec{F}$ is a multiple of $\hat{k}$ at every point.

(a) True, and I am very confident
(b) True, but I am not very confident
(c) False, but I am not very confident
(d) False, and I am very confident