The Law of Sines and the Law of Cosines

1. Given a triangle with angles $A$, $B$, and $C$ and opposite sides $a$, $b$, and $c$, find the measurements of the remaining angle and sides assuming that $B = 30^\circ$, $C = 100^\circ$ and $b = 20$ feet.

   (a) $A = 50^\circ$, $a \approx 39.39$ feet, $c \approx 30.64$ feet
   (b) $A = 50^\circ$, $a \approx 10.50$ feet, $c \approx 20.26$ feet
   (c) $A = 50^\circ$, $a \approx 30.64$ feet, $c \approx 39.39$ feet
   (d) $A = 230^\circ$, $a \approx 30.64$ feet, $c \approx 39.39$ feet

2. A big pine tree has grown so that it is tilted $3^\circ$ from vertical toward the sun. When its shadow is 20 feet long, the angle of elevation from the tip of its shadow to the top of the tree is $60^\circ$. Approximately how tall is the tree (i.e. what is its length)?

   (a) 32 feet
   (b) 38 feet
   (c) 44 feet
   (d) 331 feet

3. True or False: Two angles and a side determine a unique triangle.

   (a) True, and I am very confident.
   (b) True, but I am not very confident.
   (c) False, but I am not very confident.
   (d) False, and I am very confident.

4. True or False: Given the measurements of any two angles and one side of an oblique triangle, the triangle can be completely solved using the law of sines.

   (a) True, and I am very confident.
   (b) True, but I am not very confident.
   (c) False, but I am not very confident.
   (d) False, and I am very confident.
5. True or False: Given the measurements of any two sides and an angle, the law of sines can be used to solve any oblique triangle.

(a) True, and I am very confident.
(b) True, but I am not very confident.
(c) False, but I am not very confident.
(d) False, and I am very confident.

6. Suppose you are to construct a triangle given the lengths of two sides $a$ and $b$ and the measurement of opposite angle $A$. How many possible triangles can you construct if $A$ is acute?

(a) none
(b) one
(c) two
(d) two of the above
(e) all of the above

7. Suppose you are to construct a triangle given the lengths of two sides $a$ and $b$ and the measurement of opposite angle $A$. How many possible triangles can you construct if $A$ is obtuse?

(a) none
(b) one
(c) two
(d) two of the above
(e) all of the above

8. Given an oblique triangle with sides $a = 6$ and $b = 8$, and opposite angle $A = 30^\circ$, find the measurement of opposite angle $B$.

(a) $B \approx 42^\circ$
(b) $B \approx 138^\circ$
(c) either (a) or (b)
(d) There is no solution.
9. Given an oblique triangle with sides \( a = 3 \) and \( b = 8 \), and opposite angle \( A = 30^\circ \), find the measurement of opposite angle \( B \).

(a) \( B \approx 11^\circ \)
(b) \( B \approx 42^\circ \)
(c) either (a) or (b)
(d) There is no solution.

10. A person leaves his house and walks 2 miles west and then 4 miles northwest. Approximately how far is he from home?

(a) 31.3 miles
(b) 2.95 miles
(c) 6 miles
(d) 5.6 miles

11. Given a triangle with sides \( a = 6 \), \( b = 8 \), and \( c = 11 \), find opposite angles \( A \), \( B \), and \( C \).

(a) \( A \approx 32^\circ \), \( B \approx 103^\circ \), \( C \approx 45^\circ \)
(b) \( A \approx 45^\circ \), \( B \approx 32^\circ \), \( C \approx 103^\circ \)
(c) \( A \approx 32^\circ \), \( B \approx 45^\circ \), \( C \approx 103^\circ \)
(d) There is no solution.

12. Triangle \( ABC \) has sides of length \( a \), \( b \), and \( c \), and angles of measure \( \alpha \), \( \beta \), and \( \gamma \) opposite those sides, respectively. If \( \alpha = 42^\circ \), \( \gamma = 59^\circ \), and \( b = 45 \), find \( a \).

(a) \( \frac{45 \sin 42^\circ}{\sin 79^\circ} \)
(b) \( \frac{45 \sin 42^\circ}{\sin 59^\circ} \)
(c) \( \frac{45 \sin 79^\circ}{\sin 42^\circ} \)
(d) \( \frac{45 \sin 59^\circ}{\sin 42^\circ} \)

13. Alan at position \( A \) spots a deer bearing S43°E. Bob at position \( B \), 700 meters due east of position \( A \), spots the deer bearing S65°W. How far from Alan is the deer?
(a) \( \frac{700 \sin 43^\circ}{\sin 72^\circ} \) meters
(b) \( \frac{700 \sin 65^\circ}{\sin 72^\circ} \) meters
(c) \( \frac{700 \sin 47^\circ}{\sin 108^\circ} \) meters
(d) \( \frac{700 \sin 25^\circ}{\sin 108^\circ} \) meters

14. Triangle \( \triangle ABC \) has sides of length \( a, b, \) and \( c, \) and angles of measure \( \alpha, \beta, \) and \( \gamma \) opposite those sides, respectively. If \( \alpha = 73^\circ, b = 7.0, \) and \( c = 3.0, \) find \( a. \)

(a) \( 58 - 42 \cos 73^\circ \)
(b) \( \sqrt{58 - 42 \cos 73^\circ} \)
(c) \( 58 - 21 \cos 73^\circ \)
(d) \( \sqrt{58 - 21 \cos 73^\circ} \)

15. Triangle \( \triangle ABC \) has sides of length \( a, b, \) and \( c, \) and angles of measure \( \alpha, \beta, \) and \( \gamma \) opposite those sides, respectively. If \( a = 5, b = 4, \) and \( c = 2, \) find \( \alpha. \)

(a) \( \cos \left( \frac{5}{16} \right) \)
(b) \( \cos^{-1} \left( \frac{5}{16} \right) \)
(c) \( \cos \left( -\frac{5}{16} \right) \)
(d) \( \cos^{-1} \left( -\frac{5}{16} \right) \)